# Problem Set 1 (due Feb 8, 2014)

1. **Natural units.**

   (a) The mass of the recently discovered Higgs boson is reported to be $m_H = 126 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})\text{GeV}$. Write this mass in SI units (ignore error bars).

   (b) Rewrite the mass of the electron and the proton in GeV. The inverse of masses define corresponding length scales. Write the length scales associated with the electron, the proton and the Higgs boson in fm ($1\text{fm}{=}10^{-15}\text{m}$).

   (c) Consider a bullet of mass 1gm moving with speed 100m/s. Give its nonrelativistic momentum in GeV. Rewrite its mass in terms of GeV. Compare the two to justify the nonrelativistic approximation.

2. **Particle production from energy.** (All quantities are given in natural units) Consider the collision of two particles with mass $m$ in the centre of mass reference frame. Let the first particle be moving towards the $+z$ direction with (magnitude of the) momentum $p_1 = 2m$ and the second towards the $-z$ direction with (magnitude of the) momentum $p_2 = 2m$. Write the four momentum of the the two particles and the sum. After the collision three particles of mass $m$ are created. The first particle moves in the $+z$ direction with momentum $m$, the second particle moves in the $-z$ direction but the magnitude of its momentum is not known. Use conservation of four momenta to find the momentum of the second and the third particle. Now reconsider the collision in the case where the momenta of the initial particles $p_1,\ p_2 \ll m$. Can the final state consist of three particles with mass $m$?

3. **Creation and annihilation operators in non-relativistic theories** Srednicki Problem 1.2

4. **Special relativity, tensors**

   (a) Show that if components of any four vector $V^\mu$ are transformed by the rule $V^\mu \to (V')^\mu = \Lambda^\mu{}_\nu V^\nu$ when one goes from a reference frame/coordinate system $F$ to a new reference frame $F'$, then the condition $\eta_{\mu\nu}\Lambda^\mu{}_\sigma\Lambda^\nu{}_\lambda = \eta_{\sigma\lambda}$ implies the transformation leaves the dot product of and two four vectors unchanged. For the special cases that the two vectors are the same, this condition ensures that the norms (for eg. the space-time interval $x^\mu x_\mu$ or the mass $p_\mu p^\mu$) are Lorentz invariant

   (b) Write $\Lambda^\mu{}_\nu$ that transforms coordinates in a reference frame $F$ to a reference frame $F'$ rotated by an angle $\theta$ about the $x$ direction

(c) Write $\Lambda^\mu{}_\nu$ that transforms coordinates in a reference frame $F$ to a reference frame $F'$ boosted by speed $v$ about the $+x$ direction

(d) For $\Lambda_\nu{}^\mu$ defined by $\Lambda_\nu{}^\mu = \eta^{\mu\sigma}\eta_{\nu\lambda}\Lambda^\lambda{}_\sigma$, prove that $\Lambda_\nu{}^\mu = ([\Lambda^\lambda{}_\sigma]^{-1})^\mu{}_\nu$

(e) Write the derivatives in the new coordinate system $\partial_\mu{}' = \frac{\partial}{\partial(X')^\mu}$ in terms of the derivatives in the old coordinate system $\partial_\mu = \frac{\partial}{\partial X^\mu}$ and the transformation $\Lambda$ (Hint: The order of the upper and lower indices on $\Lambda$ is important). Show that $\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu}$ is a Lorentz scalar