The nature of the high temperature phase of QCD

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8 December, 2005

This talk: QCD at $T/T_c \simeq 0.9$–3. Much lower, and perhaps hadron gas picture may be roughly valid. Much higher, and some form of weak coupling theory is likely to work. Here: one scale.

Collaborators: Datta, Gavai, Mukherjee, Ray

Length scales

1. The microscopic structure: ($L \ll 1/T$) quarks and gluons. Quantum chromodynamics is an input.

2. The mesoscopic structure: ($L \simeq T$) long-standing confusion about counting the number of degrees of freedom from the pressure, speculations about massive quasi-particles, some current speculation about compositeness. Lattice is experiment: tests models and theories (but experiments should be experiments).

3. The macroscopic structure: ($L \gg T$) speed of sound, transport theory, compressibility— all are parameters in the hydrodynamical description of the QCD fluid. Lattice is basic theory: gives input for experiments and interpretive theory

Mesoscopic structure

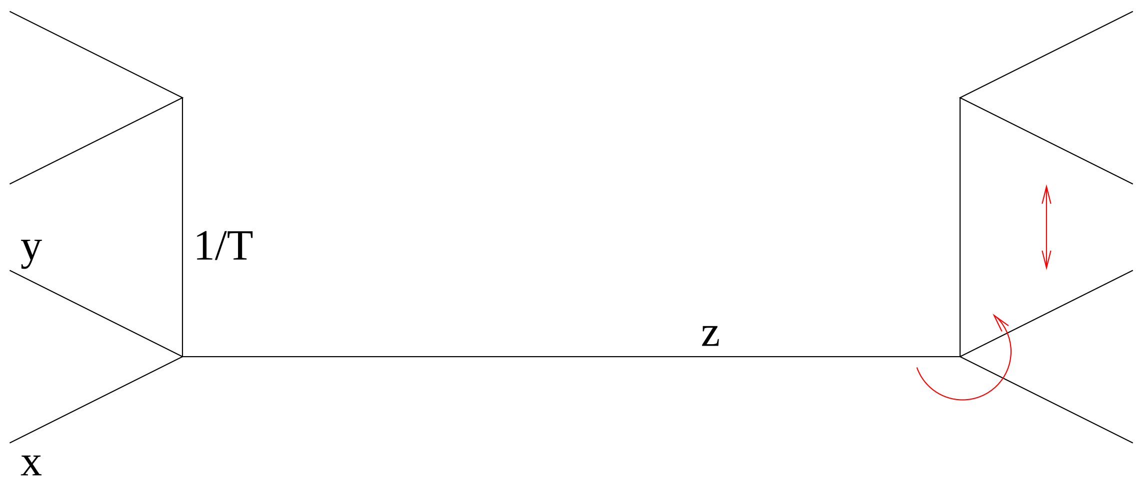
$$Z = \sum_{a,b,c,\cdots} \langle a, b, c, \cdots | \mathrm{e}^{-H/T} | a, b, c, \cdots \rangle, \qquad P = -\left(\frac{T}{V}\right) \log Z.$$

Which states saturate (or have large contributions to) the partition function? Microscopically one knows that one puts in quark and gluon states.

1. $T < T_c$: examination of the quantum numbers a, b, c etc, shows that only colour singlet composite states appear in the trace. For example, resonance gas: the states are multiparticle hadron states. This does not imply that interactions do not exist. If $H = H_{kin} + H_{int}$, then if $\langle \phi | H_{int} | \phi \rangle = 0$, i.e., the trace projects away some of the interactions. P retains only forward scattering information ($\langle \phi | H_{int}^n | \phi \rangle$). Resonance gas may fail for other observables.
2. $T > T_c$: one of the intriguing issues.

Glue sector

Glue sector investigated through screening masses of colour singlets: sometimes called "glueball" correlations.

