Fluctuations of conserved quantities

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From Strings to LHC II

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General conditions in heavy-ion collisions

- In a heavy-ion collision, almost all quantum numbers pass through without collision: valence quarks are fast particles, QCD is asymptotically free.
- When $\sqrt{S}$ is large, the central rapidity region contains significant energy with almost no flavour quantum numbers: due essentially to soft gluon and sea quark component of hadron wavefunction.
- This fireball expands and cools: strong evidence for collective (final state) effects in measurements of elliptic flow and jet quenching.
- Is there a QCD plasma formed in this region? A quantitative question: what are the properties of the plasma? Was this cup of water ever brought to boil?

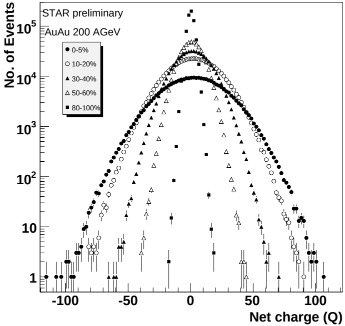
Ensembles

- **Event ensembles at colliders**: defined through various selection criteria. Values of conserved quantities can fluctuate from one event to another
- **Thermodynamic ensembles**: defined by letting certain conserved quantities fluctuate (canonical) while keeping others fixed (micro-canonical).
- **Mapping between the two**: The system is a small part of a big (ion-ion) collider event, the heat-bath is the remainder. Is the event a thermostat? Need experimental check.

Is a collider event a thermostat?

- **Energy average over events**: m_T stands in for energy, and has an exponential distribution. No other measurement of temperature performed yet. Jets at LHC could measure temperature.
- **Other conserved quantities**: If there are spatial gradients of densities, then not a thermostat. No evidence for a spatial gradient. Diffusion time could be much smaller than the lifetime of fireball. (Lattice, toy models: some "solvable" using AdS/CFT)

STAR Collaboration



Distribution of net charge is Gaussian ($p_T < 1$ GeV, central rapidity slice).

Nayak (STAR) nucl-ex/0608021

Susceptibilities

If $P(T, \mu, \cdots)$ is the pressure, then a susceptibility is

$$\chi = \frac{\partial n}{\partial \mu}, \qquad \text{where} \qquad n = \frac{\partial P}{\partial \mu}$$

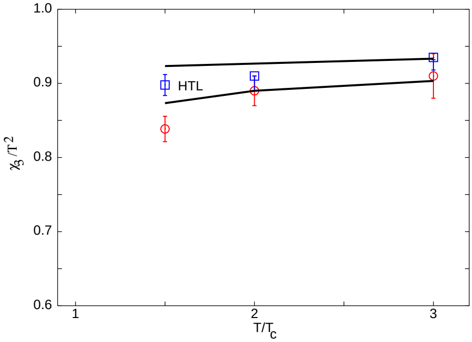
Gottlieb, Liu, Toussaint, Renken, Sugar, Phys.Rev.Lett. 59, 2247, 1987.

For each conserved charge one chemical potential: μ_B, μ_Q, μ_S *etc.*. Two kinds of susceptibilities

$$\chi_i = \frac{\partial^2 P}{\partial \mu_i^2} \qquad\qquad \chi_{ij} = \frac{\partial^2 P}{\partial \mu_i \partial \mu_j}$$

diagonal and off-diagonal. Higher derivatives are called non-linear susceptibilities. Gavai and SG, Phys.Rev. D68, 034506, 2003.

Diagonal susceptibilities



In the high temperature phase of QCD the diagonal susceptibilities (but not the off-diagonal) agree with a resummed weak coupling computation.
Blaizot, Iancu, Rebhan, Phys.Lett. B523, 143, 2001
Gavai and SG Phys.Rev. D67, 034501, 2003

Thermodynamic fluctuations

$$P(Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q}\right), \quad \text{so} \quad \langle \Delta Q^2 \rangle = VT\chi_Q$$

Bias free experimental measurement of χ_Q possible.
Asakawa, Heinz and Muller, Phys.Rev.Lett. 85, 2072, 2000;
Jeon and Koch, Phys.Rev.Lett. 85, 2076, 2000.

$$P(B,Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q} - \frac{B^2}{2VT\chi_B} - \frac{QB}{VT\chi_{BQ}}\right)$$

Variances determined by diagonal susceptibilities, covariances by off-diagonal susceptibilities.

What do fluctuations measure?

In thermodynamics

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At colliders

Many sources of fluctuations: incomplete characterization of event, beam energy fluctuations, ion charge fluctuations, volume, detector noise: all are Gaussian. Hard to disentangle from thermodynamics without proper event characterization.

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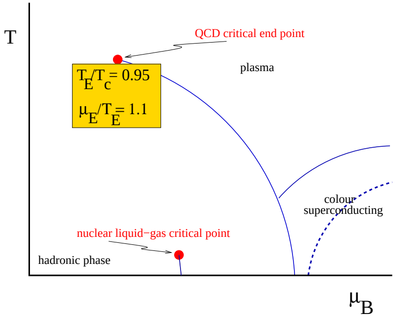
- Useful in the context of multi-particle hadron dynamics where one wants to understand correlations between particles.
- No use in thermodynamics, where correlations are irrelevant: no matter what the dynamics is (almost) always get a Gaussian. Physics is in the width of the Gaussian.
- Either find non-Gaussian thermodynamics or eliminate dependence on uncontrolled variables.

