Pressure and non-linear susceptibilities in QCD at finite chemical potential

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Taylor expansion for pressure

$$
\begin{aligned}
P(T,\{\mu\}) &= -\frac{1}{V}F(T,\{\mu\}) = \frac{T}{V}\log Z(T,\{\mu\}) \\
\chi_{fgh\cdots} &= -\left.\frac{\partial^n P}{\partial\mu_f\partial\mu_g\partial\mu_h\cdots}\right|_{\mu=0}.
\end{aligned}
$$

1st derivatives are quark number densities. 2nd derivative are the (linear) quark number susceptibilities. Odd derivatives vanish by CP symmetry. Cross derivatives are much smaller than the diagonal derivatives (later). In flavour symmetric case

$$P(T,\mu) = P(T,0) + \frac{N_f}{2!}\chi_{uu}\mu^2 + \frac{N_f}{4!}\chi_{uuuu}\mu^4 + \frac{N_f}{6!}\chi_{uuuuuu}\mu^6 + \cdots$$

Diagrammatic methods and algebraic techniques for writing the operators in S. Gupta, *Acta Phys. Pol.*, B 33 (2002) 4259, R. V. Gavai and S. Gupta, hep-lat/0303013

Lattice artifacts and continuum limit

At any finite lattice spacing, there are many equivalent ways of putting chemical potential on the lattice. For example, at $N_t = 4$ this can change the critical end point by an amount comparable to its current statistical errors—

$$\mu_E = 725 \pm 30 \,(\mathrm{stat}) \pm 35 \,(\mathrm{prescription\ ambiguity})\ \mathrm{MeV}$$

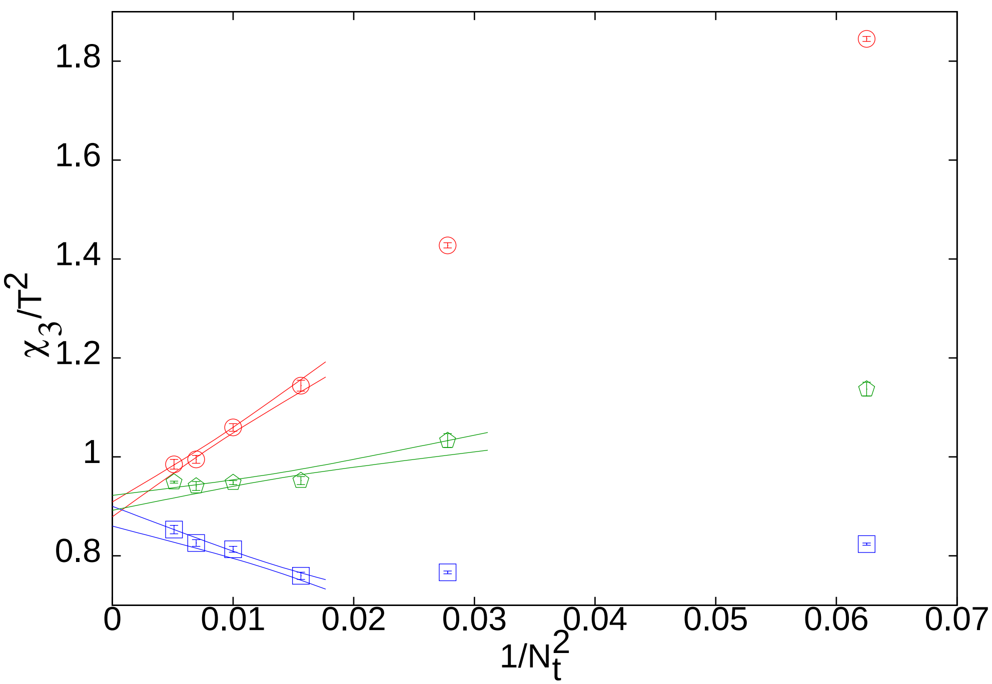
Taylor expansion allows us to take continuum limit term by term. It turns out that taking the first two terms is sufficient to get the EOS in the region of current experimental interest.

The expansion breaks down at lines of phase transitions. An estimate of the radius of convergence is a computation of the critical end point.