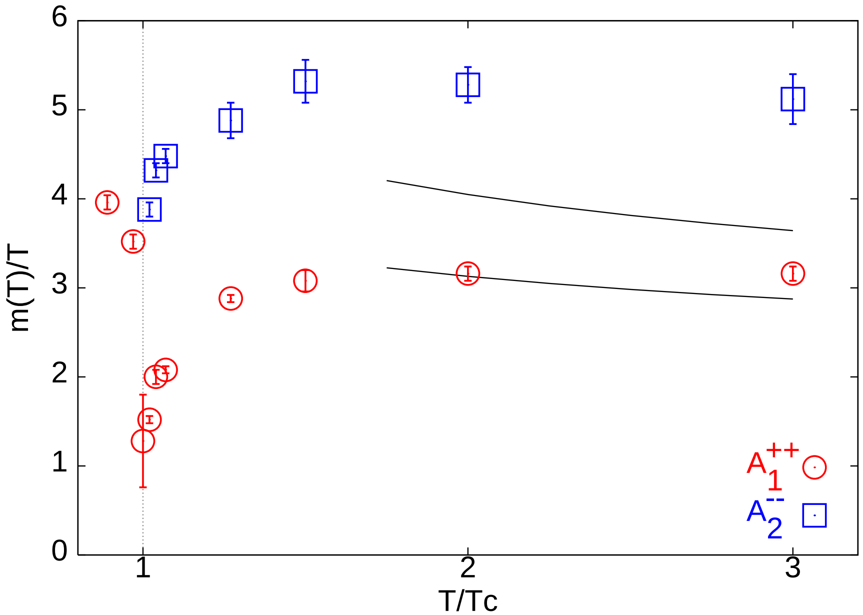


J^{PC} breaks down to $\mathcal{M}^{\mathcal{PC}}$. Measure—

$$C_{\mathcal{M}^{\mathcal{PC}}}(z) \propto \exp[-zm(\mathcal{M}^{\mathcal{PC}})].$$

Gluon sector screening masses (I)

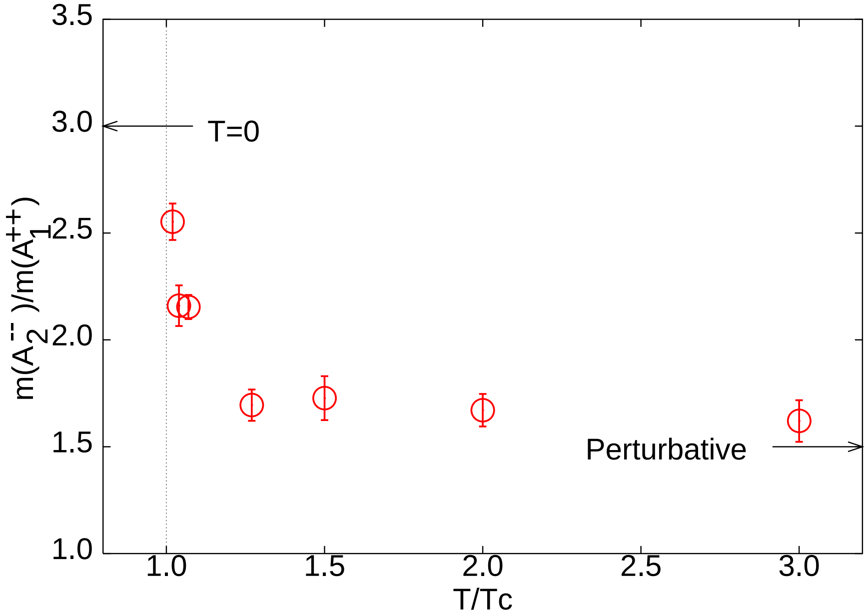
Electric gluons at finite temperature have lattice quantum number 1^{--} and colour quantum numbers. Colour singlet two gluon states have lattice quantum number 0^{++}. Colour singlet three gluon states have lattice quantum number 1^{--}.



Comparison with weak-coupling prediction $(m/T \propto g + \cdots)$ spoilt by scale uncertainty. Datta and SG, Phys.Rev.D67:054503,2003

Gluon sector (II)

Ratio of the two masses provides a test of whether weakly coupled gluons is a valid picture. Less affected by scale uncertainties if g is small enough, since $m/m' \propto 3/2 + \cdots$.



Works (qualitatively) at $T/T_c \geq 1.25$. Fails below that.
Datta and SG, Phys.Rev.D67:054503,2003

Quark sector

Since there are many conserved quantum numbers the problem becomes simpler. Look at two quantum numbers simultaneously— say U and D.

$T < T_c$: whenever $U = 1$ is excited $D = -1$ is excited along with it.

