On the critical end point of QCD

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1. The Taylor expansion: non-linear quark number susceptibilities
2. Optimizing the computation: the Steiner problem
3. Digression: simple phenomena at T_c concerning NLS
4. Radius of convergence: a limit on the end point

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The Taylor expansion for 2 flavours

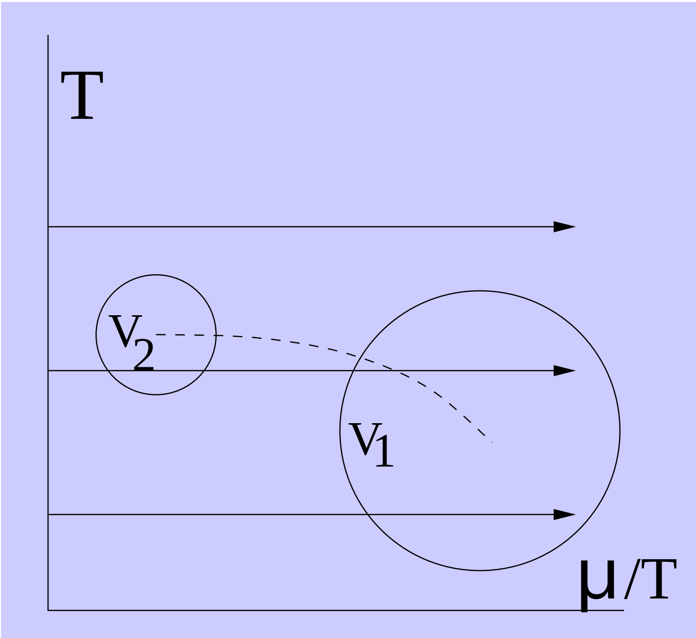
$$P(T,\mu_u,\mu_d) = \left(\frac{T}{V}\right) \log Z(T,\mu_u,\mu_d)$$

$$P(T,\mu_u,\mu_d) = P(T,0,0) + \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{\mu_u^{n_u}}{n_u!} \frac{\mu_d^{n_d}}{n_d!}$$

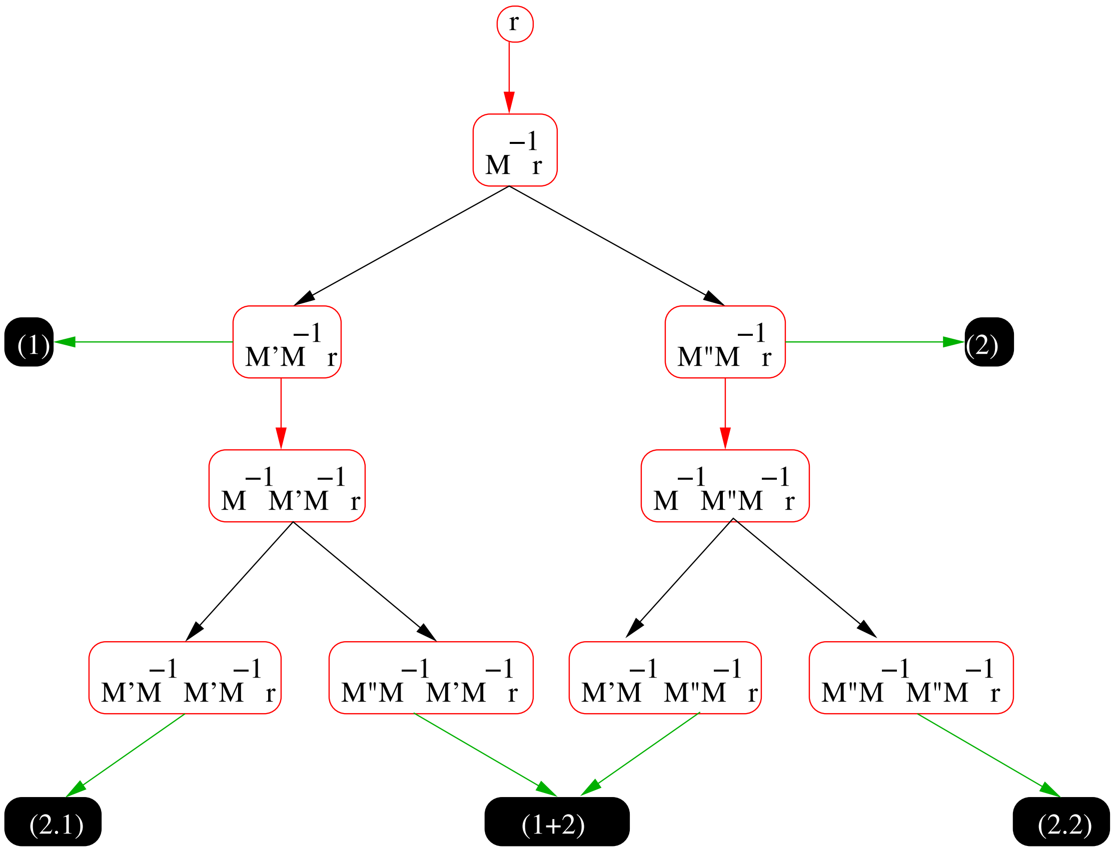
$m_u = m_d$ implies that $\chi_{n_u,n_d} = \chi_{n_d,n_u}$, for any $\mu_u = \mu_d$. One QNS is

