New results in QCD at finite μ

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ILGTI: TIFR

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Outline

Crawling towards the continuum

- Before this year: state of the art lattice computations of physics at finite chemical potential used lattice cutoff $\Lambda = 4T \simeq 800$ MeV near T_c.
- Our earlier computation used $m_\pi \simeq 230$ MeV and spatial sizes with $LT = 2$, 3, 4 and 6. This enabled extrapolation to the thermodynamic limit, *i.e.*, $L \to \infty$.
- Now: new computations with $\Lambda = 6T \simeq 1200$ MeV near T_c.
- m_π remains unchanged (230 MeV), but spatial volumes are somewhat smaller ($LT = 2$, 3 and 4). No extrapolation to $L \to \infty$ yet.
- 20000–50000 configurations at each coupling; stochastic determination of traces with 500 random vectors on each configuration. (Gavai and SG, Phys. Rev. D 68, 2003, 034506.)

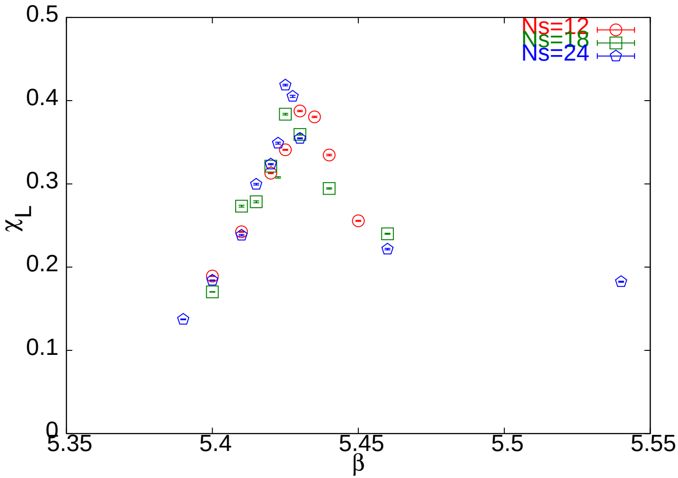
Algorithmic issues

- For $N_f = 2$ simulations with staggered quarks we used R-algorithm. Most runs used trajectory length of 1 MD time unit and $\delta T = 0.01$.
- Test case: $m/T_c = 0.1$, 6×24^3 lattice. Changed δT from 0.01 to 0.001. No change in bulk quantities: plaquettes, Re L, quark condensate.
- For same test case, changed trajectory length from 1 MD time unit to 3 MD time units. No change in bulk quantities, but with longer trajectories autocorrelation lengths decreased so that CPU time taken for generating decorrelated configs decreased.

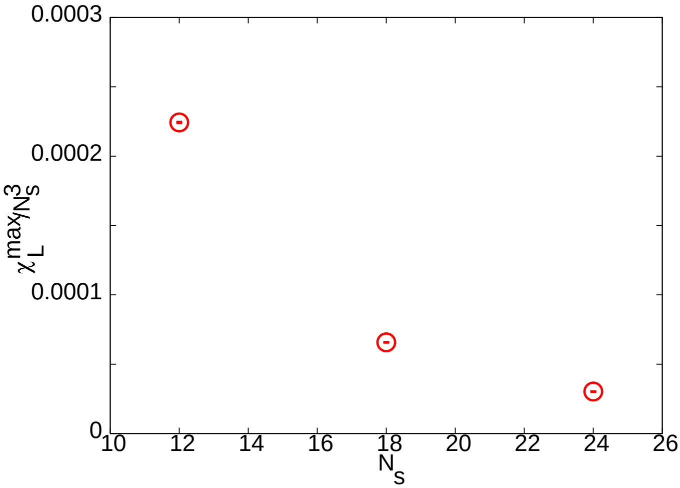
Locating the finite temperature cross over

- Cross-over coupling monitored using Polyakov Loop susceptibility: χ_L, an operator which enters fourth-order QNS: $(T/V)\langle O_{22}\rangle_c$, and an operator which enters eighth-order QNS: $(T/V)\langle O_{44}\rangle_c$. Measures consistent with each other within the precision of this work.
- For $m/T_c = 0.1$, we find $\beta_c = 5.425(5)$. Previous results bracketed this: for $m/T_c = 0.15$ one had $\beta_c = 5.438(40)$ (Gottlieb et al, PRL 59, 1987, 1513) and for $m/T_c = 0.075$ it was found that $\beta_c = 5.41$–5.43 (Bernard et al, PR D 45, 1992, 3854).
- Transition is not first order. Computations at larger volumes are required to distinguish cross over from second order transition.

Polyakov loop susceptibility



Not first order



Outline

What is a QNS?