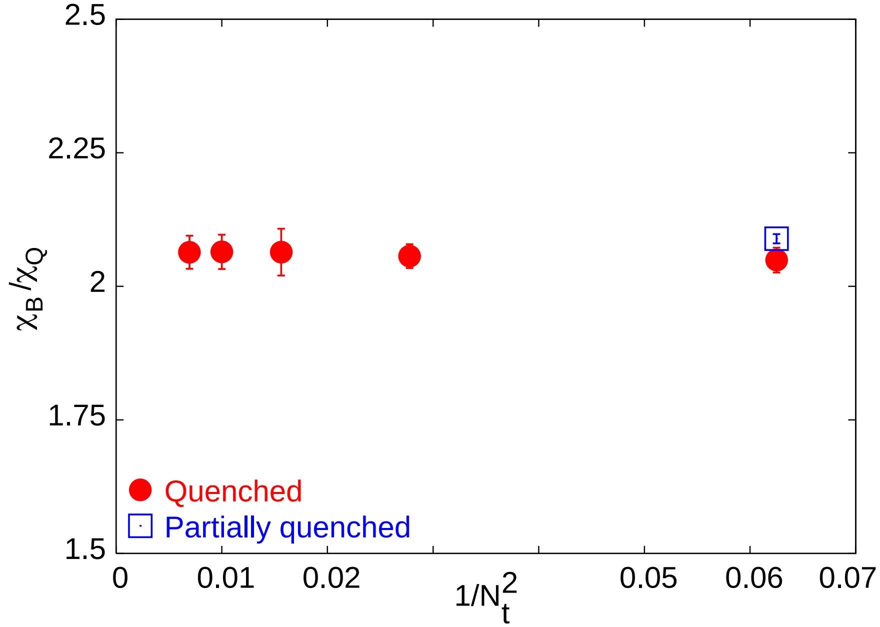
$T > T_c$: when $U = 1$ is excited $D = \pm 1$ should be excited along with it if the medium contains quarks. Otherwise, by observing what value of D is preferentially excited, you find something about the quantum numbers of the excitations.

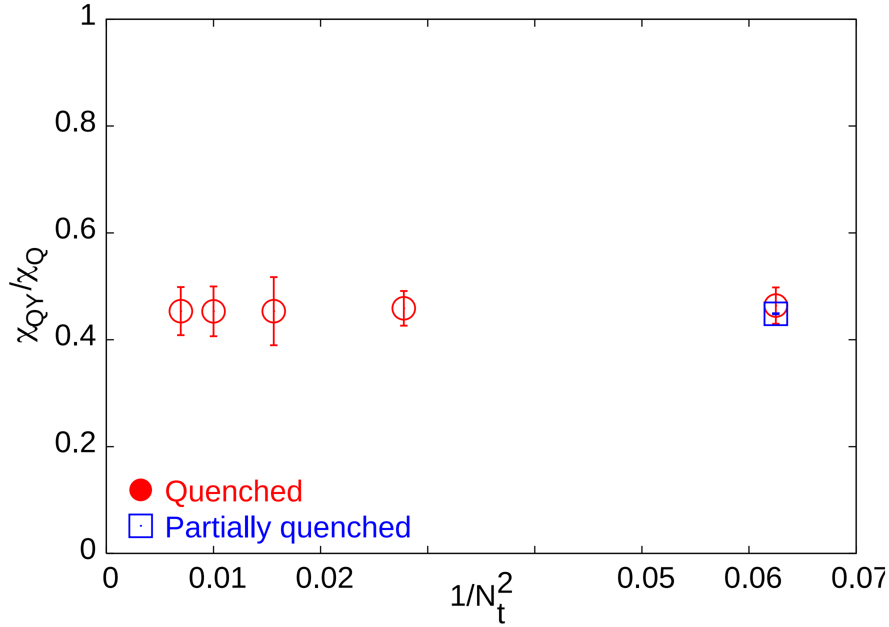
Similarly one could study the linkages $U|B$ or $U|Q$, or $D|B$ etc.

$$C_{(XY)/Y} \equiv \frac{\langle XY \rangle - \langle X \rangle \langle Y \rangle}{\langle Y^2 \rangle - \langle Y \rangle^2} = \frac{\chi_{XY}}{\chi_Y}$$

