The nature of the QCD plasma

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QCD at finite density: ECT* Trento

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The phase diagram, quasi-particles, constructing the effective theory.

Collaborators: Datta, Gavai, Lacaze, Mukherjee, Ray

Plan

1. Quantum number susceptibilities, the Taylor expansion, and various scenarios which limit the expansion.

2. Systematics of QNS/NLS— behaviour in the vicinity of T_c and mass dependence; constraints on the effective theory.

3. Quasiparticle excitations in the plasma phase of QCD, i.e., the fields in terms of which a simple effective theory may be written.

Quantum number susceptibilties (QNS)

N_f chemical potentials in QCD: μ_u, μ_d, etc.. Sometimes need fluctuations in some other conserved quantum number corresponding to a global symmetry of QCD. Generating functional is unchanged—

$$J = \mu^T \mathcal{N} = \mu^T M^{-1} M \mathcal{N} = [(M^T)^{-1}\mu]^T [M\mathcal{N}].$$

where μ is a column vector of (quark) chemical potentials and $\mathcal{N}$ is a column vector of quark number densities. Then

$$\frac{\partial}{\partial \mu_i} = \frac{\partial \mu_f}{\partial \mu_i}\frac{\partial}{\partial \mu_f} = (M^T)_{fi}\frac{\partial}{\partial \mu_f} = M_{if}\frac{\partial}{\partial \mu_f}.$$

In particular, for $N_f = 2$,

$$\chi_B = \frac{2}{9}\{\chi_{20} + \chi_{11}\}$$

Gavai and SG, Phys. Rev. D 73 (2006) 014004

The Taylor expansion for 2 flavours

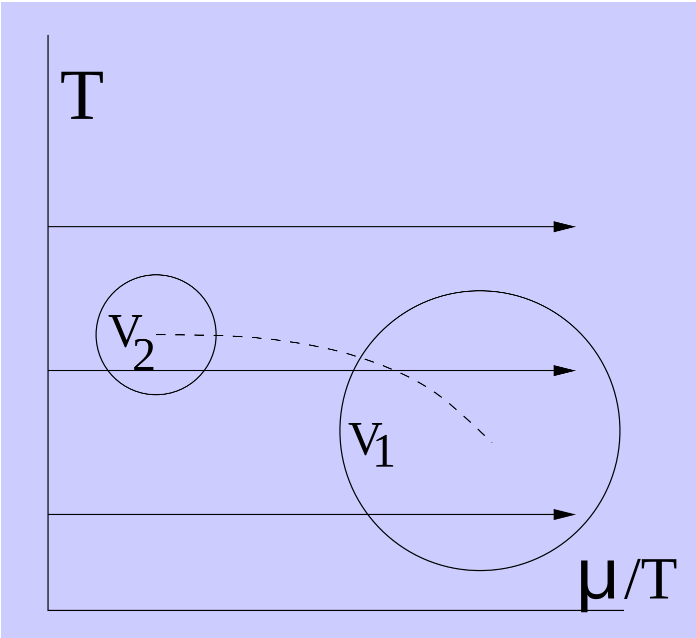
$$P(T,\mu_u,\mu_d) = \left(\frac{T}{V}\right)\log Z(T,\mu_u,\mu_d)$$

$$P(T,\mu_u,\mu_d) = P(T,0,0) + \sum_{n_u,n_d} \chi_{n_u,n_d}\,\frac{\mu_u^{n_u}}{n_u!}\,\frac{\mu_d^{n_d}}{n_d!}$$

$m_u = m_d$ implies that $\chi_{n_u,n_d} = \chi_{n_d,n_u}$, for any $\mu_u = \mu_d$. One QNS is

$$\chi_B(T,\mu_B) = \left.\frac{\partial^2 P(T,\mu_u,\mu_d)}{\partial\mu_B^2}\right|_{\mu_u=\mu_d=\mu_B/3}$$

$\chi_B(T^E,\mu_B^E)$ diverges in the infinite volume limit: pseudo critical behaviour at finite volumes.



Taylor series breakdown

The Taylor series breaks down at distances where some obstruction is encountered.

