Why the answer is 42

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February 7, 2003

1. The lab, the universe and some things

2. Gauge theories at very small couplings

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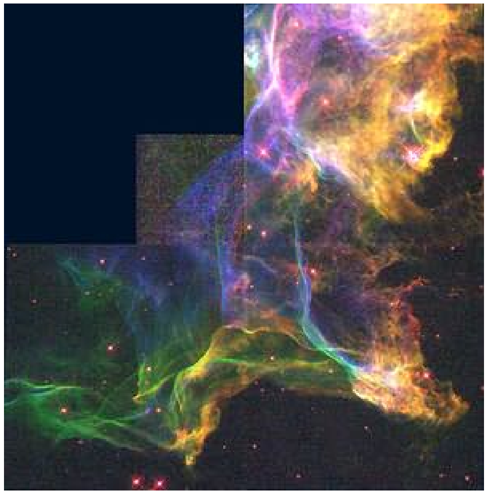
The lab, the universe and some things

HST image of the Crab nebula: 2002/09/19

The gas cloud in the Crab nebula emits polarised synchrotron radiation. There is also other evidence for large scale magnetic fields in this remnant of a 1000 year old supernova. Magnetic fields on scales $L \leq \sqrt{t/4\pi\sigma}$ are damped in time t. The transport coefficient σ is the electrical conductivity of a material.

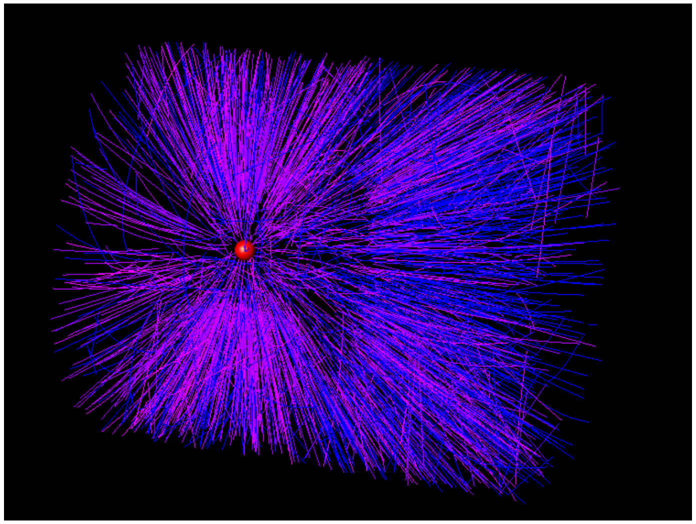
The lab, the universe and some things



HST image of the Cygnus loop: 1995-11

This is a small portion of a nebula called the Cygnus Loop. It is an expanding blast wave from a supernova explosion which occurred about 15,000 years ago. The blast wave, which is moving from left to right across the picture, has recently hit a cloud of denser-than-average interstellar gas. This collision drives shock waves into the cloud that heats interstellar gas, causing it to glow.

The lab, the universe and some things



This is the final state of a relativistic gold ion collision detected at Brookhaven. Each track is one particle, colour coded according to the ionization. The plasma is studied through the spectrum of photons and hadrons emitted by it. These rates are intimately connected with transport properties in the plasma and the time needed to equilibrate.

The rate of photon emission is 42

Gauge theories at small couplings

A gauge theory at small coupling, g, has many length/mass scales.

T	energy of typical excitations
gT	Debye screening mass (inverse of Debye screening length)
$g^2 \log\left(\frac{1}{g}\right) T$	gluon damping rate (Braaten-Pisarski resummation)
$g^2 T$	scale of non-perturbative physics (Linde problem)
$g^4 \log\left(\frac{1}{g}\right) T$	scale of transport processes (Bödeker theory)

Leading log computation of transport coefficients now available.