Gavai and SG, hep-lat/0510044

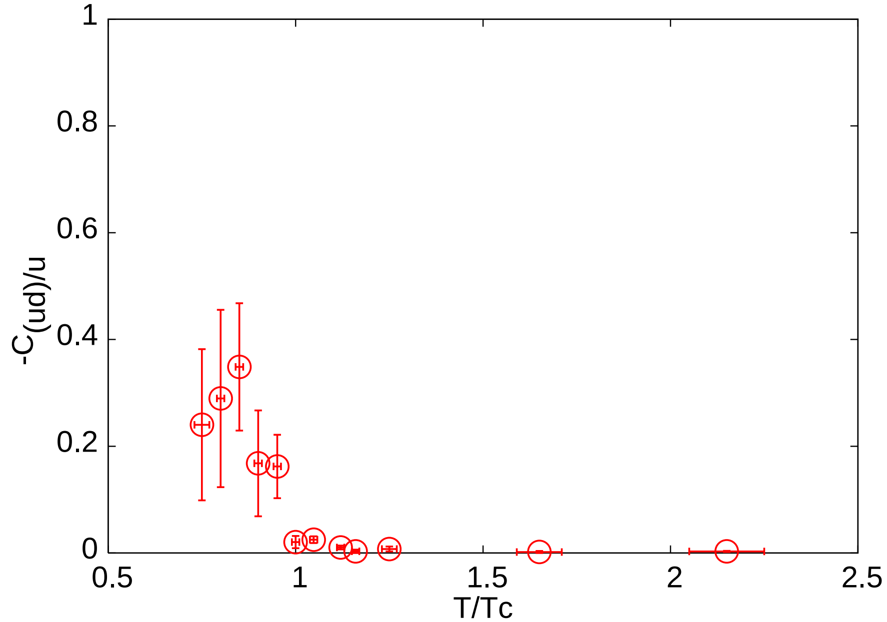
Ratios are robust





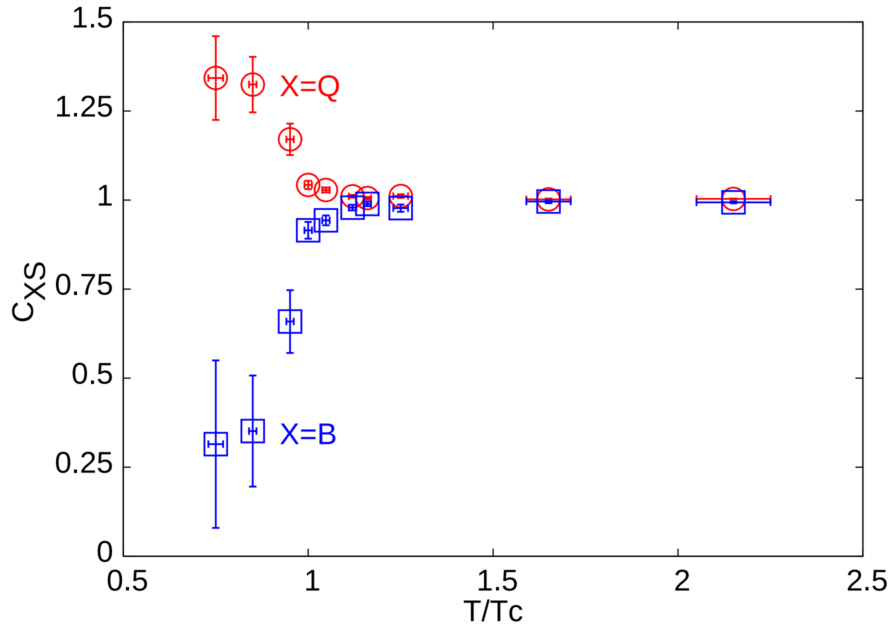
Above T_c ratios of QNS are almost independent of lattice spacing, and insensitive to quark masses (as long as $m < T$). Therefore linkage is a robust quantity above T_c. Gavai and SG, hep-lat/0510044

U and D are independent



u and $\bar{d}$ can be carried by the same particle below T_c but not above T_c.
Gavai and SG, hep-lat/0510044

Strangeness is carried by quarks



$C_{BS} = -3C_{(BS)|S}$ and $C_{QS} = 3C_{(QS)|S}$. Below T_c strange baryons are relatively heavy and therefore sparse in the plasma, but kaons are not so heavy. Above T_c: strange quarks.

Gavai and SG, hep-lat/0510044; Koch, Majumder, Randrup, hep-ph/0507???

Non-linear susceptibilties

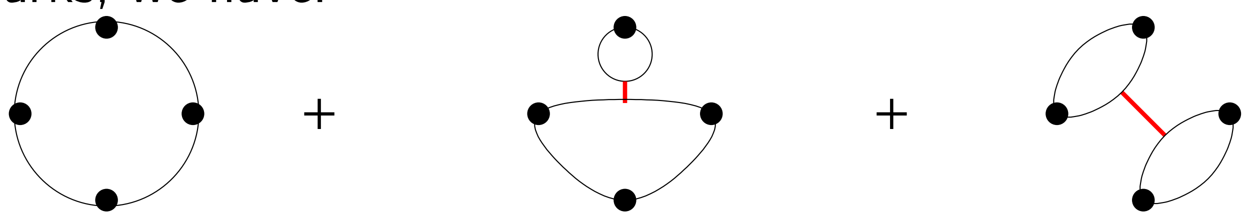
A non-linear quark number susceptibility of order n is the derivative

$$\chi_B^{(n)} = \frac{\partial^n P}{\partial \mu_B^n} \qquad \text{and} \qquad \chi_{n_u,n_d} = \frac{\partial^{n_u+n_d} P}{\partial \mu_u^{n_u} \partial \mu_d^{n_d}}$$