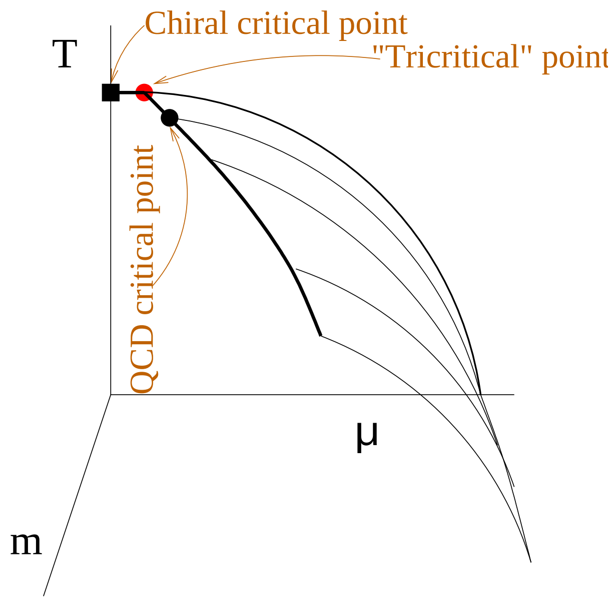
1. Imaginary chemical potential— series coefficients of the expansion in powers of μ_B will alternate in sign.

2. Isospin chemical potential— dependence on quark mass; effects should switch on rapidly as the quark mass decreases. Quadratic response coefficients (QRC): see talk by Ray. Examine the extended phase diagram.

3. Other possibilities when $m_u \neq m_d$.

Find the full phase diagram for $N_f = 2$. There are 5 parameters: T, m_u, m_d, μ_u and μ_d. Gibbs phase rule implies varieties of multicritical behaviour.

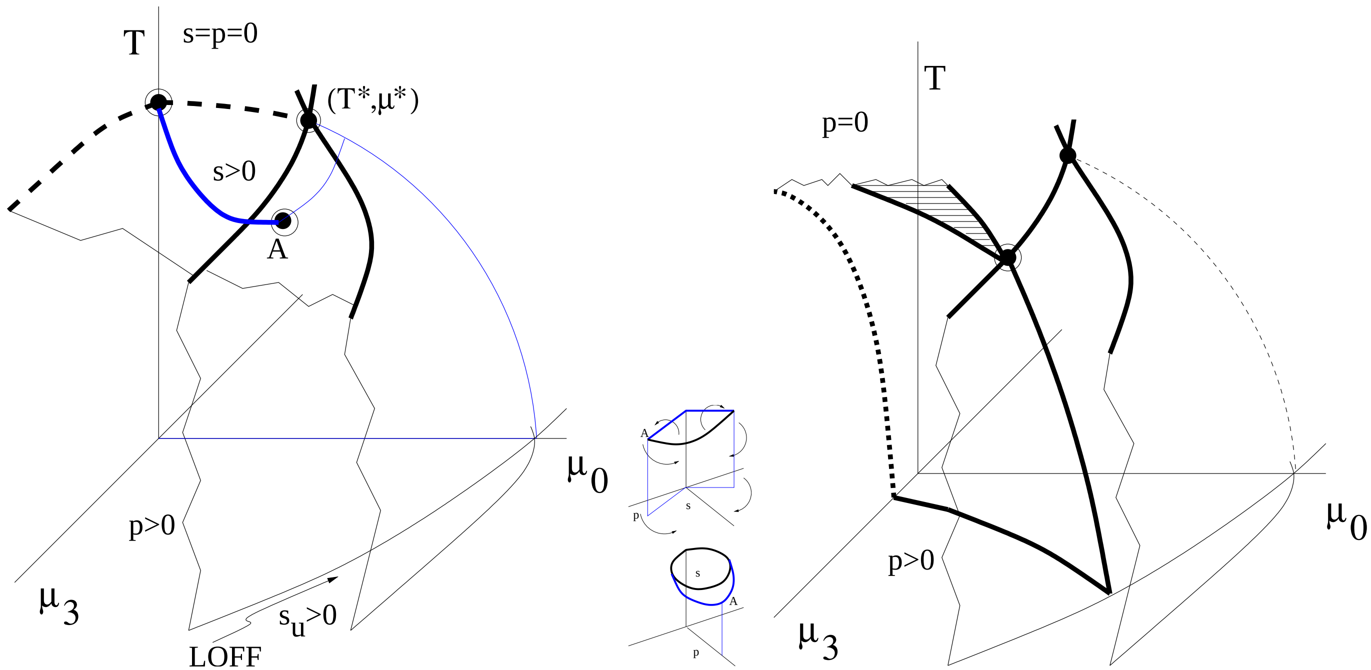
The restricted phase diagram



Berges and Rajagopal, Halasz, Jackson, Schrock, Stephanov and Verbaarschot: 1998

Part of the phase diagram restricted to symmetric 3-d parameter space with $m_u = m_d = m$ and $\mu_u = \mu_d = \mu_B/3$. Order parameter is $\langle\overline{\psi}\psi\rangle$. Other interestingly ordered phases at larger μ.