P. Arnold, G.D. Moore and L.G. Yaffe, JHEP 0111:057,2001

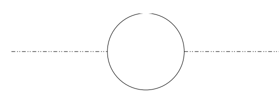
P. Aurenche, F. Gelis, R. Kobes and H. Zaraket, Phys.Rev.D58:085003,1998, Phys.Rev.D60:076002,1999

R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne, D. Schiff, Nucl.Phys.B484:265-282,1997

Gauge theories at small couplings: Debye screening

At $T = 0$ the Coulomb law is due to massless photon/gluon. This is protected by a Ward identity. At $T > 0$ the Ward identity is weakened

$$k^\mu \Pi_{\mu\nu}(k,u) = 0$$

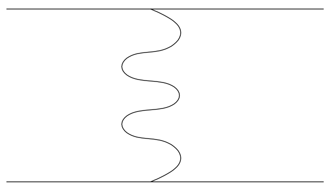
and a finite mass becomes possible: 

$$m^2 = g^2 \int \frac{d^4k}{(\text{fermion prop})^2} \propto g^2 T^2$$

Charged pairs can be popped out of the vacuum but not magnetic monopoles. Hence electric gluons screened but not magnetic gluons.

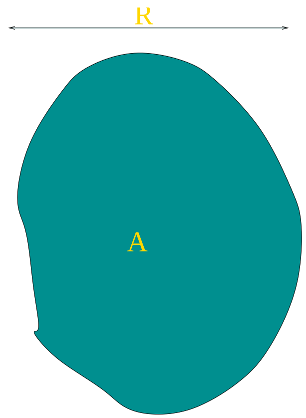
Gauge theories at small couplings: Landau damping

Landau damping rate is the typical time scale in which a hard mode scatters.



- $1/\tau = n\sigma$ where $n = T^3$ is number density, σ is cross section
- $\sigma = g^4 \int \frac{dt}{t^2}$ seems linearly divergent
- infrared singularity screened due to mass of electric gluon (Braaten-Pisarski)
- divergence turns logarithmic: $\sigma = g^4 \log(T/m)/m^2 = \frac{g^2 \log(1/g)}{T^2}$
- $1/\tau = g^2 \log\left(\frac{1}{g}\right) T$

Gauge theories at small couplings: Linde problem



- Consider a field fluctuation A in a region of size R
- $D = \nabla + igA$ is non-perturbative if $A = \frac{1}{gR}$
- then $\mathbf{B} = \frac{1}{gR^2}$ and $E = \frac{R^3}{g^2R^4} = \frac{1}{g^2R}$
- the Boltzmann factor $\exp\left[-\frac{1}{g^2RT}\right]$ is suppressed unless $R \approx \frac{1}{g^2T}$
- since electric fields screened in distance $1/gT$, these are magnetic.

Gauge theories at small couplings: Hydrodynamic modes

Fast modes steal energy from soft modes and convert it into entropy

D. Bödeker, Phys.Lett.B516:175-182,2001

- Divide fields into soft (slow, classical) modes and hard (fast, quantum) modes
- $\mathbf{D} \times \mathbf{B} = D_t\mathbf{E} + \mathbf{J}_h$ where $\mathbf{J}_h = \sigma\mathbf{E}$ (consider σ later)
- select $A_0 = 0$ gauge: $\sigma d_t\mathbf{A} = -\mathbf{D} \times \mathbf{B}$ (neglecting slow D_t)
- dissipative: soft modes accelerate hard modes and lose energy
- stochastic because fast (hard) modes provide noise
- $\sigma d_t\mathbf{A} = -\mathbf{D} \times \mathbf{B} + \xi$ where $\langle\xi\rangle = 0$ and $\langle\xi_i(t)\xi_j(0)\rangle = 2\sigma T\delta_{ij}\delta(t)$.
- $\sigma\mathbf{A}/t = \mathbf{A}/R^2$ (since $\mathbf{B} = \mathbf{D} \times \mathbf{A}$); hence $t = \frac{\sigma}{g^4T^2} = \frac{1}{g^4\log(1/g)T}$