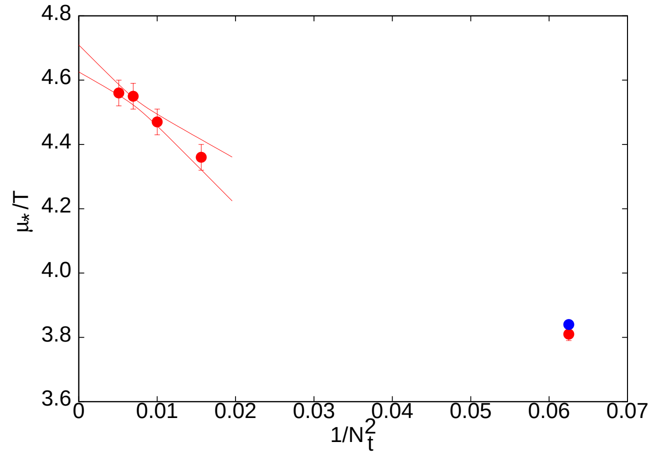
How large should N_t be?

In earlier work we have seen that different Fermion formulations (staggered and Naik) extrapolate to the same limit on using data for $N_t \geq 8$. Non-singlet QNS used for this computation.

R. V. Gavai and S. Gupta, *Phys. Rev.* D 67 (2003) 034501



Continuum limit of μ^*, *i.e.*, χ_{uuuu}



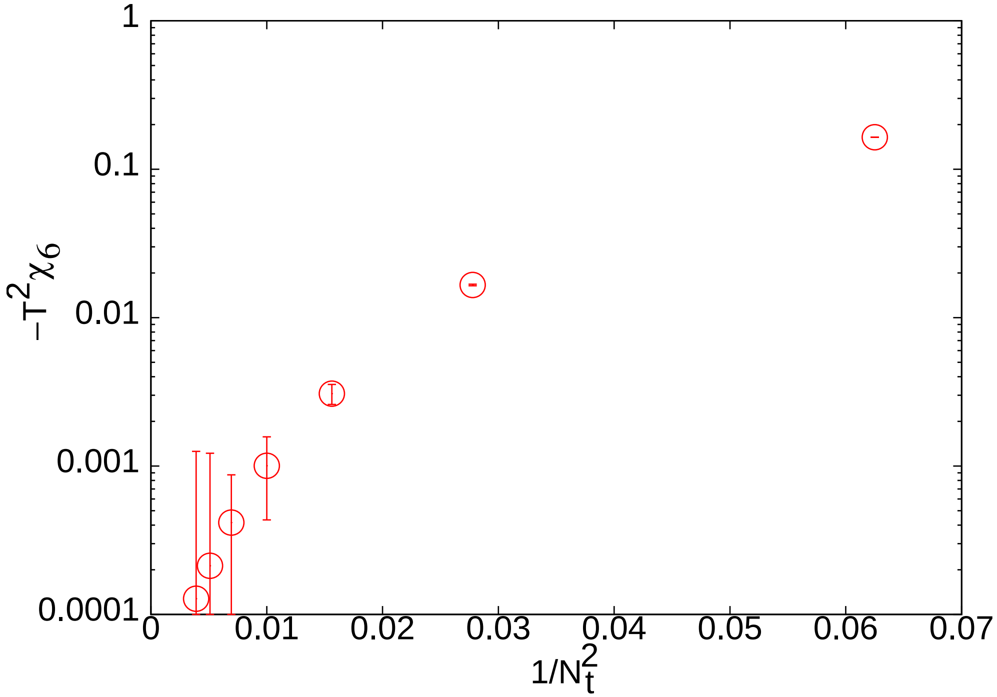
$\mu_* = \sqrt{12\chi_3/\chi_{uuuu}}$ at $T = 1.5T_c$. At finite N_t, the series is insensitive to prescription when $\mu \ll \mu_*$. In the continuum μ_* is the first estimate of the radius of convergence of the series.

