Problem Set 1 (due Feb 11, 2014)

1. **Special relativity, tensors**

   (a) Show that if components of any four vector V^μ are transformed by the rule $V^\mu \to (V')^\mu = \Lambda^\mu{}_\nu V^\nu$ when one goes from a reference frame/coordinate system F to a new reference frame F', then the condition $\eta_{\mu\nu}\Lambda^\mu{}_\sigma\Lambda^\nu{}_\lambda = \eta_{\sigma\lambda}$ implies the transformation leaves the dot product of and two four vectors unchanged. For the special cases that the two vectors are the same, this condition ensures that the norms (for eg. the space-time interval $x^\mu x_\mu$ or the mass $p_\mu p^\mu$) are Lorentz invariant

   (b) Write $\Lambda^\mu{}_\nu$ that transforms coordinates in a reference frame F to a reference frame F' rotated by an angle θ about the x direction

   (c) Write $\Lambda^\mu{}_\nu$ that transforms coordinates in a reference frame F to a reference frame F' boosted by speed v about the $+x$ direction

   (d) For $\Lambda_\nu{}^\mu$ defined by $\Lambda_\nu{}^\mu = \eta^{\mu\sigma}\eta_{\nu\lambda}\Lambda^\lambda{}_\sigma$, prove that $\Lambda_\nu{}^\mu = ([\Lambda^\lambda{}_\sigma]^{-1})_\nu{}^\mu$

   (e) Write the derivatives in the new coordinate system $\partial_\mu{}' = \frac{\partial}{\partial(X')^\mu}$ in terms of the derivatives in the old coordinate system $\partial_\mu = \frac{\partial}{\partial X^\mu}$ and the transformation Λ (Hint: The order of the upper and lower indices on Λ is important). Show that $\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu}$ is a Lorentz scalar

2. **Reading assignment** Read Chapter 3., Srednicki's QFT book

3. **Quantization of a complex field** Srednicki Problem 3.5