Advanced Quantum Mechanics

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Lecture #19

Path Integrals and QM

Hamiltonian Formulation of QM

- An isolated quantum system at time t: $|\psi(t)\rangle$, a normalized vector in an abstract Hilbert space over complex no.s.

- Any observable (measurable quantity): a linear Hermitian operator in this Hilbert space. With a choice of basis they can be rep. by matrices.

- A special operator, the Hamiltonian, generates the time evolution of the state through the Schrodinger eqn.

$$i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$$

How do we write down the Hamiltonian operator?

- Write down the Hamiltonian of the system assuming classical mechanics

defines the co-ordinates and conjugate momenta

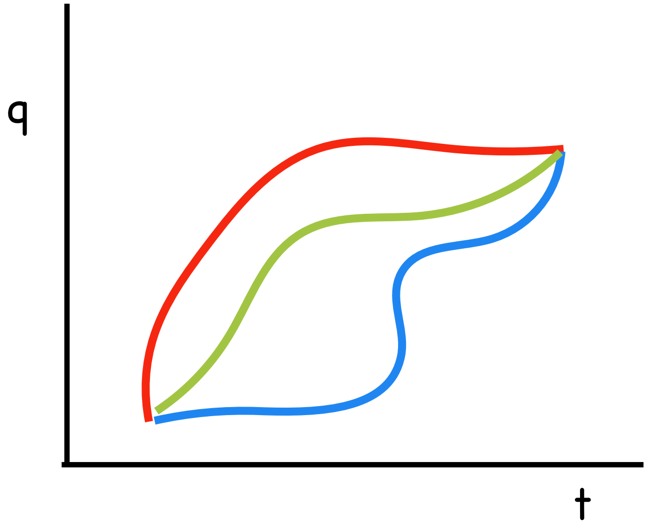
- Elevate the co-ordinates and momenta to operators and replace the Poisson Brackets by commutators to obtain the QM Hamiltonian

Lagrangian formulation of CM

In Classical Mechanics, one defines a quantity called action for every trajectory of a system.

$$S = \int dt \mathcal{L}(q, \dot{q}, t) \qquad \mathcal{L}(q, \dot{q}, t) = \frac{1}{2} m \dot{q}^2 - V(q)$$

for a particle in a potential



The actual path taken by the classical system is the path of extremal action, which leads to the Euler Lagrange eqn.s

$$\frac{\delta \mathcal{L}}{\delta q} - \frac{d}{dt}\left(\frac{\delta \mathcal{L}}{\delta \dot{q}}\right) = 0$$

Advantages of Lagrangian formalism:

- Easier to deal with constraints/ generalized co-ordinates.

- The form of Euler Lagrange equations is invariant under co-ordinate transforms

- Easier to generalize to Relativistic (Lorentz invariant) situations.

- Standard way to generalize to classical field theories via Lagrangian density (e.g. Electromagnetism)

- New formulation of QM, along Lagrangian formalism ----- the Path Integral formalism

QM and time evolution operator

- Let us start with the familiar Schrodinger eqn. for QM $i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$

  (For the moment, stick to time indep. H)

Formal Soln.: $|\psi(t)\rangle = \hat{U}(t,0)|\psi(0)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$

Time-Evolution Operator

If the matrix elements of U are known, the QM problem is solved.

$$\psi(x,t) = \langle x|\psi(t)\rangle = \langle x|\hat{U}(t,0)|\psi(0)\rangle = \int d^3x' \langle x|\hat{U}(t,0)|x'\rangle\langle x'|\psi(0)\rangle \;= \int d^3x' \langle x|\hat{U}(t,0)|x'\rangle\psi(x',0)$$

More generally: $\psi(x,t) = i\int d^3x' G(x,t;x',t')\psi(x',t')$ $\qquad G(x,t;x',t') = -i\langle x|U(t,t')|x'\rangle$

Green's fn/ Propagator: propagates the influence of things at x' across time to x