For $N_t = 4$ and $N_f = 2$ with $m = 0.1T_c$ we estimate

$$\mu^*/T = 3.38 \pm 0.05 \qquad i.e. \qquad \mu^* = 591 \pm 9 \text{ MeV}$$

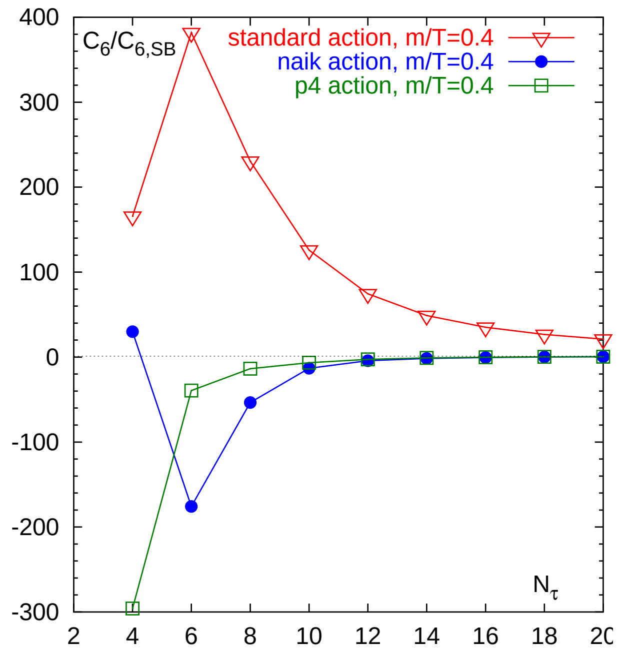
Previous estimate of end-point (Fodor and Katz) gives $\mu^*/T \simeq 4.4$.

Continuum limit of χ_{uuuuuu}



χ_{uuuuuu} at $2T_c$ for $m = 0.1T_c$. This is indistinguishable from zero in the continuum limit. Large artifacts at coarse lattice spacing may influence determination of end-point at finite spacing.

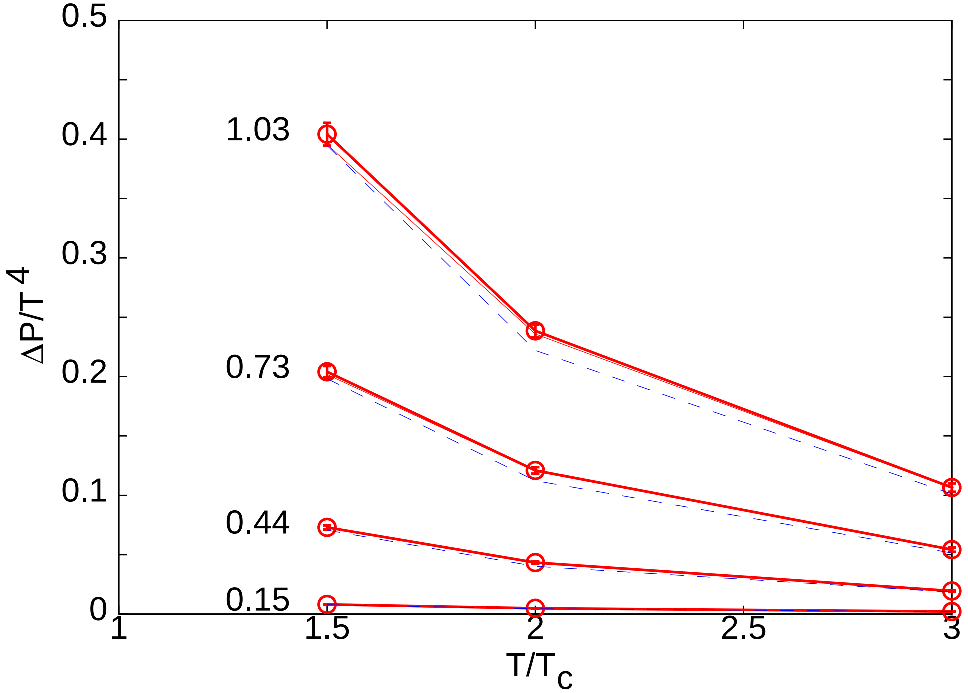
Free field theory



For χ_{uuuuuu} improved quarks do not perform significantly better than standard actions for the same expense in CPU time.