Completed phase diagram



Extending work by Son and Stephanov (2000) Klein, Toublan and Verbaarschot (2003) Nishida (2003) Barducci, Casalbuoni, Pettini and Ravagli (2004).

(T^*, μ^*) is a tetra-critical point (tri-critical if one considers the highly symmetric case $m_u = m_d$ and $\mu_u = \mu_d$). **A** joins a penta-critical point for arbitrary $m_{u,d}$.

Current estimates

Current estimates are— $T^E/m_\rho \approx 0.17$, $\mu^E/m_\rho \approx 0.19$

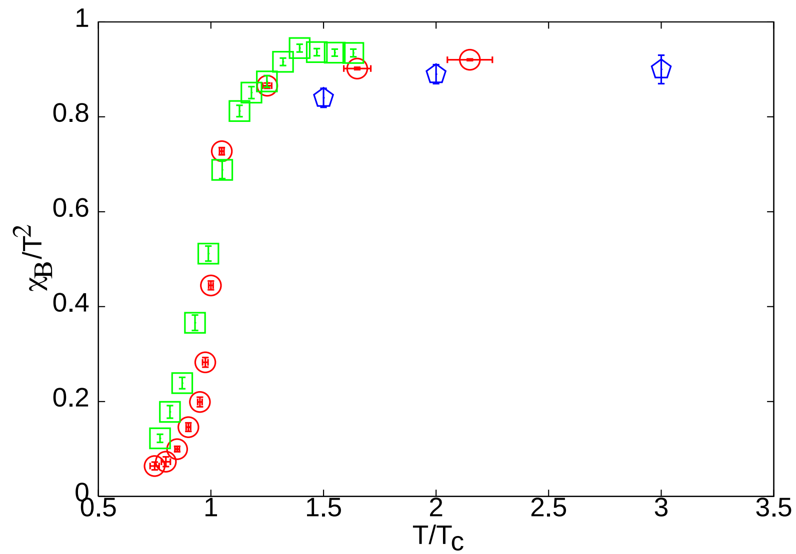
1. The transition surface for pion condensation "leans back" in model computations. Casalbuoni et al, Phys. Rev. D 69 (2004) 096004. In agreement with this QRC's indicate that mass dependence in the μ_3 direction is larger in magnitude SG and Ray, Phys. Rev. D D70 (2004) 114015

2. Since $m_\pi/m_\rho = 0.31$ (for this computation), we find $T^E/m_\pi \approx 0.57$ and $\mu^E/(3m_\pi) \approx 0.21$ Pion condensation expected for $\mu_3^c/m_\pi \geq 1$ at finite T. Even better for Bi-Sw, since they have larger m_π.

3. Chiral limit may be approached non-analytically. Argument worth following up.

4. At μ^E, the Taylor coefficients of χ_B all have the same sign.

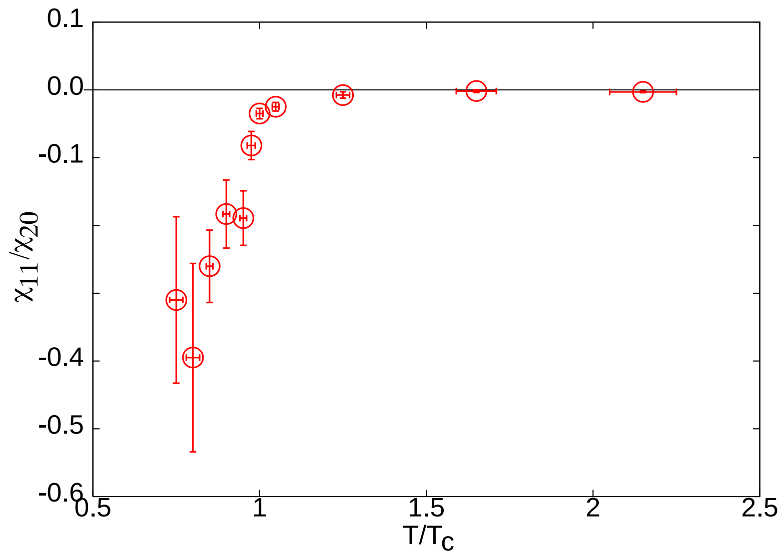
Fluctuations



Quenched continuum (Mumbai), MILC $a = 1/8T$, $N_f = 2$, Mumbai $N_f = 2$ continuum extrapolation following quenched

Weak coupling expansion works above $2T_c$: Blaizot, Iancu and Rebhan, Vuorinen, Chakraborty, Mustafa and Thoma.