Properties of U / G

- For time-translation invariant systems $G(x,t;x',t') = G(x,x',t-t')$
- If the system is homogeneous in space $G(x,t;x',t') = G(x-x',t-t')$
- The propagator satisfies the equation $(i\partial_t - H)G(t-t') = \delta(t-t')$

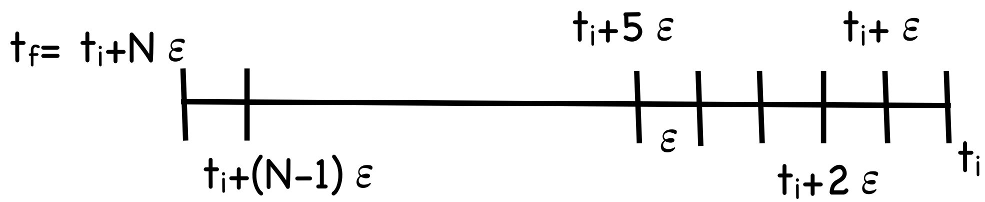
This is why it is called Green's fn

- One could have taken any complete basis to define $G(\alpha,t;\beta,t') = -i\langle\alpha|U(t,t')|\beta\rangle$

E.g.: $G(\vec{k},t-t') = \int d^3r e^{i\vec{k}\cdot r}G(\vec{r},t-t')$

Breaking up the time evolution operator: $\hat{U}(t,t_0) = \hat{U}(t,t_1)\hat{U}(t_1,t_0)$ $\quad t < t_1 < t_0$

Break up the time interval (t_f-t_i) into a large number N of small intervals of width ε , $t_f=t_i+N\ \varepsilon$



In the end we will take N --> ∞, ε --> 0,

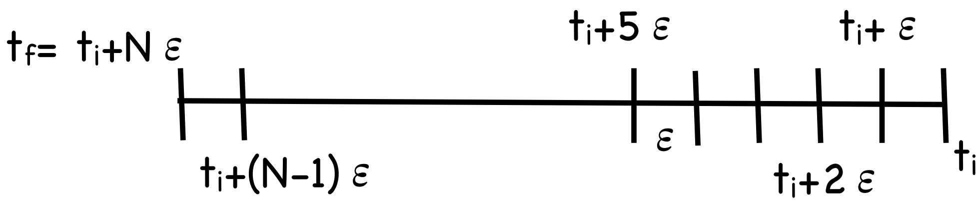
so that N ε = t_f-t_i is const

$$\hat{U}(t_f,t_i) = \hat{U}[t_f,t_i+(N-1)\epsilon]\hat{U}[t_i+(N-1)\epsilon,t_i+(N-2)\epsilon]...\hat{U}[t_i+2\epsilon,t_i+\epsilon]\hat{U}[t_i+\epsilon,t_i]$$

Matrix Elements of U

Let us now consider the matrix element of $U(t_f,t_i)$ between a complete basis set.

For concreteness, we choose x basis,
but this can be done with any basis.



$$U(x_f,t_f;x_i,t_i)=\langle x_f|\hat{U}[t_f,t_i+(N-1)\epsilon]\hat{U}[t_i+(N-1)\epsilon,t_i+(N-2)\epsilon]...\hat{U}[t_i+2\epsilon,t_i+\epsilon]\hat{U}[t_i+\epsilon,t_i]|x_i\rangle$$

$U(x_f,t_f;x_i,t_i)$ is the probability amplitude for the particle to propagate from x_i at t_i to x_f at t_f

We introduce identity operators between each U in the product above

$$\hat{U}(x_f,t_f;x_i,t_i)=\int dx_1dx_2..dx_{N-1}\,[$$
$$\boxed{\langle x_f|\hat{U}(t_f,t_i{+}(N{-}1)\epsilon)|x_{N-1}\rangle}\langle x_{N-1}|\hat{U}(t_i{+}(N{-}1)\epsilon,t_i{+}(N{-}2)\epsilon)|x_{N-2}\rangle\langle x_{N-2}|...$$
$$|x_2\rangle\langle x_2|\hat{U}(t_i+2\epsilon,t_i+\epsilon)|x_1\rangle\boxed{\langle x_1|\hat{U}(t_i+\epsilon,t_i)|x_i\rangle]}$$