Use non-Gaussian fluctuations

Phase diagram of QCD



At a critical point fluctuations are non-gaussian.
Stephanov, Rajagopal, Shuryak, Phys.Rev. D60, 114028, 1999

Sign problem

Monte Carlo

$\mathrm{Det}\ D$ is real if $QDQ^\dagger = D^\dagger$ for unitary Q. $Q = \gamma_5$ for the Dirac operator (free and minimally coupled to a gauge field). Hence Monte Carlo evaluation of partition function possible.

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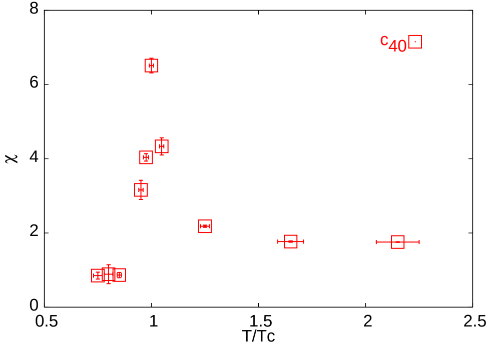
Taylor expansion

Phase diagram explored using Taylor expansion of free energy around $\mu = 0$. Taylor coefficients are non-linear susceptibilities. Taylor series fails to converge at the phase boundary: non-gaussian fluctuations. Gavai and SG, Phys.Rev. D71, 114014, 2005

Non-Gausssianity

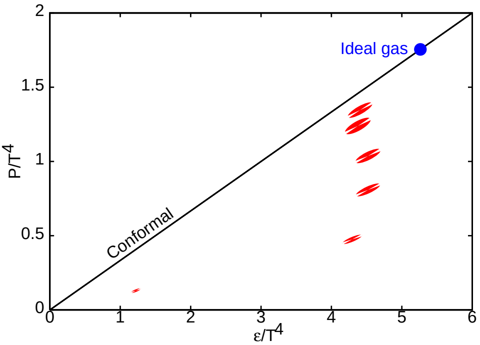


Gavai, SG, Phys.Rev. D72, 054006, 2005

Strong peak in the fourth order susceptibility near T_c signals strongly non-Gaussian fluctuations. Residual non-Gaussian behaviour in the high-temperature phase: true even for an ideal gas.

Eliminate uncontrolled variables

Equation of state of hot matter



Gavai, SG, Mukherjee, Phys.Rev. D71, 074013, 2005

The EOS of an ideal gas says how many degrees of freedom there are. In an interacting gas?

Linkage

- **Linkage** between two quantum numbers:

$$C_{KL/L} = \frac{\langle KL\rangle - \langle K\rangle\langle L\rangle}{\langle L^2\rangle - \langle L\rangle^2} = \frac{\chi_{KL}}{\chi_L}$$

- **Example**: $C_{BS/S}$ is zero at low temperatures, because the lightest strange particle is the K, which has no baryon number. It is -1/3 for quarks because $S = 1$ goes with $B = -1/3$.
- **Robust**: systematic lattice effects cancel out in the ratio, systematic experimental effects also expected to reduced tremendously in the ratio: factors of VT cancel.

Quarks in matter

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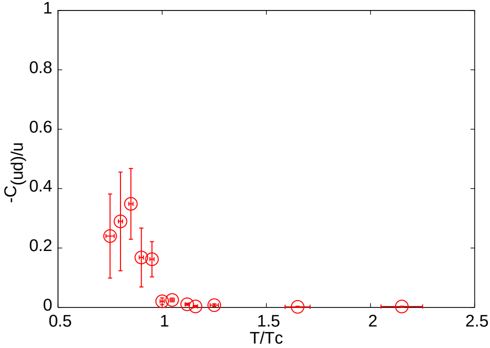
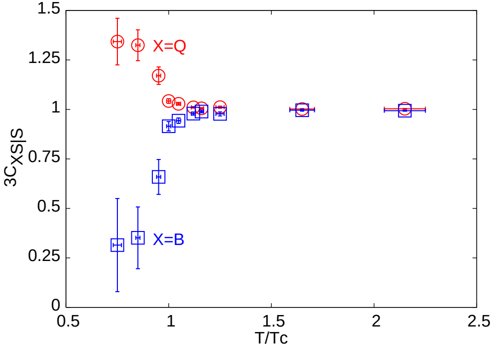
Outline
Introduction
Fluctuations
Critical end point
Linkage
Summary



Flavours are decoupled from each other: up and down shown, others similiar. Gavai and SG, Phys.Rev. D73, 014004, 2006

Quarks in matter



Strangeness is carried by quarks. Similarly for up and down flavours. Gavai and SG, Phys.Rev. D73, 014004, 2006

At the LHC

- Measurement of linkage possible in principle since the factors of VT in two fluctuation measures cancel out. Need to characterize event volume and centrality in greater detail. (Possible: STAR preliminary)
- Influence of hadronization mechanism remains to be studied. Flow measurements (v_2) throw light on this: quark coalescence dynamics seems to determine v_2.
- Hadronization will definitely disrupt certain linkages: for example, $C_{BS|S}$. However, if quark coalescence works then $C_{QS|S}$ will not be disrupted. Such off-equilibrium linkages between flavour quantum numbers can be frozen in at the freezeout time.

Summary

Non-Gaussian fluctuations

Non-Gaussian fluctuations are visible near a phase transition. The QCD critical point can be studied through the energy ($\sqrt{S}$) dependence of measures such as the kurtosis (Binder cumulant) of the event-to-event baryon number fluctuations.

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Linkage between fluctuations

Some of the uncertainties in going from observed fluctuations to QCD predictions cancel out in a robust measure called Linkage. Some linkages seem to persist during the hadronization of the fireball and can be directly used to look for quark-like quasiparticles in the plasma.