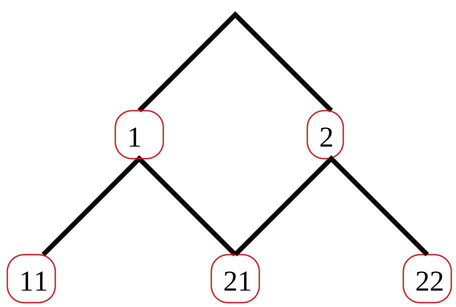
$$\chi_{20}(T,\mu_B) = \left.\frac{\partial^2 P(T,\mu_u,\mu_d)}{\partial \mu_u^2}\right|_{\mu_u=\mu_d=\mu_B/3}$$

$\chi_{20}(T^E,\mu_B^E)$ diverges in the infinite volume limit: pseudo critical behaviour at finite volumes.



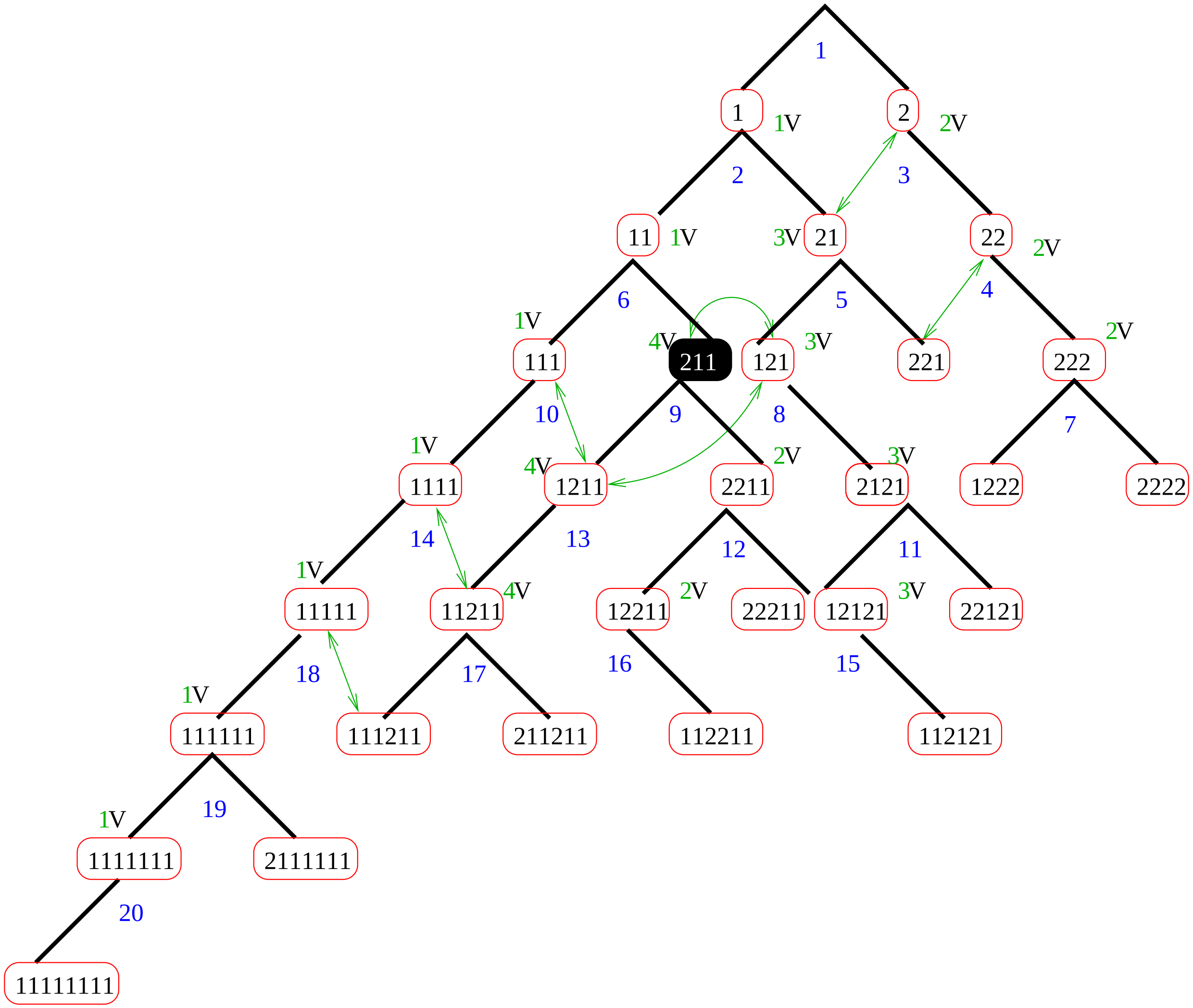
Steiner problem: efficient stochastic evaluation of traces