C. R. Allton *et al.*, hep-lat/0305007

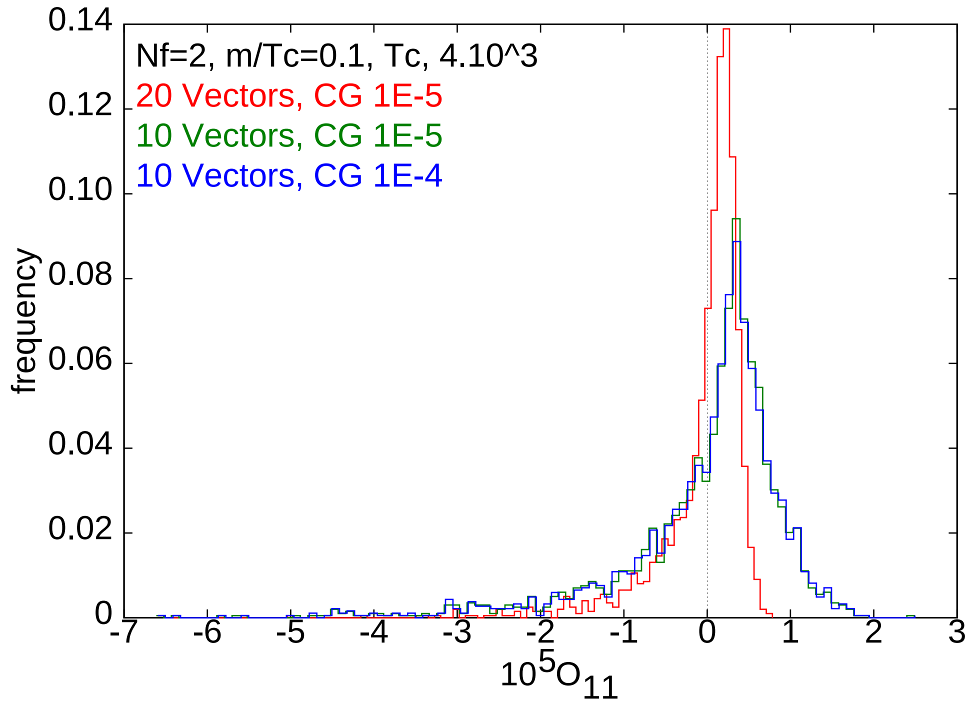
The pressure



$$\Delta P(T) = P(T,\mu) - P(T,0)$$

(Reweighting) Z. Fodor, S. D. Katz and K. K. Szabo, hep-lat/0208078,
(Taylor expn) R. V. Gavai and S. Gupta, hep-lat/0303013

