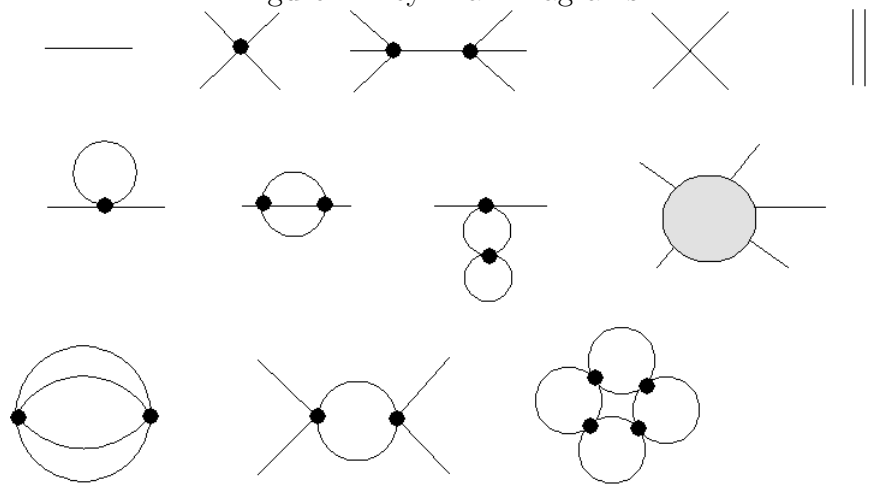
Problem Set 5 (due Mar 16, 2015)

1. **Fun with Feynman diagrams** Write the expressions for the 12 Feynman diagrams for (use correlation function Feynman rules) given below for the ϕ^4 theory. Give results both in position and momentum space. Introduce appropriate symbols for external and internal space-time locations (four momenta for momentum space) and explicitly show the symmetry factors. You can write the results in terms of the position space Feynman propagator D_F and the momentum space propagator $\frac{i}{p^2-m^2+i\epsilon}$ (Peskin conventions) but give the correct arguments and appropriate integrals. You need not evaluate the integrals

Figure 1: Feynman Diagrams



2. **(Writing Feynman rules)** In a previous problem set you quantized the free theory for the complex scalar field. Now consider the theory with interactions

$$\begin{aligned}\mathcal{L} &= \partial_\mu \Phi^* \partial^\mu \Phi - M_\Phi^2 \Phi^* \Phi \\ &\quad - \frac{\lambda_3}{1!}[\Phi^* \Phi^2 + h.c] - \frac{\lambda_4}{y!}[\Phi \Phi^*]^2 \, .\end{aligned} \tag{1}$$

   (a) If the interacting largangian is to be invariant under phase transformations of Φ, what is the allowed value of λ_3. From now on assume that λ_3 is this value

   (b) Rescale and shift the fields appropriately to rewrite the lagrangian in terms of renormalized fields and constants. Do you need to shift Φ?

   (c) Separate out the free renormalized lagrangian, the interaction terms and the conterterms and give the Feynman rules

(d) What is the convenient choice for y so that the symmetry factor is given by the number of automorphisms of the Feynman diagram

(e) Give renormalization conditions for the one-point correlators, $\langle\Phi\rangle$, $\langle\Phi^*\rangle$, and the two point correlators, $\langle\Phi\Phi\rangle$, $\langle\Phi\Phi^*\rangle$, $\langle\Phi^*\Phi^*\rangle$