We have reduced the matrix element to a product over matrix elements of infinitesimal time evolution operators. The price we pay is a large no. of integrals over intermediate co-ord.s

Matrix Elements of infinitesimal U

$$\langle x_{k+1}|\hat{U}(t_i + (k+1)\epsilon, t_i + k\epsilon)|x_k\rangle = \langle x_{k+1}|e^{-i\hat{H}(\hat{p},\hat{x})\epsilon}|x_k\rangle$$

Not very convenient since p --> ∇ in the exponential.

Introduce identity with momentum states $\int_{-\infty}^{\infty} \frac{dp}{2\pi}|p\rangle\langle p| = 1$

$$\langle x_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int \frac{dp_{k+1}}{2\pi}\langle x_{k+1}|p_{k+1}\rangle\langle p_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int \frac{dp_{k+1}}{2\pi} e^{ix_{k+1}\cdot p_{k+1}} \underline{\langle p_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle}$$

This would be easy to evaluate if all the p operators appeared to the left of all x operators

Such an operator where all p operators appear to the left of all x operators is called a normal ordered operator.

Then the p operator can act on the bra to give the eigenvalue p and the x operators can act on the ket to give x eigenvalues. We get rid of operators in favour of no.s

However, with $H = \frac{\hat{p}^2}{2m} + V(\hat{x})$ $\quad e^{-iH\epsilon}$ is not normal ordered

$$e^{-iH\epsilon} = 1 - i\epsilon\left[\frac{\hat{p}^2}{2m} + V(\hat{x})\right] - \frac{\epsilon^2}{2}\left[\left(\frac{\hat{p}^2}{2m}\right)^2 + V^2(\hat{x}) + \frac{\hat{p}^2}{2m}V(\hat{x}) + V(\hat{x})\frac{\hat{p}^2}{2m}\right] + \ldots$$

Normal Ordering and Errors

$$e^{-iH\epsilon} = 1 - i\epsilon\left[\frac{\hat{p}^2}{2m} + V(\hat{x})\right] - \frac{\epsilon^2}{2}\left[\left(\frac{\hat{p}^2}{2m}\right)^2 + V^2(\hat{x}) + \frac{\hat{p}^2}{2m}V(\hat{x}) + V(\hat{x})\frac{\hat{p}^2}{2m}\right] + \ldots$$

Now $$V(\hat{x})\frac{\hat{p}^2}{2m} = \frac{\hat{p}^2}{2m}V(\hat{x}) + \left[V(\hat{x}), \frac{\hat{p}^2}{2m}\right] \longrightarrow V'(\hat{x})\frac{\hat{p}}{m}[\hat{x}, \hat{p}] = iV'(\hat{x})\frac{\hat{p}}{m}$$

What happens if we simply replace the infinitesimal evolution operator by its normal ordered form?

The leading order correction is O(ε^2): $\frac{-i\epsilon^2}{2m}\frac{\partial V}{\partial_x}(\hat{x})\hat{p}$

Assumption: This and higher order terms are not singular

$$e^{-i\hat{H}\epsilon} = \sum_{n=0}^{\infty}\frac{(-i\epsilon)^n}{n!}\left(\frac{\hat{p}^2}{2m} + V(\hat{x})\right)^n \longrightarrow :e^{-i\hat{H}\epsilon} := \sum_{n=0}^{\infty}\frac{(-i\epsilon)^n}{n!}\sum_{k=0}^{n}\frac{n!}{n-k!k!}\left(\frac{\hat{p}^2}{2m}\right)^k [V(\hat{x})]^{n-k}$$

So, in calculating each matrix element, the leading error that we are making is O(ε^2).

Since N matrix elements are being calculated and multiplied, the leading error in matrix element of U is O(N ε^2).