The susceptibilities: sign fluctuations



χ_{20} measures the spread of quark numbers in the real direction, and χ_{11} its spread in the imaginary direction. Ratio χ_{11}/χ_{20} measures severity of the sign problem.

In the region of interest, the sign problem is under reasonable control. Ambiguities due to square root? Investigate QRCs of $\langle\overline{\psi}\psi\rangle$, for example.

Golterman, Shamir and Svetitsky, hep-lat/0602026

Non-linear susceptibilties

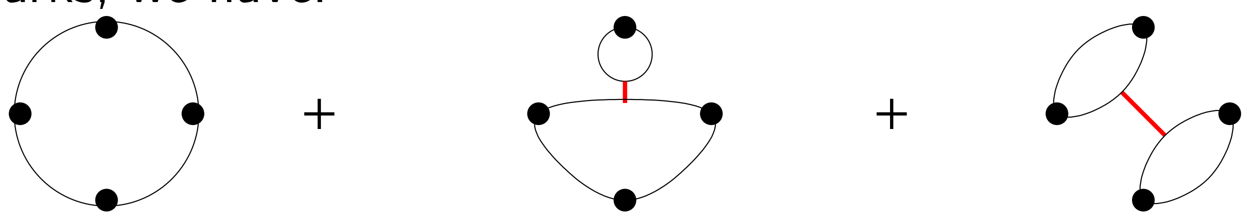
A non-linear quark number susceptibility of order n is the derivative

$$\chi_B^{(n)} = \frac{\partial^n P}{\partial \mu_B^n} \qquad \text{and} \qquad \chi_{n_u,n_d} = \frac{\partial^{n_u+n_d} P}{\partial \mu_u^{n_u} \partial \mu_d^{n_d}}$$

Now,

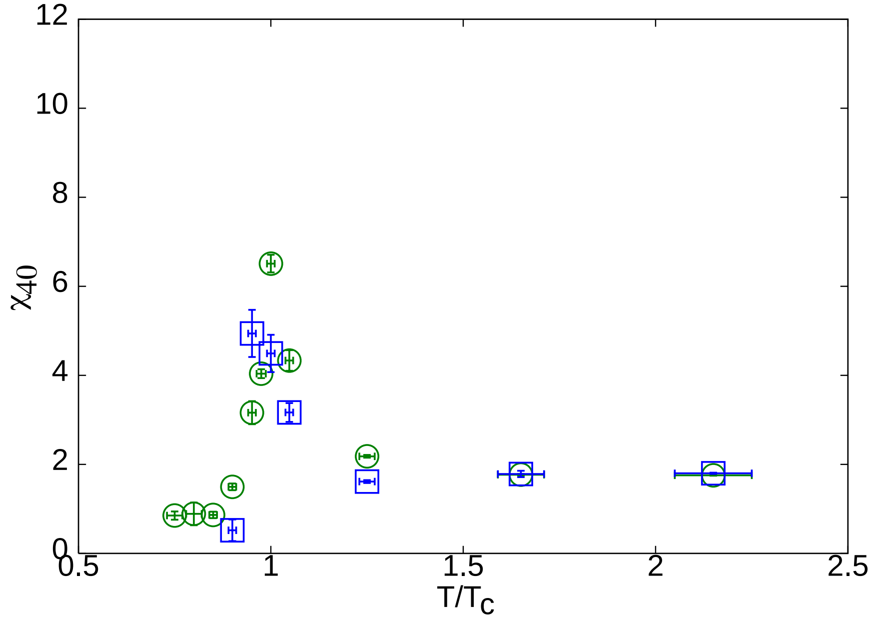
$$\chi_B^{(4)} = \frac{1}{2} \left[\chi_{40} + 2\chi_{31} + \chi_{22}\right]$$

In terms of quarks, we have:



