Problem Set 7 (due Dec 7, 2018)

1. **Lorentz transformations** Problem 1, Chapter 3, Lowell Brown
2. **Lie algebra of the lorentz group** Problem 6, Chapter 3, Lowell Brown
3. **Lie algebra of the lorentz group** Problem 8, Chapter 3, Lowell Brown
4. **Spinor representations of Lorentz transformations** Consider a four vector $A^\mu = (A^0, A^1, A^2, A^3)$. Consider four tuples of 2×2 matrices

$$\begin{aligned} \sigma^\mu &= (1, \sigma^i) \\ \bar{\sigma}^\mu &= (1, -\sigma^i)\ . \end{aligned} \tag{1}$$

   Now consider 2×2 matrices

$$\begin{aligned} [A] &= A_\mu \sigma^\mu \\ [\bar{A}] &= A_\mu \bar{\sigma}^\mu\ . \end{aligned} \tag{2}$$

   The metric $\{-1, +1, +1, +1\}$ can be used to lower the indices of A.

   (a) Calculate $\det[A]$, $\det[\bar{A}]$. Are they lorentz invariant?

   (b) Consider the transformations,

$$\begin{aligned} [A] &\to [M][A][M]^\dagger \\ [\bar{A}] &\to [M][\bar{A}][M]^\dagger\ . \end{aligned} \tag{3}$$

   What is the condition on M for the Minkowski norm of A to be invariant

   (c) Show that

$$[M] = e^{i\sigma^z \theta_z / 2} \tag{4}$$

   corresponds to a rotation by θ_z about the z axis in both cases

   (d) Show that

$$[M] = e^{i\sigma^z (i\eta_z / 2)} \tag{5}$$

   corresponds to a boost by η_z along the $+z$ or $-z$ axis

5. **Pauli-Lubansky vector** Define the vector $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu J_{\rho\sigma}$.

(a) Find $[W^\mu, W^\nu]$

(b) Find $[W^\mu, P^\nu]$

6. **Massive spin 1/2 particles** For spin 1/2 particles with mass $m > 0$, the "standard momentum vector" can be chosen to be $k^\mu = (0, 0, 0, m)$. We can take the spin states in the "standard reference frame" to be eigenstates of σ^z. Find the transformed states by boosting the states for the following two cases.

   (a) $p^\mu = (0, 0, p^z, \sqrt{p_z^2 + m^2})$

   (b) $p^\mu = (p^x, 0, 0, \sqrt{p_x^2 + m^2})$

7. **Massless spin 1 particles** For spin 1 particles with mass $m = 0$, the "standard momentum vector" can be chosen to be $k^\mu = (0, 0, k, k)$. We can take the spin states in the "standard reference frame" to be states of helicity $+1$ and -1. Find the transformed states by boosting the states for the following two cases.

   (a) $p^\mu = (0, 0, p^z, p^z)$

   (b) $p^\mu = (p^x, 0, 0, p^x)$