Now,

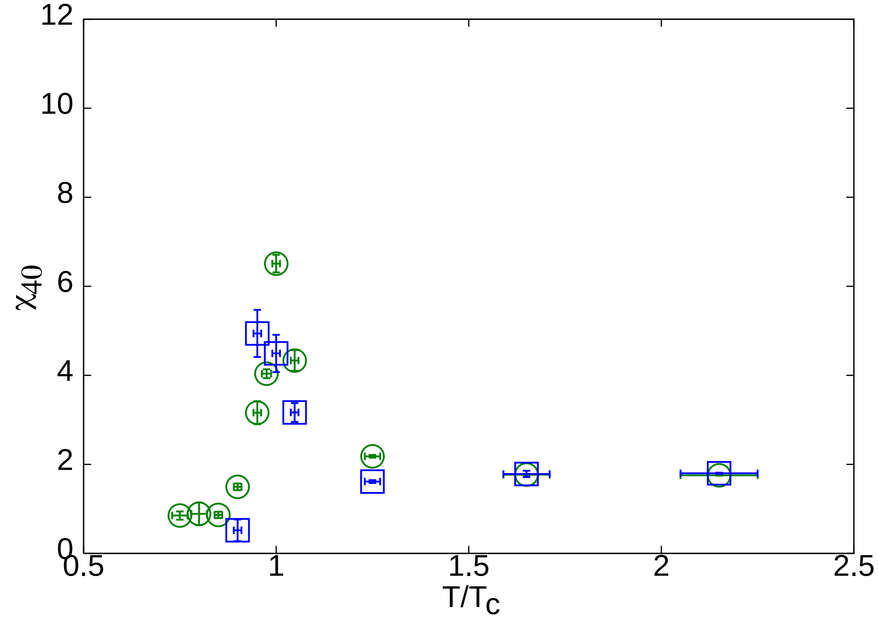
$$\chi_B^{(4)} = \frac{1}{2}\left[\chi_{40} + 2\chi_{31} + \chi_{22}\right]$$

In terms of quarks, we have:



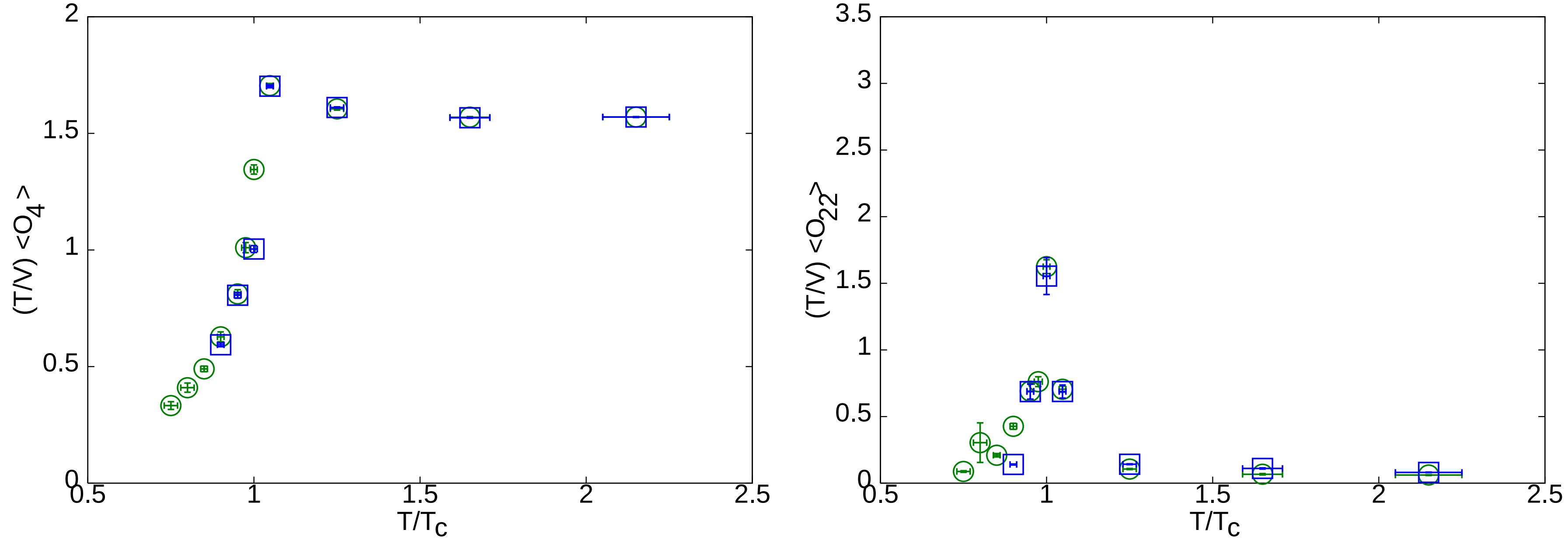
Flavour disconnected pieces are absent for free quarks, i.e., vanish when $g \to 0$. In a resonance gas one has vertices such as $\pi^+ + \pi^- \to \pi^0 + \pi^0$ which allow the flavour disconnected pieces to add up.