Normal Ordering and Errors

Define $t_m = t_i + m\varepsilon$, $t_N = t_f$

We want to calculate

$$U(x_f, t_f; x_i, t_i) = \int dx_1 dx_2 .. dx_{N-1} U[x_f, t_f; x_{N-1}, t_{N-1}] U[x_{N-1}, t_{N-1}; x_{N-2}, t_{N-2}] ... U[x_2, t_2; x_1, t_1] U[x_1, t_1; x_i, t_i]$$

We are replacing this by

$$\int dx_1 dx_2 .. dx_{N-1} \{: U[x_f, t_f; x_{N-1}, t_{N-1}] : + \mathcal{O}(\epsilon^2)\} \{: U[x_{N-1}, t_{N-1}; x_{N-2}, t_{N-2}] : + \mathcal{O}(\epsilon^2)\} ...$$

$$... \{: U[x_2, t_2; x_1, t_1] : + \mathcal{O}(\epsilon^2)\} \{: U[x_1, t_1; x_i, t_i] : + \mathcal{O}(\epsilon^2)\}$$

The leading error in calculating matrix element of U is thus $O(N\varepsilon^2)$.

(Implicit assumption : integrals involved in calculating infinitesimal U do not diverge)

In the end we want to take the limit: N --> ∞, ε --> 0, so that $N\varepsilon = t_f - t_i$ is const.

i.e. $N\varepsilon^2$ --> 0 in this limit. So the error due to normal ordering vanishes in this limit.

Once we normal order, we can simply replace $\langle p| : e^{-i\hat{H}(\hat{x},\hat{p})\epsilon} : |x\rangle = e^{-iH(x,p)\epsilon} \langle p|x\rangle$

and work with the real numbers x and p

Matrix Elements of infinitesimal U

$$2\pi\langle x_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int dp_{k+1}\langle x_{k+1}|p_{k+1}\rangle\langle p_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle$$

Introduce identity with p states

$$= \int dp_{k+1}e^{ip_{k+1}\cdot x_{k+1}}\langle p_{k+1}|e^{-i\hat{H}(\hat{p},\hat{x})\epsilon}|x_k\rangle$$

$$= \int dp_{k+1}e^{ip_{k+1}\cdot x_{k+1}}\langle p_{k+1}|:e^{-i\hat{H}(\hat{p},\hat{x})\epsilon}:|x_k\rangle$$

Normal Ordering

$$= \int dp_{k+1}e^{ip_{k+1}\cdot x_{k+1}}e^{-iH(p_{k+1},x_k)\epsilon}\langle p_{k+1}|x_k\rangle$$

Note that in H, p is with index k+1
x is with index k

$$= \int dp_{k+1}e^{ip_{k+1}\cdot(x_{k+1}-x_k)}e^{-iH(p_{k+1},x_k)\epsilon}$$

$$= \int dp_{k+1}e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon}-H(p_{k+1},x_k)\right]}$$

Exponent looks like $p\dot{x}$ -H = L
where L is the classical Lagrangian
for the infinitesimal path

Continuum Limit and Path Integrals in Phase Space

We have

N matrix elements

$$U(x_f,t_f;x_i,t_i) = \int dx_1 dx_2..dx_{N-1} \prod_{n=0}^{N-1} < x_{n+1}|e^{-i\epsilon\hat{H}(\hat{x},\hat{p})}|x_n\rangle \qquad x_0 = x_i, \qquad x_N = x_f$$

N-1 x integrals

$$\langle x_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int \frac{dp_{k+1}}{2\pi} e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon} - H(x_k,p_{k+1})\right]}$$

dt

$$U(x_f,t_f;x_i,t_i) = \int \frac{dx_1 dp_1}{2\pi}\int \frac{dx_2 dp_2}{2\pi}..\int \frac{dx_{N-1}dp_{N-1}}{2\pi}\int \frac{dp_N}{2\pi}\prod_{n=0}^{N-1} e^{i\epsilon\underline{\left[p_{n+1}\cdot\frac{x_{n+1}-x_n}{\epsilon} - H(x_n,p_{n+1})\right]}}$$

Lagrangian L

N-1 x integrals N p integrals

Now take the continuum limit, i.e. N --> ∞, ε --> 0, so that N ε = $t_f - t_i$ = constant .