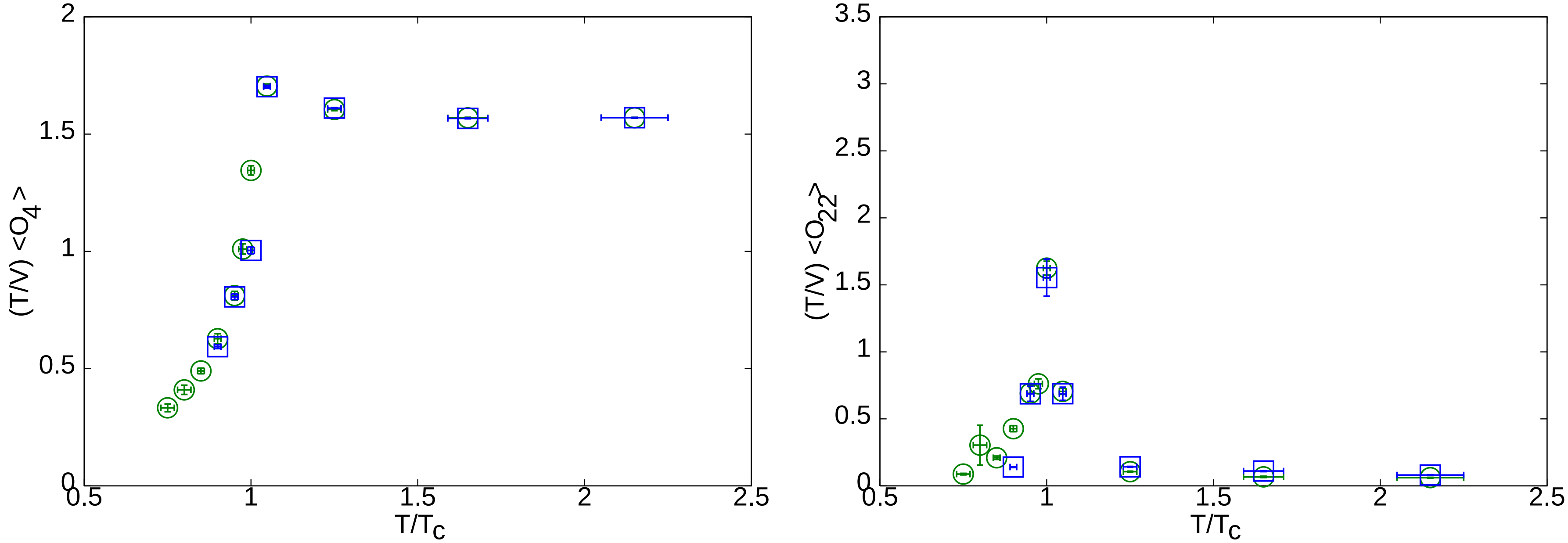
Flavour disconnected pieces are absent for free quarks, i.e., vanish when $g \to 0$. In a resonance gas one has vertices such as $\pi^+ + \pi^- \to \pi^0 + \pi^0$ which allow the flavour disconnected pieces to add up.

Peaking at T_c



Peak in $\chi_B^{(4)}$ implies decrease in radius of convergence $r_{2/4} = \sqrt{\chi_B^{(2)}/\chi_B^{(4)}}$.

Resolving the peak

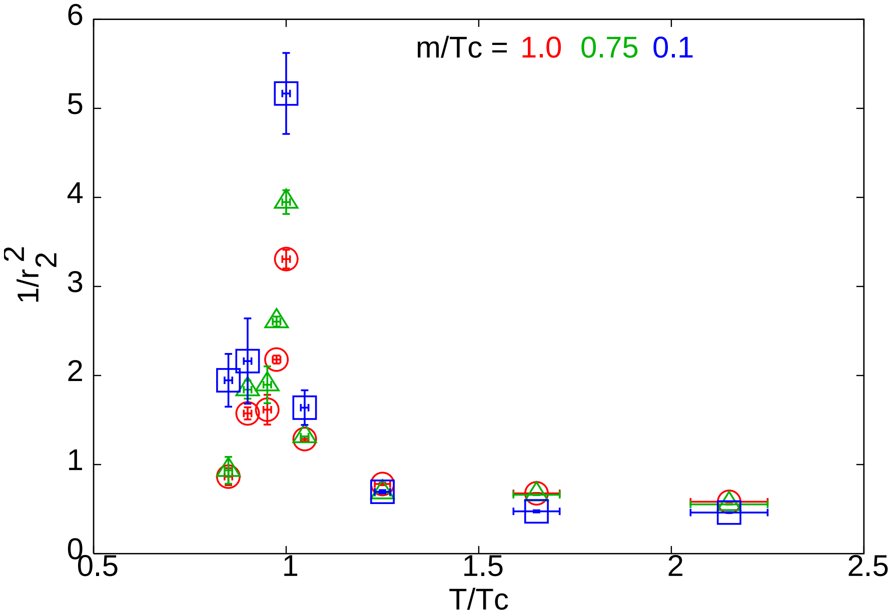


Peaks in other susceptibilties also found very close to T_c. Always in O_{22}, O_{222}, O_{2222}— i.e., quark line disconnected operators $(\mathrm{Tr}\,\gamma_0 D \gamma_0 D)^n$.

R. V. Gavai and SG, PRD 72 (2005) 054007

Quark mass dependence

Peak in χ_{40} etc., implies decreasing radius of convergence. The radius of convergence seems to be very sensitive to quark mass in some region of T.