Peaking at T_c



Peak in $\chi_B^{(4)}$ implies decrease in radius of convergence $r_{2/4} = \sqrt{\chi_B^{(2)}/\chi_B^{(4)}}$.

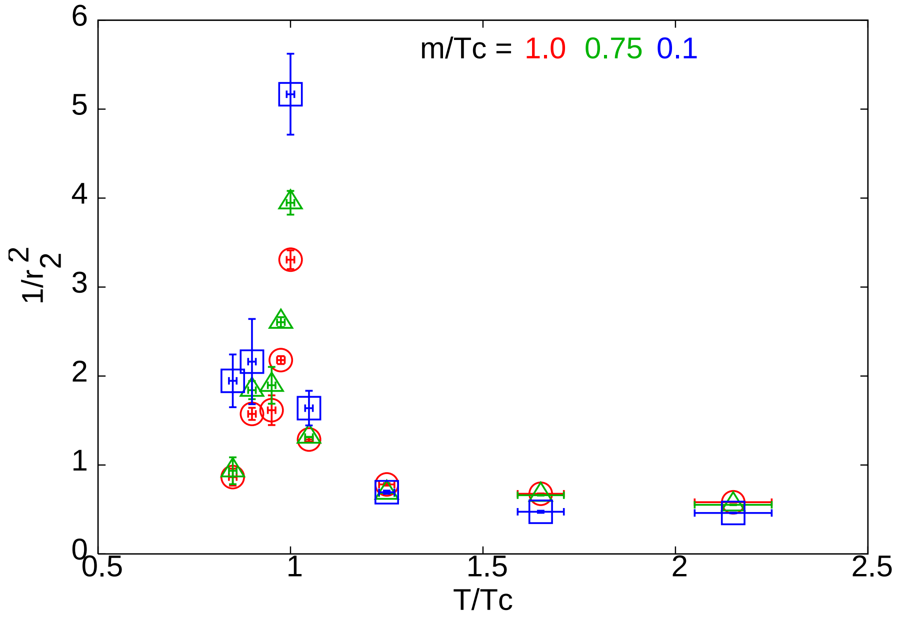
Resolving the peak



Peaks in other quantities also found very close to T_c. Always in O_{22}, O_{222}, O_{2222}.

Radius of convergence

Karsch, Ejiri, Redlich, hep-lat/0510126 suggest that the ratio $\chi_Q^{(2)}/\chi_Q^{(4)}$ can test bound state picture. In accord with weak coupling theory above T_c, and with resonance as below T_c.



Pion mass dependence is very strong. In the chiral limit?

Summary: mesoscopic structure

1. For $T \geq 1.25T_c$, electric gluons are a "good" concept. Magnetic gluons remain to be examined in equal detail. Closer to T_c the notion of a weakly interacting gluon seems to break down. Is there a finer test? What replaces it?

2. Clear evidence of elementary u, d and s quarks above T_c, and something different below T_c (consistent with hadron resonance gas?). Some evidence of structure very close to T_c.

Macroscopic structure: $L \gg 1/T$

Spatial homogeneity expected in equilibrium, i.e., QCD matter is a fluid. At very long distances gluons are screened— electric and magnetic mass both exist.

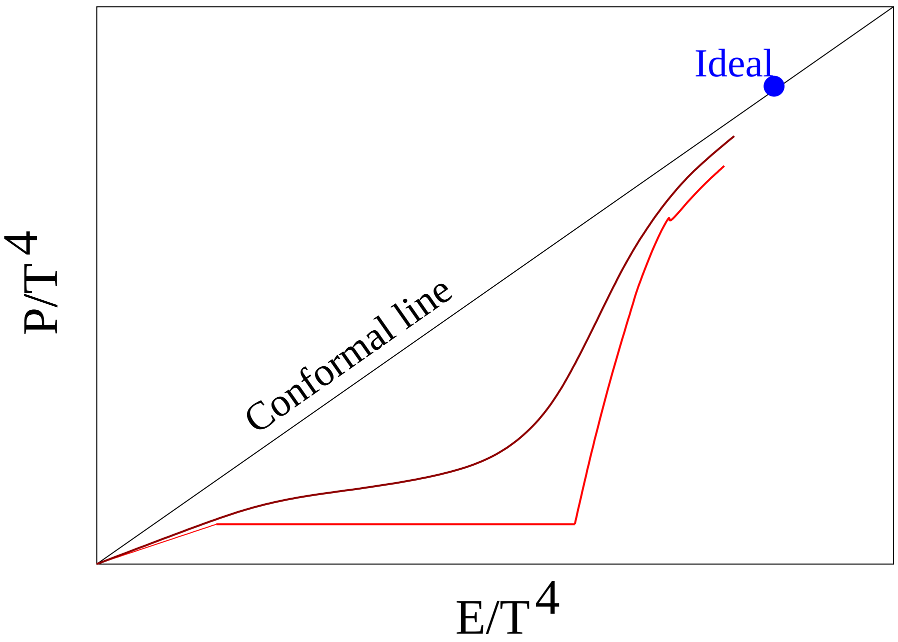
Therefore expect macroscopic physics involves flavour hydrodynamics: coupled hydrodynamics and diffusion equations follow from the conservation of energy-momentum and conserved flavour currents. EOS known, speed of sound computed.