In this limit, $(x_{k+1}-x_k)/\varepsilon = \dot{x}(t)$, and ε =dt.

The exponents add up to give the action $S = \int dt\, L$ in the final exponent.

Continuum Limit and Path Integrals in Phase Space

We have finally

$$U(x_f, t_f; x_i, t_i) = \int \frac{dp_N}{2\pi} \int \frac{dx_1 dp_1}{2\pi} \int \frac{dx_2 dp_2}{2\pi} .. \int \frac{dx_{N-1} dp_{N-1}}{2\pi} e^{iS}$$

$$= \int \frac{dp_N}{2\pi} \int \frac{dx_1 dp_1}{2\pi} \int \frac{dx_2 dp_2}{2\pi} .. \int \frac{dx_{N-1} dp_{N-1}}{2\pi} e^{i \int_{t_i}^{t_f} dt [p\dot{x} - H(x,p)]}$$

If we remove the additional p integral, the rest of the integrals involve summing over all possible phase space trajectories.

$$U(x_f, t_f; x_i, t_i) = \int \frac{dp_N}{2\pi} D[x] D[p] e^{i \int_{t_i}^{t_f} dt [p\dot{x} - H(x,p)]}$$

$$= \int D^{'}[x] D[p] e^{i \int_{t_i}^{t_f} dt [p\dot{x} - H(x,p)]}$$

Each phase space trajectory which starts and ends at definite co-ordinates x_i and x_f contribute a pure phase to the propagator between these positions.

The phase is the classical action along that trajectory.

The phase space trajectories can reach the final point with any momenta $\longrightarrow$ additional momentum integral

Path Integrals in Config. Space

$$2\pi\langle x_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int dp_{k+1}\langle x_{k+1}|p_{k+1}\rangle\langle p_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle \;\; = \int dp_{k+1}e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon}-H(p_{k+1},x_k)\right]}$$

Exponent looks like $p\dot{x}$ -H = L. However, we need to integrate p out to get L=L(x,$\dot{x}$,t)

At this point, we assume that H =p^2/2m+V(x). So the p-integral is a Fresnel integral.

$$\int_{-\infty}^{\infty} dp_{k+1}e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon}-\frac{p_{k+1}^2}{2m}\right]}$$

Gaussian and Fresnel Integrals.

1-D Gaussian integral: $\int_{-\infty}^{\infty} dx e^{-ax^2/2} = \sqrt{\frac{2\pi}{a}}$ Re[a] >0

Complete the square:

$$\int_{-\infty}^{\infty} dx e^{-ax^2/2-bx} = e^{b^2/2a}\sqrt{\frac{2\pi}{a}}$$

Multidimensional Gaussian integral: $\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 .. \int_{-\infty}^{\infty} dx_N e^{-\sum_i a_i x_i^2/2} = \frac{(2\pi)^{N/2}}{\sqrt{\prod_i a_i}}$

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 .. \int_{-\infty}^{\infty} dx_N e^{-\frac{1}{2}\sum_{ij} x_i A_{ij} x_j} = \frac{(2\pi)^{N/2}}{\sqrt{DetA}}$$

To see this, work with linear combination of x_i which diagonalizes A. The Jacobian for this transformation is 1.

In these co-ord, exponent —> $-(1/2) \Sigma \lambda_i q_i^2$. Finally note that Det[A]= $\Pi \lambda_i$

Complete the square:

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 .. \int_{-\infty}^{\infty} dx_N e^{-\frac{1}{2}\sum_{ij} x_i A_i j x_j + \sum_i J_i x_i} = \frac{(2\pi)^{N/2}}{\sqrt{DetA}} e^{\frac{1}{2}\sum_{ij} J_i A_{ij}^{-1} J_j}$$

Gaussian and Fresnel Integrals.