Charikar et al, STAN-CS-TN-97-56.

The Steiner tree for order 8 Taylor expansion



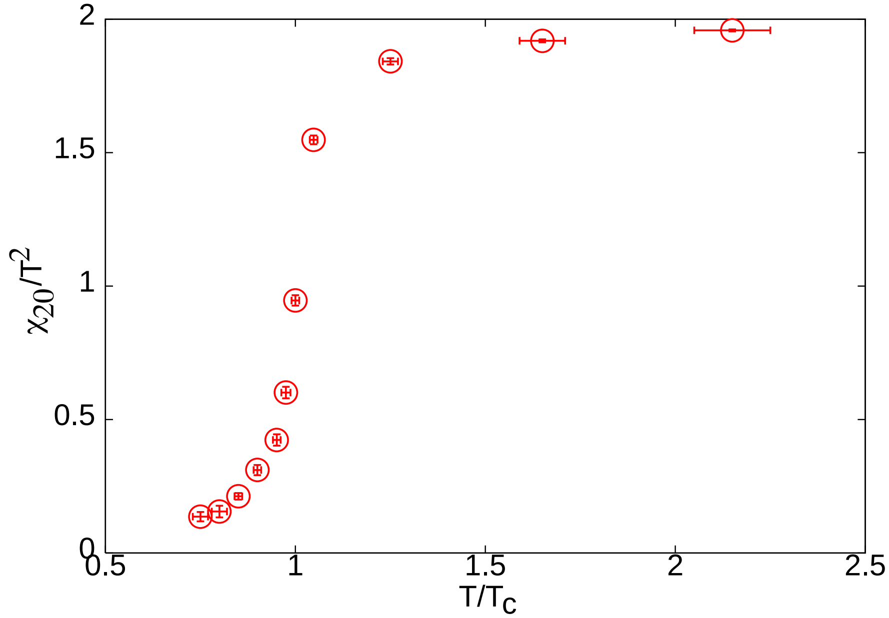
Simulation parameters

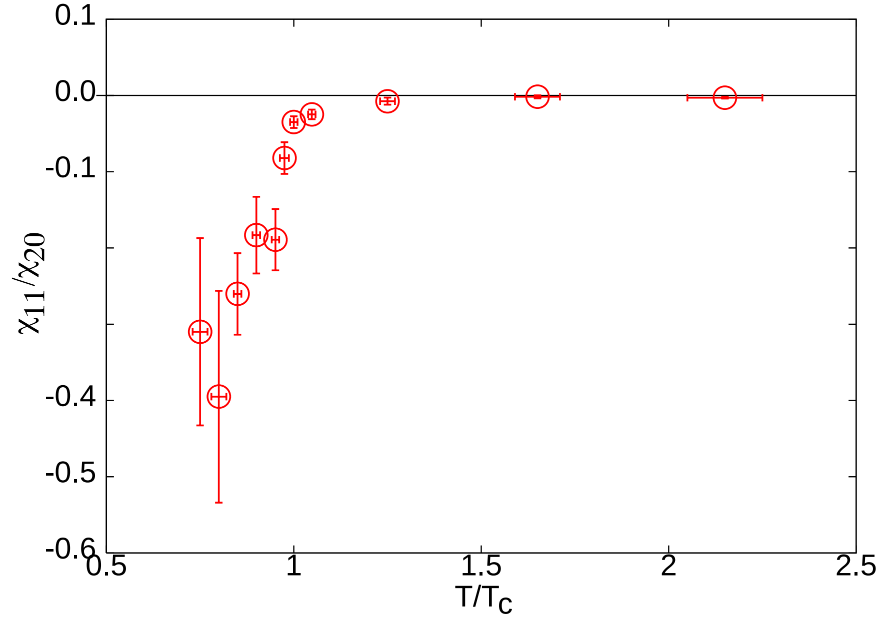
1. Taylor expansion to 8th order
2. $\mathbf{N_t = 4}$: $m_\rho/T_c = 5.6$, $m_\pi/m_\rho = 0.3$ (Fodor Katz, de Forcrand Philipsen)
3. $\mathbf{N_s}$: $3.3 \le m_\pi V^{1/3} \le 10.0$ (interpolates to Bielefeld-Swansea)
4. Statistics: more than $50\tau_{int}$ of configs at each T and V

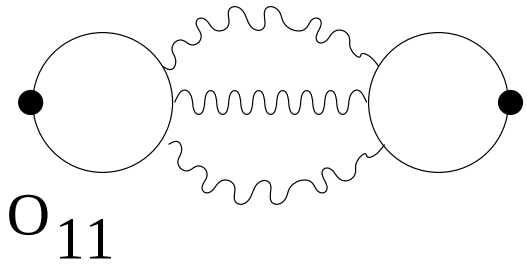
$$\frac{\chi_{20}}{m_\rho^2} = \frac{T}{m_\rho^2 V}\left[\sum_{ij}\frac{|\langle i|\gamma_0|j\rangle|^2}{\lambda_i\lambda_j} + \left(\sum_i \frac{\langle i|\gamma_0|i\rangle}{\lambda_i}\right)^2\right]$$

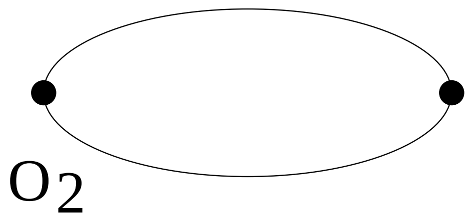
$$= \frac{(m_\pi/m_\rho)^2}{m_\pi^3 V}\left[\sum_{ij}|\langle i|\gamma_0|j\rangle|^2\,\frac{Tm_\pi}{\lambda_i\lambda_j} + \left(\sum_i \langle i|\gamma_0|i\rangle\,\frac{\sqrt{Tm_\pi}}{\lambda_i}\right)^2\right]$$