With increasing order larger there is a slower approach to the infinite volume limit, and a threshold $Lm_\pi \approx 5$ is needed to study the thermodynamic limit.

Quasiparticles: linkage of quantum numbers

Since there are many conserved quantum numbers the problem becomes simpler. Look at two quantum numbers simultaneously— say U and D.

$T < T_c$: whenever $U = 1$ is excited $D = -1$ is excited along with it.

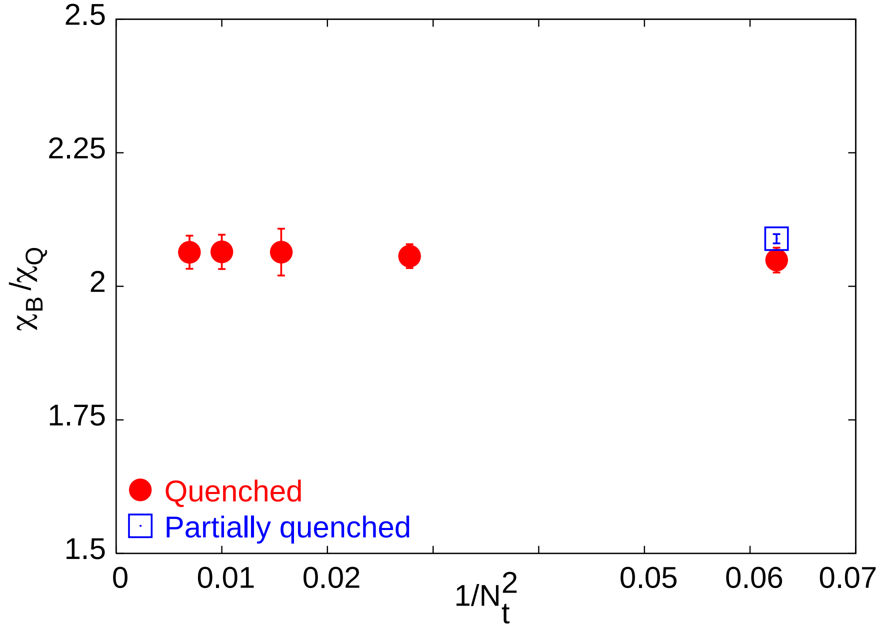
$T > T_c$: when $U = 1$ is excited $D = \pm 1$ should be excited along with it if the medium contains quarks. Otherwise, by observing what value of D is preferentially excited, you find something about the quantum numbers of the excitations.

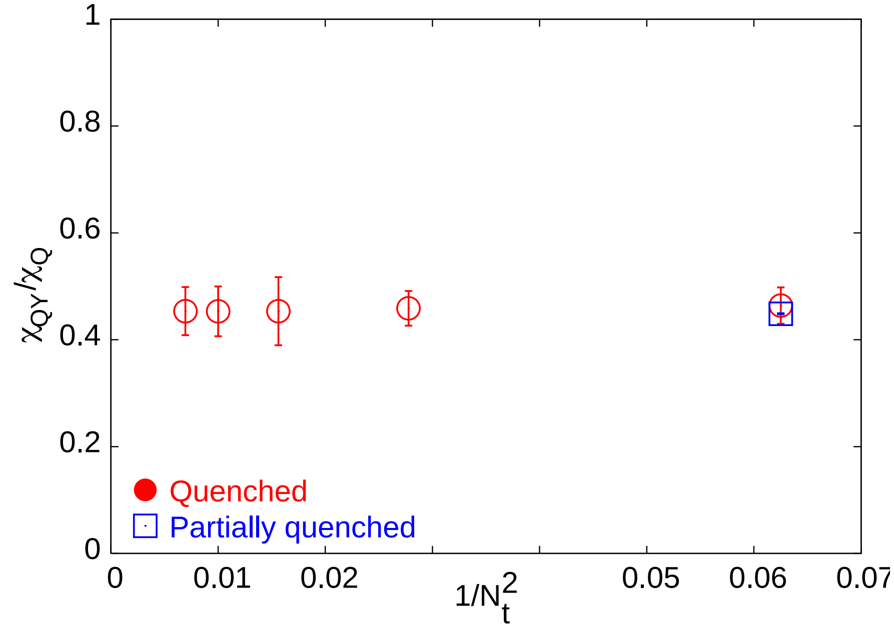
Similarly one could study the linkages $U|B$ or $U|Q$, or $D|B$ etc.