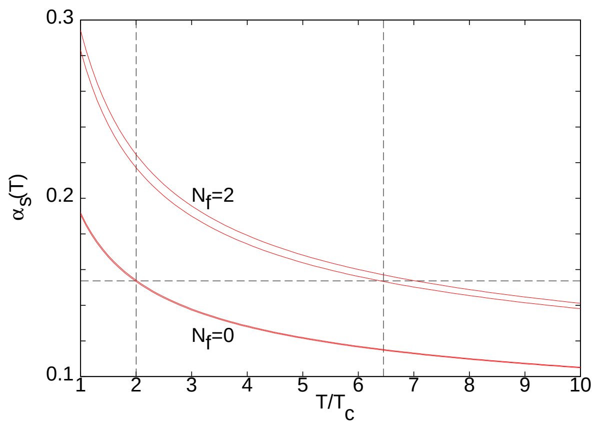
Gauge theories at small couplings: Colour conductivity

Transport: multiple small angle scattering or single large angle scattering

- $d\mathbf{p} = g\mathbf{E}dt$ where $d\mathbf{p} = d\mathbf{v}p_0 = d\mathbf{v}T$ and hence $d\mathbf{v} = (gdt/T)\mathbf{E}$
- current $\mathbf{J} = ngd\mathbf{v} = (g^2T^2dt)\mathbf{E}$ and hence $\sigma = g^2T^2dt$
- collision-less plasma: spatial oscillation of electric field will stop growth of current (since temporal oscillation are slow) $dt = 1/k$ and hence $\sigma = g^2T^2/k$.
- Drude theory: collisions cut off the growth at a time scale of the gluon damping rate, $1/dt = g^2 \log\left(\frac{1}{g}\right) T$, giving $\sigma = \frac{T}{\log(1/g)}$.

Gauge theories at any coupling

For long distance physics the coupling g can become small. In QCD at experimentally accessible heavy-ion collider energies $g \geq 1$.



1. Linear response theory
2. Electrical conductivity and photon emissivity
3. Euclidean correlators and spectral densities
4. Extraction of spectral density on the lattice
5. Why the answer is 42

Linear Response Theory

The response, $\mathbf{A}(t)$, of a system to a force $\mathbf{F}(t)$ if non-linear terms are neglected—

$$\mathbf{A}(t) = \int_{-\infty}^{\infty} dt' \chi(t-t')\mathbf{F}(t') \qquad \text{hence} \qquad \mathbf{A}(\omega) = \chi(\omega)\mathbf{F}(\omega).$$

The zero frequency limit of χ is related to transport coefficients, which in turn is related to dissipative processes. A microscopic computation with the perturbation term $\mathbf{A}\cdot\mathbf{F}$ relates the retarded correlator to the transport coefficient—

$$\chi \propto \lim_{\epsilon\to 0} \int d^3x' \int_{-\infty}^{t} dt'' \mathrm{e}^{\epsilon(t''-t)} \int_{-\infty}^{t''} dt' \langle \mathbf{A}(\mathbf{x},t)\mathbf{A}(\mathbf{x}',t')\rangle.$$

This is a consequence of the fluctuation-dissipation theorem.

Electrical conductivity and photon emissivity

The differential photon emissivity is given by—

$$\omega \frac{d\Omega}{d^3p} = \frac{C_{EM}}{8\pi^3} n_B(\omega;T)\rho^\mu_\mu(\omega,\mathbf{p};T) \qquad \text{where} \qquad C_{EM} = 4\pi\alpha\sum_f e_f^2.$$

In terms of the DC electrical conductivity ($\mathbf{j} = \sigma\mathbf{E}$)

$$\sigma(T) = \frac{C_{EM}}{6}\; \frac{\partial}{\partial\omega}\rho^i_i(\omega,\mathbf{0};T)\bigg|_{\omega=0}, \qquad \frac{8\pi^3\omega}{C_{EM}T^2}\frac{d\Omega}{d^3p} = 6\,\frac{\sigma}{T}.$$

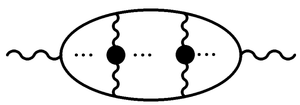
Since $k^\mu\rho_{\mu\nu} = 0$, we have $\rho_{00} = 0$ along the line $\mathbf{p} = 0$. However,