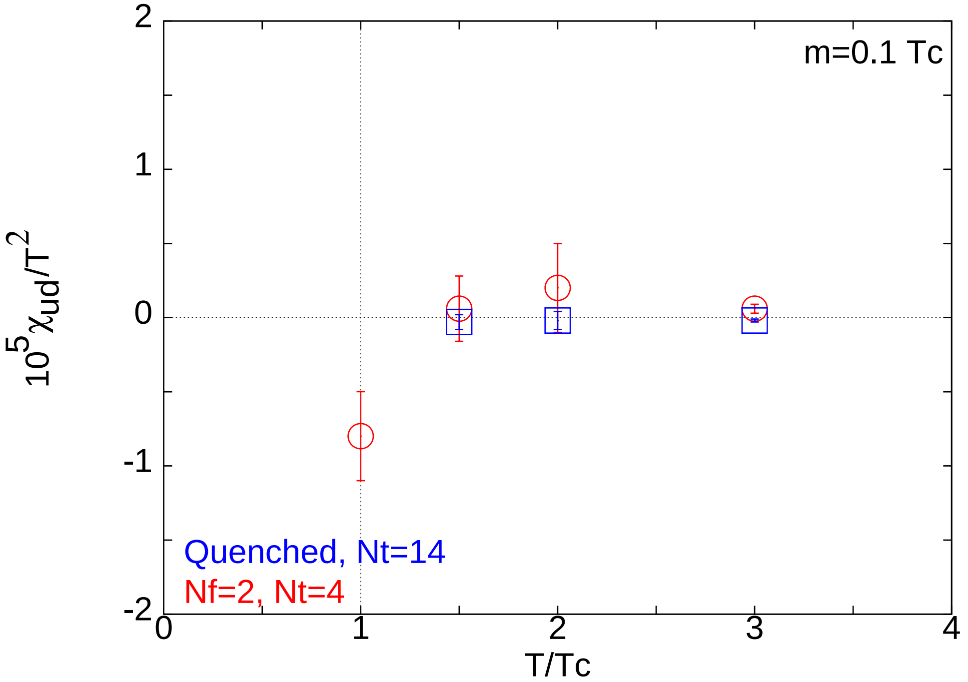
The off-diagonal susceptibility χ_{ud}



$$\mathrm{Tr}\ A^2 = \overline{\langle r|A|r\rangle\langle s|A|s\rangle}$$

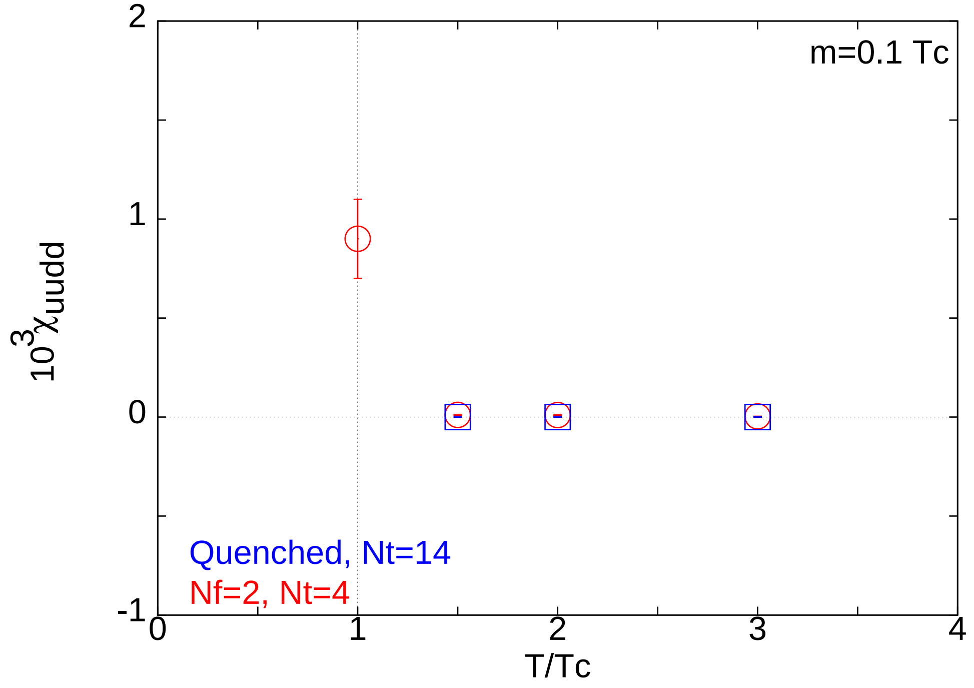
For χ_{ud} the matrix A is anti-hermitean. Reduced variance operator uses $\mathrm{Im}\,\langle r|A|r\rangle$ in the above formula.

$\chi_{ud}(T)$



R. V. Gavai and S. Gupta, hep-lat/0303013. See also C. Bernard *et al.*, (MILC), hep-lat/0209079.

$\chi_{uudd}(T)$



For $T/T_c \geq 1.5$ χ_{uudd} very small, decreasing with T, but non-vanishing within errors.

Summary of Results

- Pressure can be extrapolated to finite chemical potential by Taylor expansion. The expansion coefficients are the (linear and non-linear) quark number susceptibilities.

- Taking the continuum limit term by term is computationally straightforward, and allows us to find the EOS in all regions of interest to experiments.

- Computation of several high order susceptibilities may allow estimation of the critical end point by series extrapolation methods.

- Some of the off-diagonal susceptibilities (χ_{ud}, χ_{uudd} *etc.*) may not vanish (mainly near T_c) but are much smaller than the diagonal susceptibilities.

- Near T_c, since $\chi_{ud} < 0$, the pressure in iso-vector chemical potential is slightly more than that in comparable baryon chemical potential.