1D Fresnel Integral:

$$\int_{-\infty}^{\infty} dx e^{iax^2/2} = \sqrt{\frac{2\pi i}{a}}$$

Complete the square:

$$\int_{-\infty}^{\infty} dx e^{i(ax^2/2+bx)} = e^{-ib^2/2a}\sqrt{\frac{2\pi i}{a}}$$

Multidimensional Fresnel Integral:

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 .. \int_{-\infty}^{\infty} dx_N e^{\frac{i}{2}\sum_{ij} x_i A_i j x_j} = \frac{(2\pi i)^{N/2}}{\sqrt{DetA}}$$

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 .. \int_{-\infty}^{\infty} dx_N e^{i\left[\frac{1}{2}\sum_{ij} x_i A_i j x_j + \sum_i J_i x_i\right]} = \frac{(2\pi i)^{N/2}}{\sqrt{DetA}} e^{\frac{i}{2}\sum_{ij} J_i A_{ij}^{-1} J_j}$$

$$2\pi\langle x_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int dp_{k+1}\langle x_{k+1}|p_{k+1}\rangle\langle p_{k+1}|e^{-i\hat{H}(\hat{x},\hat{p})\epsilon}|x_k\rangle = \int dp_{k+1} e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon} - H(p_{k+1},x_k)\right]}$$

At this point, we assume that H =p^2/2m+V(x). So the p-integral is a Fresnel integral.

$$\int_{-\infty}^{\infty} dp_{k+1} e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon} - \frac{p_{k+1}^2}{2m}\right]}$$

Path Integrals in Config. Space

So the Fresnel integral in the matrix element is evaluated as

$$\int \frac{dp_{k+1}}{2\pi} e^{i\epsilon\left[p_{k+1}\cdot\frac{(x_{k+1}-x_k)}{\epsilon}-\frac{p_{k+1}^2}{2m}\right]} = \frac{1}{2\pi}\sqrt{\frac{2\pi m}{i\epsilon}}e^{\frac{im}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon}} = \sqrt{\frac{m}{2\pi i\epsilon}}e^{\frac{im}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon}}$$

Hence: $$\langle x_{k+1}|e^{-i\hat{H}\epsilon}|x_k\rangle = \sqrt{\frac{m}{2\pi i\epsilon}}e^{i\epsilon\left[\frac{m}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon^2}-V(x_k)\right]}$$

Considering the product of all the matrix elements (remember there are N p-integrals)

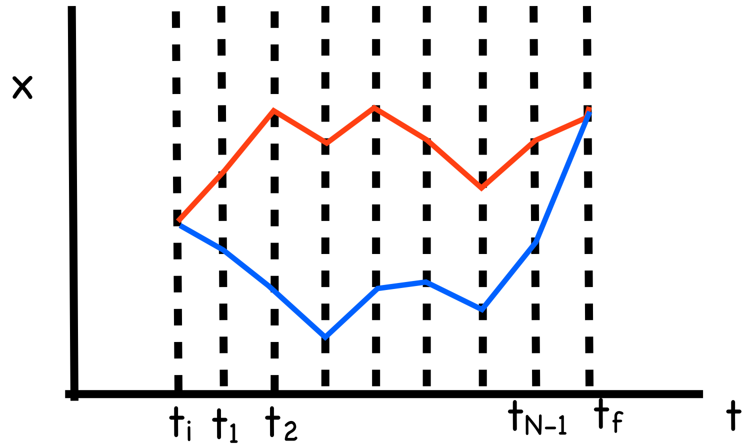
$$U(x_f,t_f;x_i,t_i) = \left(\frac{m}{2\pi i\epsilon}\right)^{N/2}\prod_{k=1}^{N-1}\int dx_k e^{i\epsilon\left[\frac{m}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon^2}-V(x_k)\right]}$$