The QNS: sign fluctuations

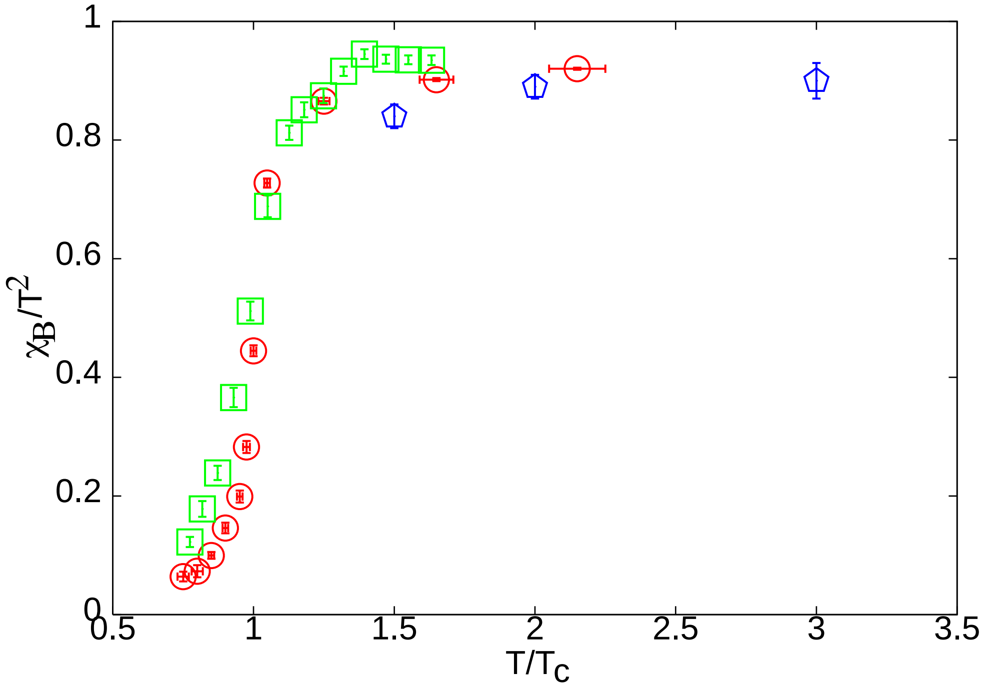








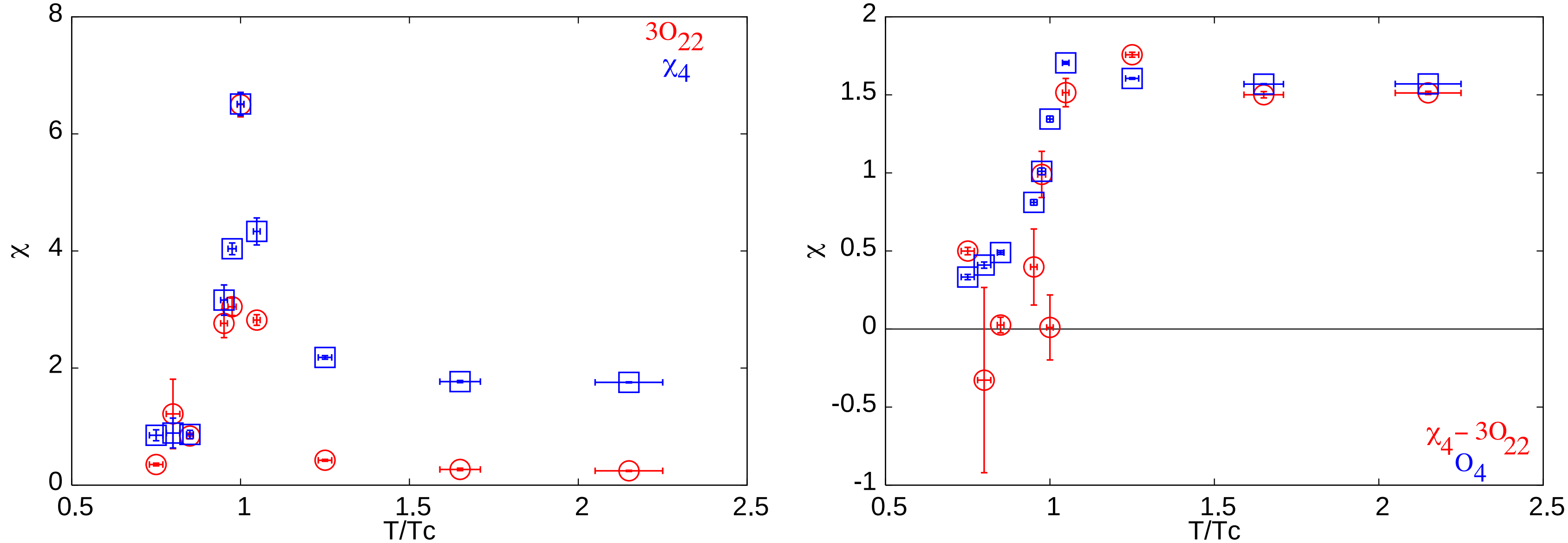
Fluctuations

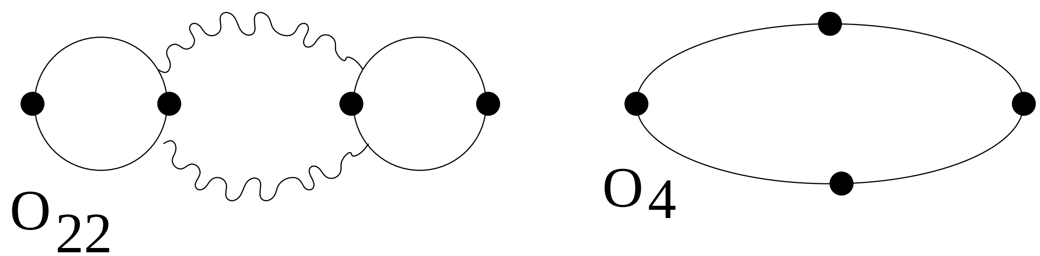


Quenched, MILC $N_t = 8$, this work, scaled by quenched

Weak coupling:Blaizot et al, Mustafa et al, Vuorinen

4th order NLS





... and similiar systematics at higher orders