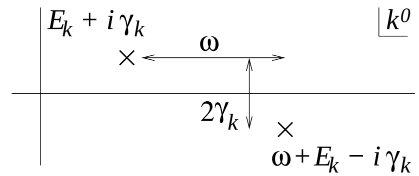
$$\rho_{00}(\omega,\mathbf{0};T) = 2\pi\chi_Q\omega\delta(\omega),$$

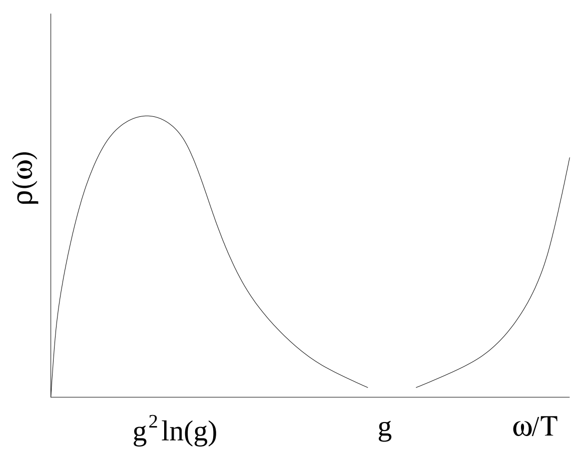
where χ_Q is the charge susceptibility. This gives virtual soft photon rate.

Pinch singularities and transport

There are pinch singularities at small external energy, ω, from ladder diagrams. These ladder diagrams correspond to multiple scatterings off particles in the plasma.







G. Aarts and J.M.M. Resco JHEP 0204:053,2002

Euclidean Correlators

Equilibrium correlation functions are most easily constructed in the continuum theory. In that case one has the relation—

$$G(\tau, \mathbf{p}; T) = \int_0^\infty \frac{\omega}{2\pi} K(\omega, \tau; T) \rho(\omega, \mathbf{p}; T).$$

Inverse is ill-defined

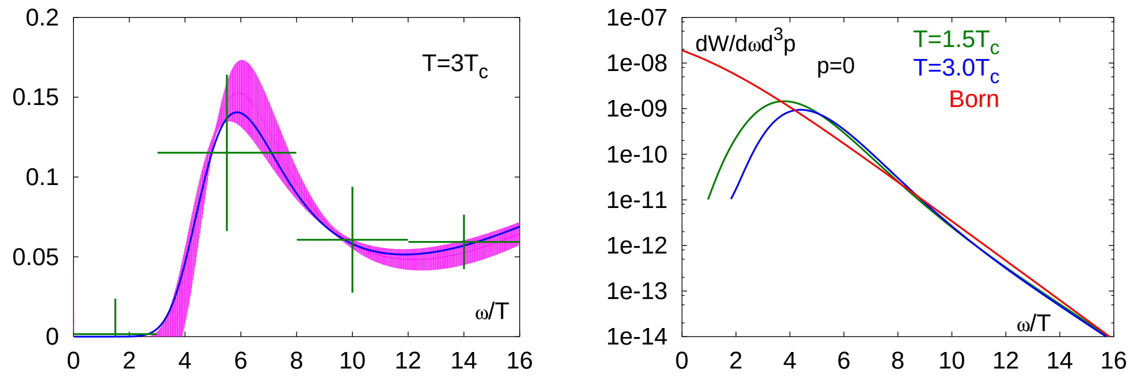
Convert to a minimisation/Bayesian problem—

$$F(\rho) = (G - K\rho)^T \Sigma (G - K\rho) + \beta U(\rho) \qquad \text{Bayesian probability } = \exp F(\rho).$$