Defining $$\mathcal{D}[x] = \prod_{k=1}^{N-1} dx_k\left(\frac{m}{2\pi i\epsilon}\right)^{1/2}$$

$$U(x_f,t_f;x_i,t_i) = \left(\frac{m}{2\pi i\epsilon}\right)^{1/2}\int \mathcal{D}[x]e^{i\epsilon\sum_{k=0}^{N-1}\left[\frac{m}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon^2}-V(x_k)\right]}$$

Continuum Limit and Path Integrals

$$U(x_f,t_f;x_i,t_i) = \left(\frac{m}{2\pi i\epsilon}\right)^{N/2} \prod_{k=1}^{N-1} \int dx_k e^{i\epsilon\left[\frac{m}{2}\frac{(x_{k+1}-x_k)^2}{\epsilon^2} - V(x_k)\right]}$$



Let us first relabel $x_1=x(t_1)$, $x_2=x(t_2)$,... $x_k=x(t_k)$, etc.

A specific sequence $(x_1,x_2,....x_{N-1})$ defines a discretized version of a path in time with fixed endpoints (x_i,t_i) and (x_f,t_f).

The integral over all the intermediate co-ord. is equivalent to a sum over all discretized paths.

Now as N --> ∞, ε --> 0, these become continuous paths. In this limit, $(x_{k+1}-x_k)/\varepsilon = \dot{x}(t)$,

and ε =dt.

$$U(x_f,t_f;x_i,t_i) = \left(\frac{mN}{2\pi i(t_f-t_i)}\right)^{1/2} \int \mathcal{D}[x(t)]e^{iS[x(t)]}$$

$$S[x(t)] = \int_{t_i}^{t_f} dt\left[\frac{1}{2}m\dot{x}^2(t) - V(x(t))\right] = \int_{t_i}^{t_f} dt\mathcal{L}[x(t),\dot{x}(t)]$$

The prob. amplitude of a particle traveling from x_i to x_f through any path is equal in magnitude and has a phase proportional to the classical action along the path.

To calculate the amplitude of the particle traveling from x_i to x_f, we must sum over the amplitude contribution from all paths.

Path Integral Formulation of QM

- The fundamental quantity in QM is the probability amplitude for a process to occur, e.g. prob. amplitude for a particle to move from x_i at time t_i to x_f at time t_f.

- The probability amplitude for the process to occur along different routes (defined by the states of the system at intermediate times), has same magnitude. E.g. prob amplitude of the particle moving through different paths has same magnitude.

- The probability amplitude for the process to occur along a definite route has a phase which is given by the classical action along that route. The action (like the Hamiltonian in the usual formulation of QM) is the external input defining the system. E.g. For a particle moving in an external potential $S=\int dt\ (1/2)\ m\ \dot{x}^2 - V(x)$

- To get the probability amplitude of a process, the probability amplitudes for occurence through different routes has to be added. For the particle, this takes the form of a path integral or a sum over all paths. The modulus square of the total prob. amplitude is the desired probability.

- If additional information about intermediate states of the system is available (say through measurements at intermediate times), the routes have to be restricted accordingly. E.g. if it is known through measurement that the particle is at x_k at intermediate time t_k, one should sum up prob. ampl. contributions from only those paths which pass through (x_k,t_k)

Path Integrals and Quantum Mechanics

Intuitive picture of contribution of many paths. Easiest way to explain quantum interference (say double slit experiment) and its vanishing on intermediate measurements.

Usual benefits of Lagrangians --- easier to treat non-local dynamics.
--- easier to generalize to relativistic situations
--- generalizes to field theories.

Close relation to partition function in statistical mechanics ---- imaginary time path (functional) integrals and partition function.

Easier to take the classical limit, which corresponds to the case where only the classical path of extremal action contributes. Starting point of small quantum fluctuations approximations.

Starting point for several approximations ---- perturbation theory and Feynman diagrams, semi classical (WKB like) approximations, instantons etc.

We will soon generalize this formalism for QM of many particles and from there to description of relativistic quantum systems..