Systematics of susceptibilities

1. $\mathbf{T \geq 1.5T_c}$ perturbative power counting roughly sets the right scale of various operators. More detailed tests, comparisons with weak-coupling theory, might be useful.

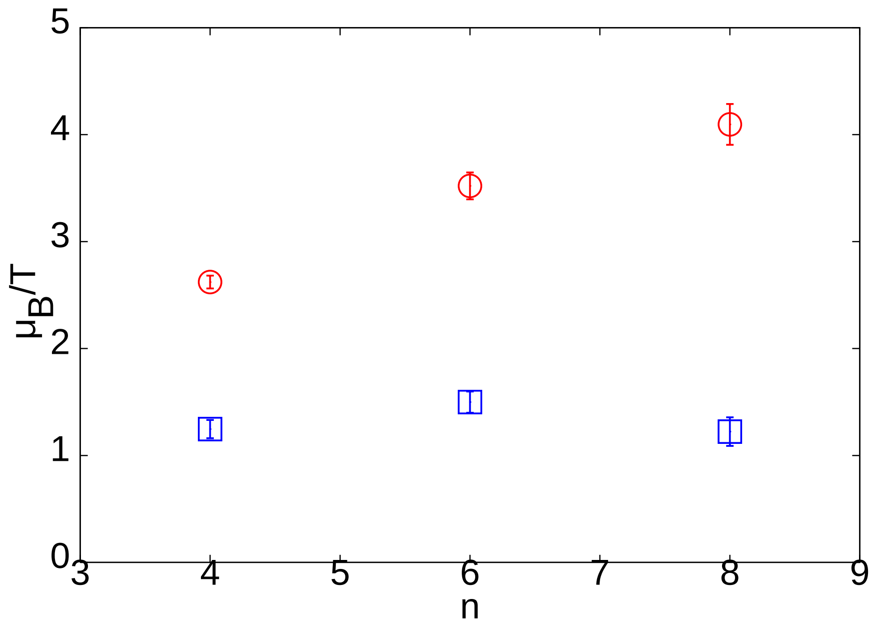