More complicated if equilibrium is not reached— i.e., if finite volume effects and short times dominate. Need estimates of relaxation times (diffusion constants) and mean free paths.

Estimate of mean free path

1. Evaluate a correlation function. Often stress-tensor: $\langle T_{\mu\nu}(0) T_{\lambda\rho}(x)\rangle$. Noisy, smoothing must be done preserving the meaning of the operator. Better to investigate electromagnetic current: $\langle J_\mu(0) J_\nu(x)\rangle$. Statistical error under better control. Needed to extract long-distance physics.
2. Extract transport coefficient. $\rho(\omega) \propto \omega^N$, small ω related to transport. Subtract high ω tail, inspect low ω behaviour: look for transport peak. Bayesian methods needed: MEM or beyond.
3. From transport coefficient extract mean free path. $\tau \simeq 0.3$ fm (at 1.5–2T_c). SG Phys.Lett.B597:57-62,2004 Factor two larger than AdS/CFT. Close to Teany's analysis.
4. Is ideal hydro a good appproximation? Yes if $\tau/L \ll 1$. In the early stages of the fireball when elliptic flow is being set up $\tau/L > 0.1$. Flow lines smeared over 10% of fireball volume, material 10% inside the fireball can leave without taking part in further dynamics. So effects expected on elliptic flow and Bjorken estimate of energy density.

The equation of state



1. Pressure is continuous.
2. At first order transition energy density jumps: latent heat.
3. At cross over, the energy density is continuous, but C_V and $1/C_s^2$ may peak.

Gavai, SG, Mukherjee, hep-lat/0506015: Swagato's talk.

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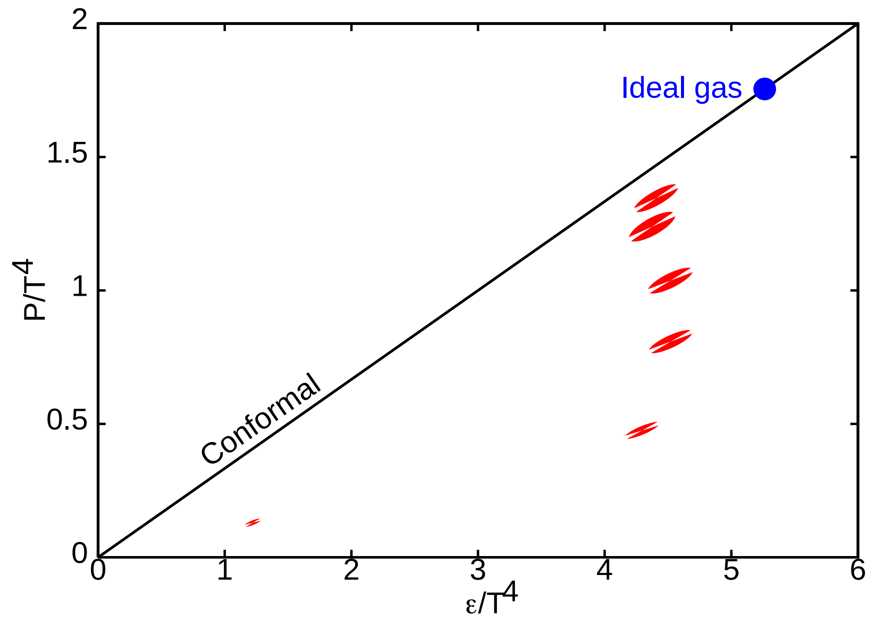