1. Linear: $U = (L\rho)^2$, $L = 1$, D or D^2

2. Maximum Entropy Method: $U = \sum \rho_0 \log(\rho/\rho_0)$

MEM: The first lattice computation



F. Karsch et al, Phys.Lett.B530:147,2002

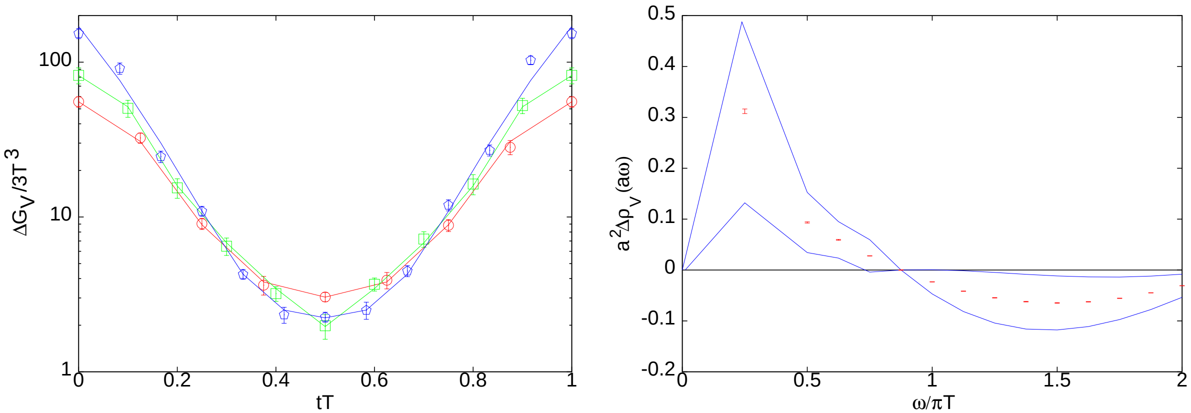
Full agreement with Born for $\omega/T \geq 4$. But vanishes at $\omega = 0$!

Beyond MEM: The second lattice computation

Since the problem is linear, work with

$$\Delta G(\omega, \mathbf{p}; T) = G_{full}(\omega, \mathbf{p}; T) - G_{ideal}(\omega, \mathbf{p}; T).$$

This shows a bump at small ω.



S. Gupta, hep-lat/0301006

Beyond lattice: the continuum limit

Assume

$$\frac{\Delta\rho}{T^2} = \frac{z\sum_{n=0}^{N}\gamma_n z^{2n}}{1+\sum_{m=1}^{M}\delta_m z^{2m}}.$$

Each parameter has an a priori distribution which we don't know. If we can discover some combinations of ΔG which are independent of some of the parameters, then they can be integrated out. For example, for $N = 0$ and all M, we find

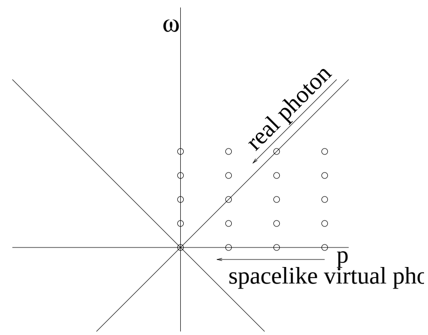
$$\frac{\sigma}{T} = C_{EM}\frac{\Delta\chi}{T^2}, \qquad \text{where} \qquad \Delta\chi = \int_0^{1/T} d\tau \Delta G(\tau).$$