2. $\mathbf{T \leq 0.9T_c}$ most NLS of given order are of the same order of magnitude. No special simplification.

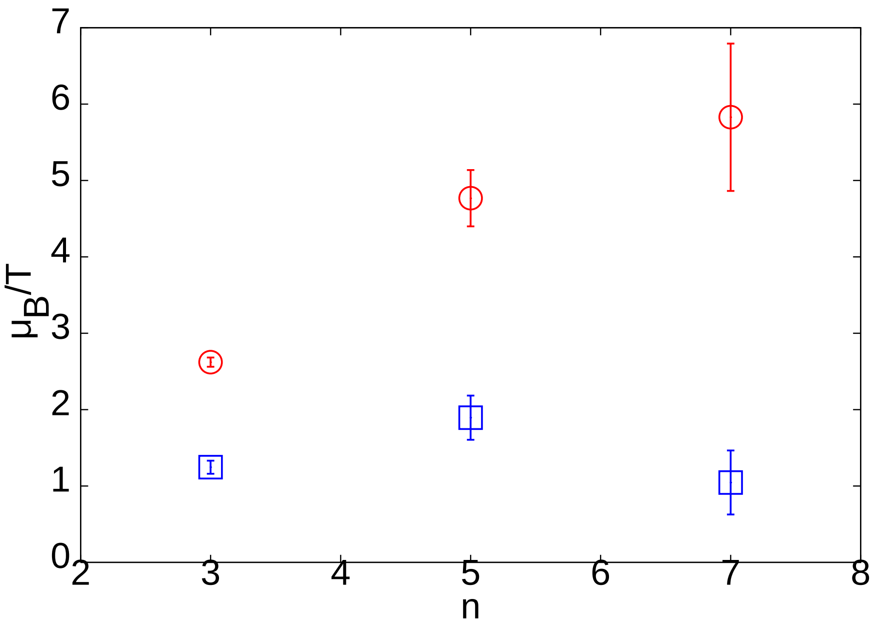
3. **neighbourhood of $\mathbf{T_c}$** Susceptibilities of the composite operator O_2 dominate and cause all the peaks that are seen. Shapes are consistent with $(T/V)\langle O_2 \rangle$ behaving as an order parameter.