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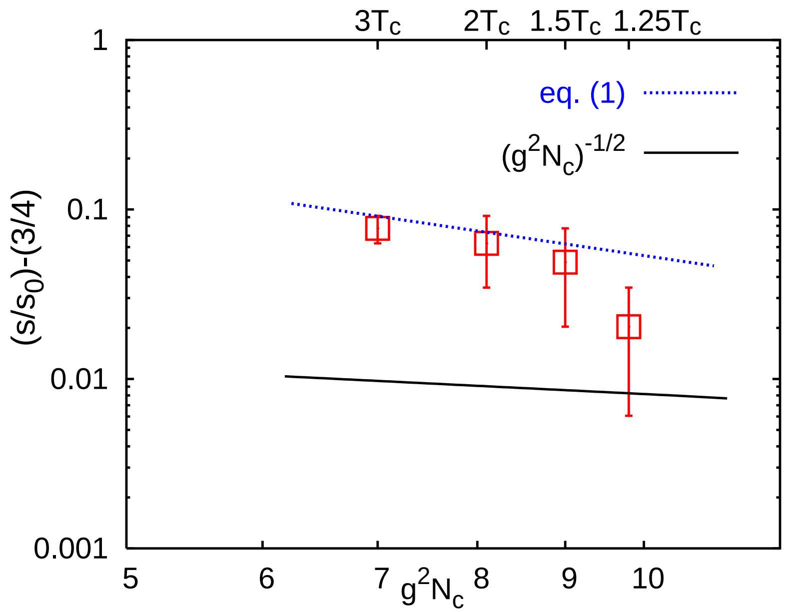
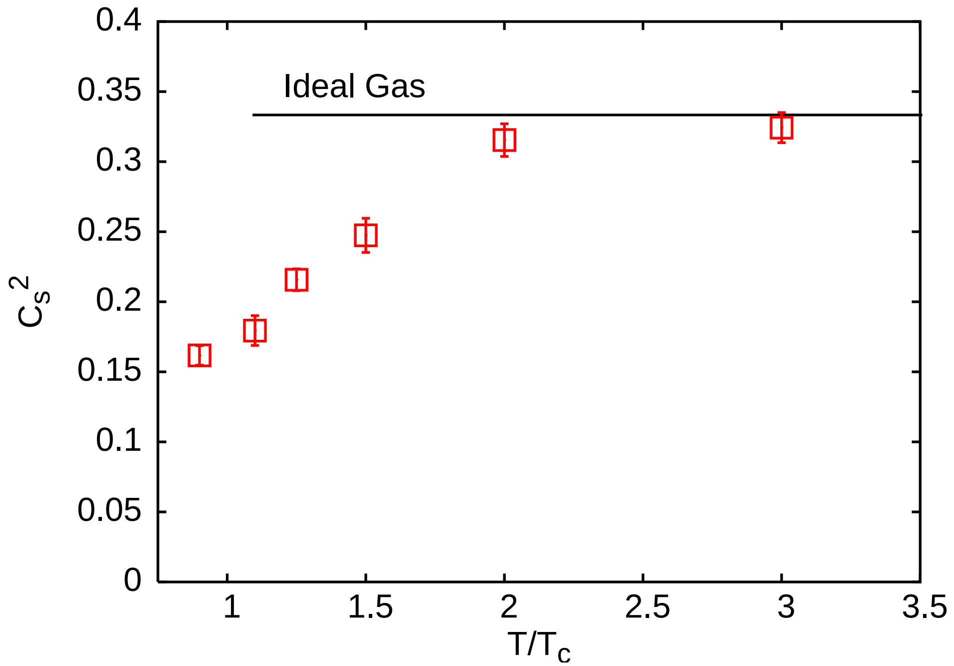
Conformal symmetry



$$\frac{s}{s_0} - \frac{3}{4} = \frac{45}{32}\zeta(3)(2g_{t'h})^{-3/2} + \cdots$$

Gubser, Klebanov, Tseytlin, NP B 534 (1998) 202. GGM hep-lat/0506015

The speed of sound



Gavai, SG, Mukherjee, hep-lat/0506015: Swagato's talk.

Summary: macroscopic physics

1. Equation of state usually computed using an integral method: requires input of pressure at some temperature. Specific heat and speed of sound can be obtained through numerical differentiation. Conformal limit is approached if $(E-3P)/E$ goes to zero. Input to the hydro equations.

2. A modified operator method of computing EOS suggested. C_V and C_s^2 can be computed as operator expectation values.

3. Is equilibrium reached? Can ideal hydro be used? Even if the mean free path is as short as suggested by the AdS/CFT limit, the Knudsen number $\tau/L \approx 0.1$, suggesting slow approach to equilibrium and departure from ideal hydro.

4. Non-ideal hydro required. Lattice must give more than the EOS. First studies have started.

Take home message

1. For $T > 1.25T_c$ the plasma consists of quarks and gluons. For $T > 2T_c$ it may be possible to deal with some aspects in weak coupling.

2. For $T \approx T_c$ our understanding is still incomplete, but some systematics are emerging.

3. If the fireball created in RHIC obeys ideal hydro, then only the EOS is required from the lattice. This is now only a matter of computing power.

4. First indications (from lattice, analysis of RHIC data and models) are that the mean free path length is almost comparable to initial fireball size, and hence out of equilibrium dynamics needs to be considered. More work on the lattice is called for.