$$C_{(XY)/Y} \equiv \frac{\langle XY \rangle - \langle X \rangle \langle Y \rangle}{\langle Y^2 \rangle - \langle Y \rangle^2} = \frac{\chi_{XY}}{\chi_Y}$$

Gavai and SG, PRD 73 (2006) 014004

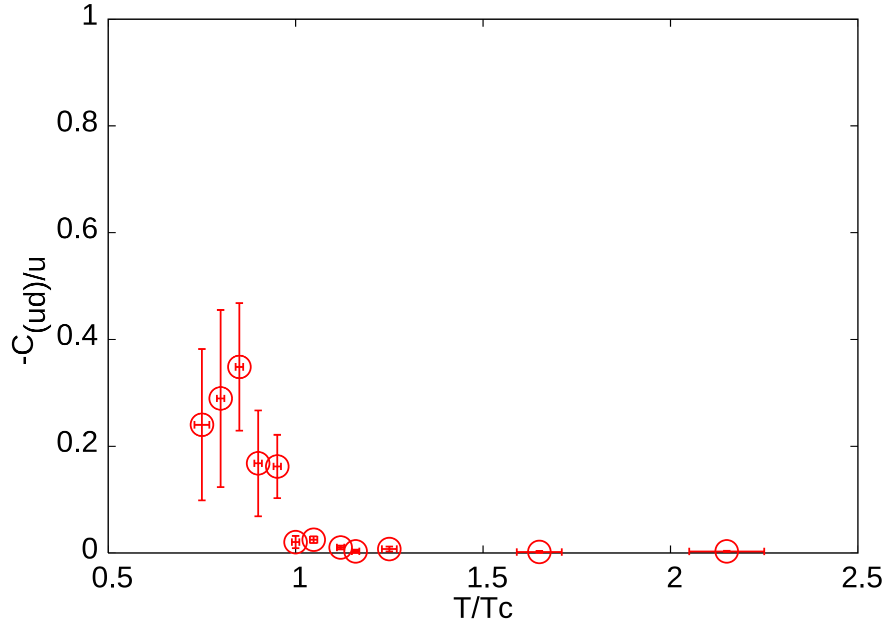
Linkage is robust





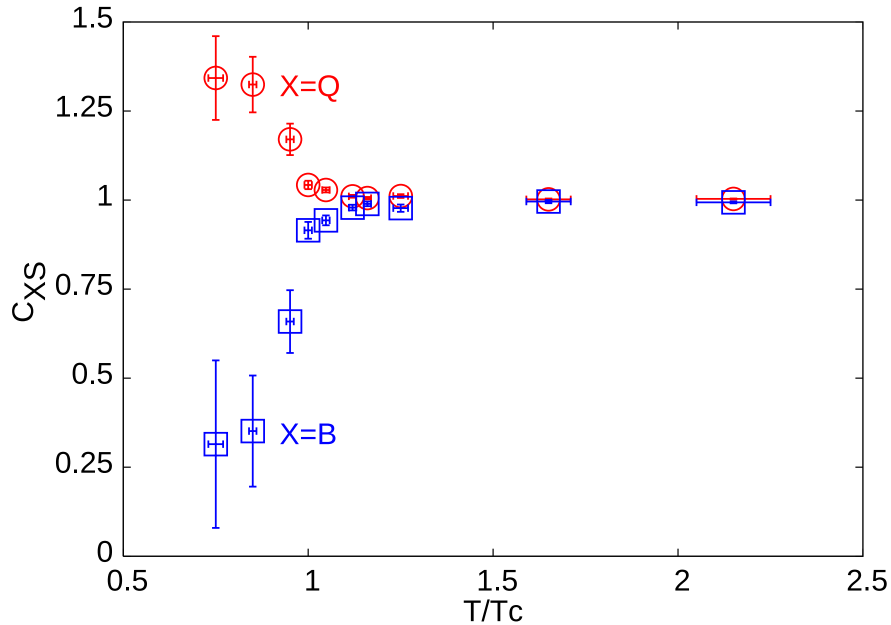
Above T_c ratios of QNS are almost independent of lattice spacing, and insensitive to quark masses (as long as $m < T$). Therefore linkage is a robust quantity above T_c.

U and D are not linked



u and $\bar{d}$ can be carried by the same particle below T_c but not above T_c.