Volume dependence under control at fixed mass. Mass dependence? Chiral limit? Is there an effective theory of O_2 near T_c? What is the role of O_{11} in the chiral limit?

Radius of convergence

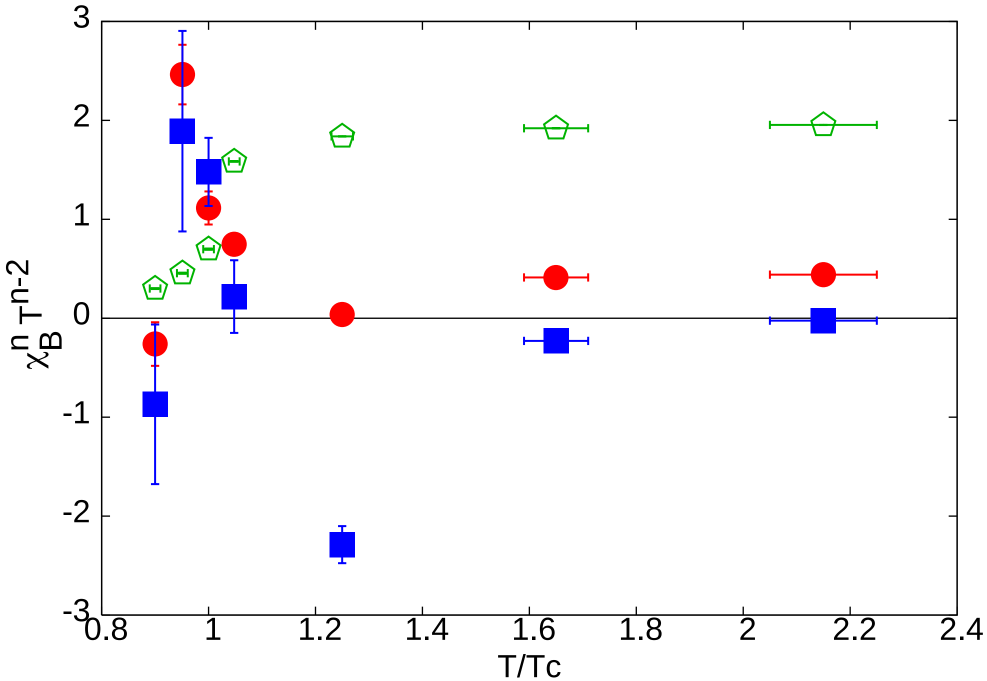
Notation: If $f(x) = \sum_n f_{2n} x^{2n}$ then $\rho_{2n} = \left|\frac{f_0}{f_{2n}}\right|^{1/2n}$ and $r_{2n} = \sqrt{\left|\frac{f_{2n}}{f_{2n+2}}\right|}$