In the continuum limit

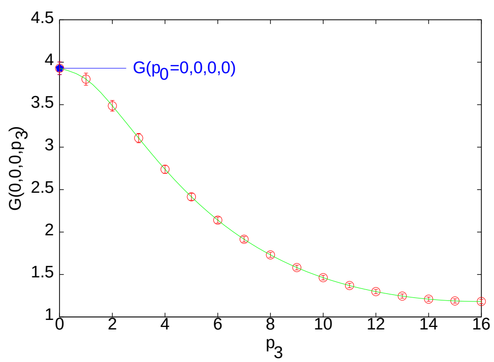
$$\frac{\sigma}{T} \approx 7C_{EM}.$$

Approaching $\omega = \mathrm{p} = 0$ in the continuum limit

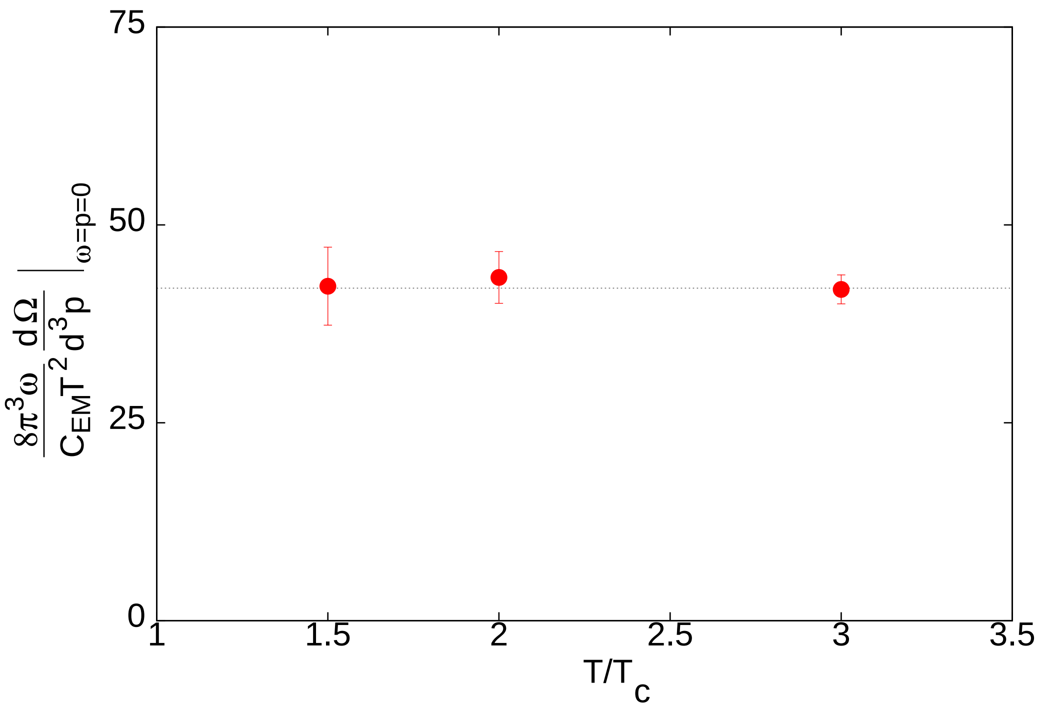
Many different possible limits of the soft photon cross section—



—but they are the same on every lattice. The limit is singular.



$42 = 6 \times 7$



75
50
25
0
1
1.5
2
2.5
3
3.5
T/T c
$\frac{8\pi^3\omega}{C_{EM}T^2}\frac{d\Omega}{d^3p}\Big|_{\omega=p=0}$

The QCD plasma at Brookhaven and CERN

1. A photon emitted in the plasma is reabsorbed if its path length is

$$\ell = \frac{1}{\sigma} \approx \frac{1}{7C_{EM}T} \approx 3 \text{ fm}.$$

   Typical fireball dimensions at RHIC are a few fm, so the fireball is marginally transparent to photons. This may no longer be so at LHC.

2. Typical hadronic length/time scales in the plasma are

$$\tau \approx \frac{1}{7T} \approx 0.15 \text{ fm}.$$

   Hydrodynamic description of the final state in the plasma work if its thermalisation time is less than 1 fm. Hydrodynamics may work at both RHIC and LHC.

3. Spontaneous thermal fluctuations of flavour even out by diffusion. The diffusion constants, D, are related to the electrical conductivity as

$$\sigma = \sum_f e_f^2 D_f \chi_f,$$

where χ_f is the thermal susceptibility for particle number. We find $D \approx \frac{1}{7T} \approx 0.15\ \mathrm{fm}$, and hence the only visible chemical fluctuations are those at freeze out. Strong implications for strangeness production.

R.V. Gavai and S. Gupta, Phys.Rev.D65:094515,2002

4. Jets traversing the plasma are quenched. This calls for high shear viscosity, η. No reason, now, not to expect this. Watch for forthcoming computation of η.