Strangeness is carried by quarks



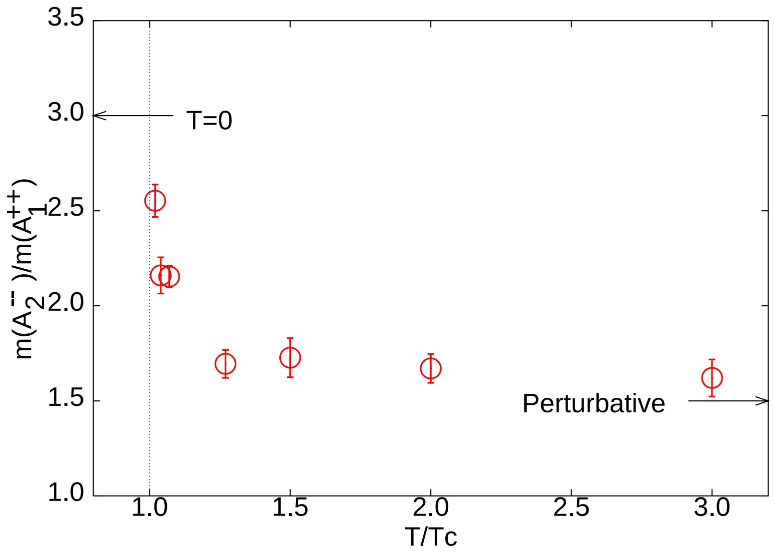
$C_{BS} = -3C_{(BS)|S}$ and $C_{QS} = 3C_{(QS)|S}$. Below T_c strange baryons are relatively heavy and therefore sparse in the plasma, but kaons are not so heavy. Above T_c: strange quarks.

Gavai and SG, Koch, Majumder, Randrup, PRL 95 (2005) 182301

Gluon screening masses

Since gluons do not carry any global quantum numbers, linkage cannot be used. Investigated screening masses of colour singlets: sometimes called "glueball" correlations.

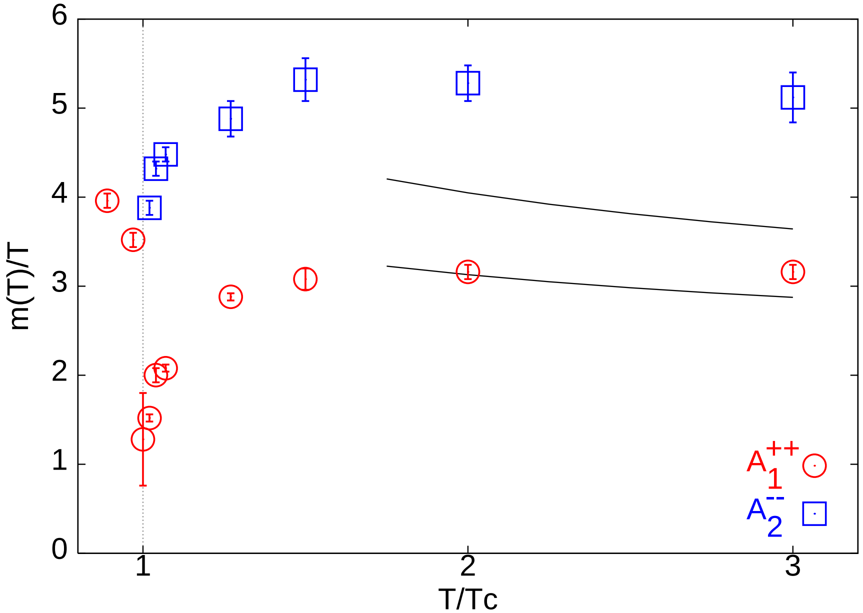
Ratio of the two masses provides a test of whether weakly coupled gluons is a valid picture. Less affected by scale uncertainties if g is small enough, since $m/m' \propto 3/2 + \cdots$.



Datta and SG, Phys.Rev.D67 (2003) 054503

...but not perturbation theory

Electric gluons at finite temperature have lattice quantum number 1^{--} and colour quantum numbers. Colour singlet two gluon states have lattice quantum number 0^{++}. Colour singlet three gluon states have lattice quantum number 1^{--}.



Band includes scale uncertainty; non-perturbative contributions crucial. Helsinki group

Different effective theories

Many phases, many transitions, many effective theories ...

Near the chiral critical point, quarks cannot be integrated out of the theory. The effective theory at this point is expected to be a bosonized theory involving the pseudo-Goldstone modes. This is true whenever the quark mass is such that pions are lighter than the lightest glueball.

Near the critical end point quarks are light degrees of freedom but the QCD coupling is large. (Weak coupling computations of CEP are quantitatively inadequate, as they are also for pressure, glue screening masses)

At very large temperature the Euclidean thermal field theory is expected to contain Fermion modes with $\omega \approx \pi T$, and these modes can be integrated away, leaving a dimensionally reduced theory with gauge fields (magnetic modes) coupled to adjoint Higgs (electric modes).