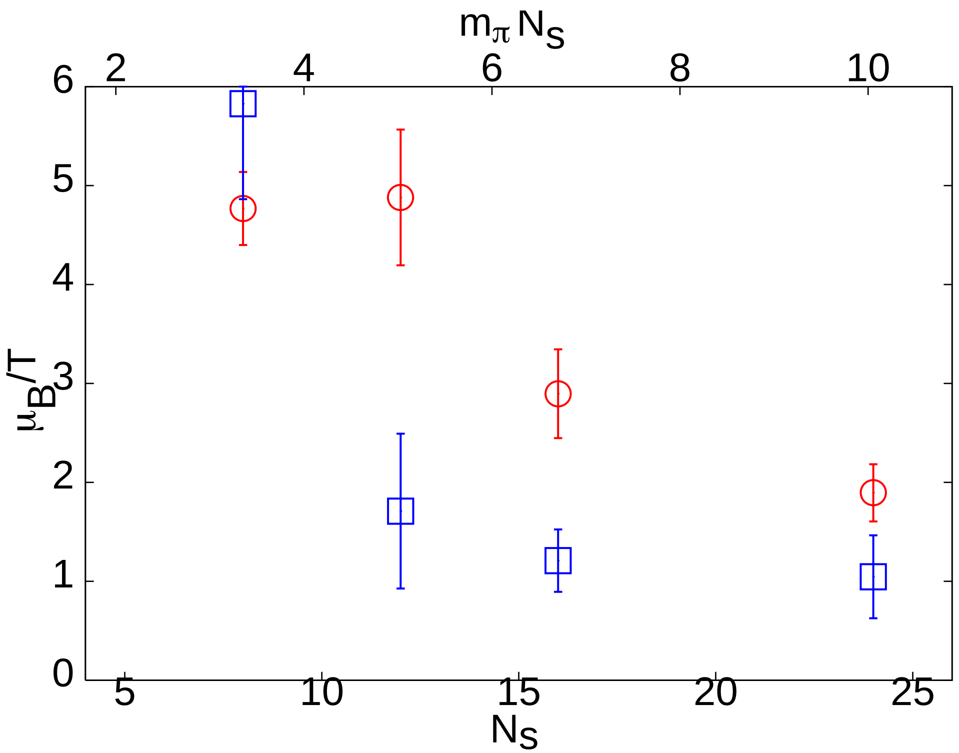
Pseudocritical behaviour: larger volumes required at larger order

Sign and convergence of subexpressions



4×24^3 lattice: χ_B^0/T^2, $\chi_B^4/2$, $\chi_B^6 T^2/12$.

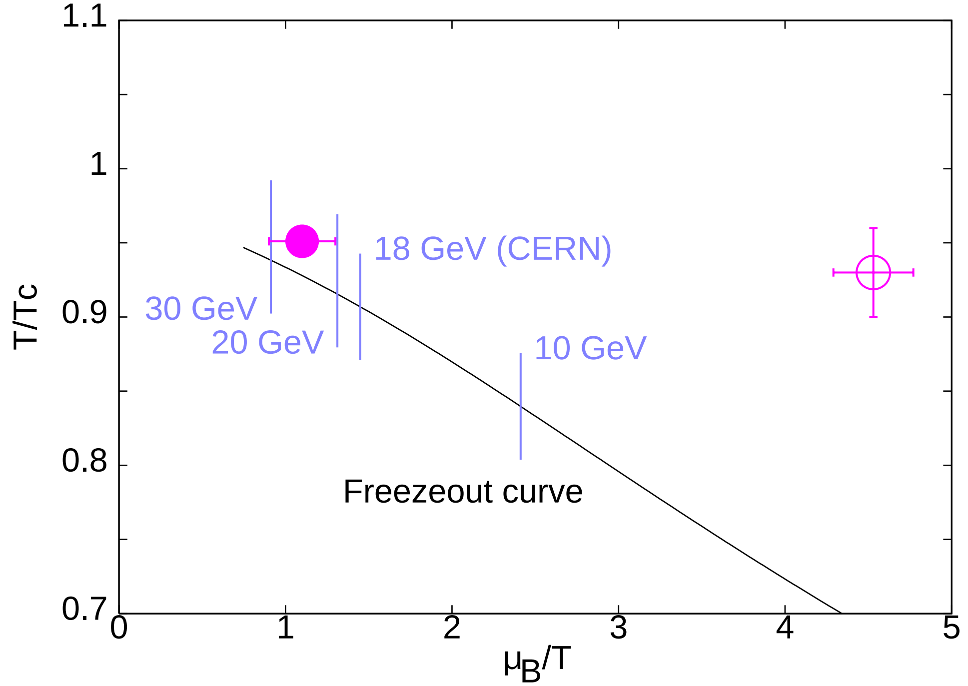
Finite size scaling



$T/T_c = 0.95$: r_4 (circles), r_6 (boxes).

FS shift in μ_B^E estimated assuming CEP is in Ising universality class.

Summary: Critical end point



Volume effects under control. Examine mass dependence and continuum limit.
Use more volumes to refine FSS and find critical exponents.