Problem Set 3 (due Nov 9, 2015)

1. **Lienard-Wiechert fields** Begin from Eq. 14.6 of Jackson.

$$A^{\mu}(x) = \frac{eV^{\alpha}(\tau)}{V \cdot (x - r(\tau))} \, . \tag{1}$$

(a) Show that

$$V \cdot (x - r(\tau)) = \gamma c R (1 - \beta \cdot \hat{\mathbf{n}}) \tag{2}$$

and therefore obtain Eq. 14.8 of Jackson

$$\begin{aligned} \Psi(\mathbf{x}, t) &= \left[\frac{e}{(1 - \beta \cdot \hat{\mathbf{n}}) R} \right]_{\text{Ret}} \\ \Psi(\mathbf{x}, t) &= \left[\frac{e\beta}{(1 - \beta \cdot \hat{\mathbf{n}}) R} \right]_{\text{Ret}} \end{aligned} \tag{3}$$

(b) Show

$$\frac{\partial \mathbf{x}'^{j}}{\partial \mathbf{x}^{i}}|_{t} = -\frac{\beta^{j} \hat{\mathbf{n}}^{i}}{1 - \beta \cdot \hat{\mathbf{n}}} \tag{4}$$

(c) Show

$$\frac{\partial R}{\partial \mathbf{x}^{i}}|_{t} = \frac{\hat{\mathbf{n}}_{i}}{1 - \beta \cdot \hat{\mathbf{n}}} \tag{5}$$

(d) Show

$$\frac{\partial \hat{\mathbf{n}}^{j}}{\partial \mathbf{x}^{i}}|_{t} = \frac{1}{R(1 - \beta \cdot \hat{\mathbf{n}})} [\delta^{ij}(1 - \hat{\mathbf{n}} \cdot \beta) + \beta^{j} \hat{\mathbf{n}}^{i} - \hat{\mathbf{n}}^{i} \hat{\mathbf{n}}^{j}] \tag{6}$$

(e) Show

$$\frac{\partial \beta^{j}}{\partial \mathbf{x}^{i}}|_{t} = \frac{1}{c(1 - \beta \cdot \hat{\mathbf{n}})} [-\dot{\beta}^{j} \hat{\mathbf{n}}^{i}] \tag{7}$$

(f) Find

$$\frac{dt'}{dt}|_{\mathbf{x}} \tag{8}$$

(g) Find

$$\frac{dR}{dt}|_{\mathbf{x}} \tag{9}$$

(h) Find

$$\frac{d\beta}{dt}|_{\mathbf{x}} \tag{10}$$

(i) Find

$$\frac{d\hat{\mathbf{n}}^i}{dt}|_{\mathbf{x}} \tag{11}$$

(j) Use the results found above to find $\mathbf{E}^i$ explicitly (without using Eq. 14.11) but by directly using

$$\mathbf{E}^i = -\partial_0 A^i - \partial_i A^0 \tag{12}$$

2. **Non-relativistic harmonic oscillator** Problem 14.12

3. **Relativistic harmonic oscillator** Problem 14.14

4. **Helical motion** Problem 14.17

5. **Helical motion** Problem 14.18