Taylor coefficient of the pressure in $N_f = 2$ QCD is

$$P(T, \mu_u, \mu_d) = \sum_{n_u, n_d} \frac{1}{n_u! n_d!} \chi_{n_u, n_d}(T) \mu_u^{n_u} \mu_d^{n_d},$$

and, since the two quark flavours are degenerate, $\chi_{n_u,n_d} = \chi_{n_d,n_u}$.
Diagonal QNS have either $n_u = 0$ or $n_d = 0$.
In two flavour QCD trade $\mu_{u,d}$ for $\mu_{B,Q}$. Then

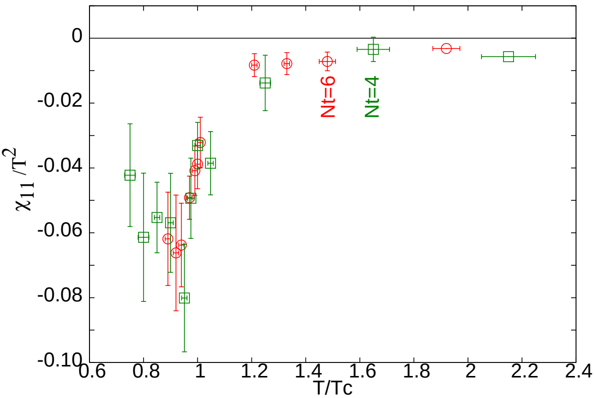
$$\chi_B = \frac{2}{9}(\chi_{20} + \chi_{11}) = 2\chi_{BQ}$$

$$\chi_Q = \frac{1}{9}(5\chi_{20} - 4\chi_{11}).$$

Transforming to μ_{B,I_3}, one has

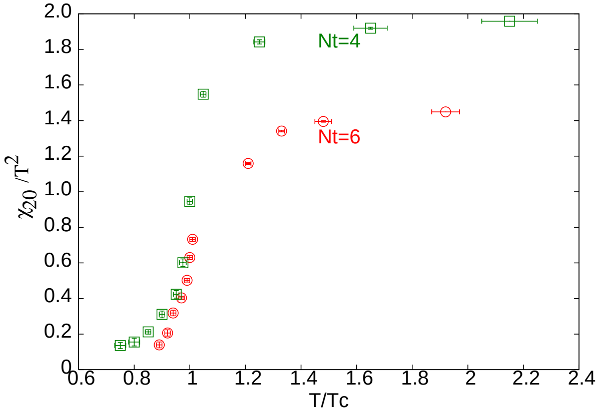
$$\chi_{BI_3} = 0, \qquad \chi_{I_3} = \frac{1}{2}(\chi_{20} - \chi_{11}).$$

Off-diagonal QNS



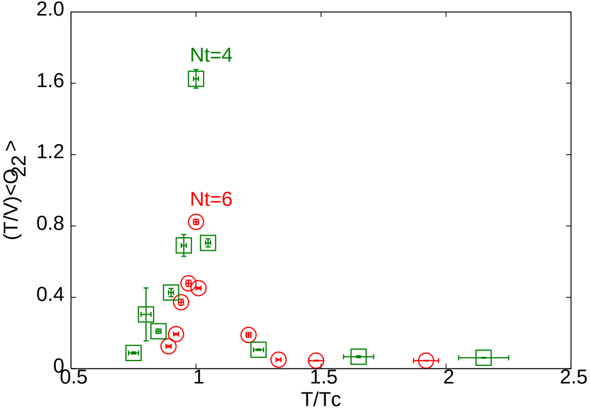
Sees only $\langle O_{11} \rangle$. No evidence for lattice spacing effects.

Diagonal QNS



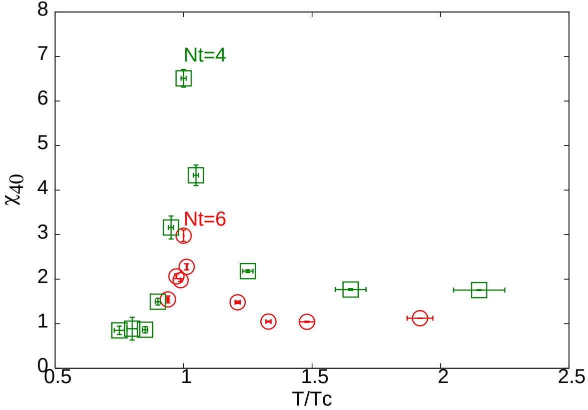
Sees $\langle O_{11} \rangle$ and $\langle O_2 \rangle$. Second expectation value is cutoff dependent. Also, has a cross over. We look at its susceptibility $\langle O_{22} \rangle_c$ to identify T_c.

"Susceptibility" of QNS: $\langle O_{22}\rangle_c$— 4th order QNS



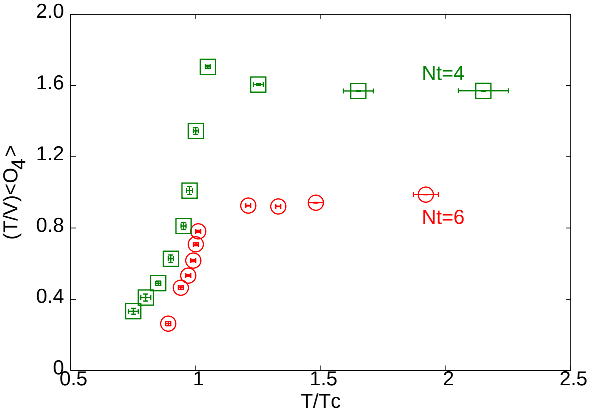
Peak at the same coupling as peak of χ_L. Within the 1% precision of T/T_c, the two quantities peak at the same coupling. See Gavai and Gupta, PR D72 (2005) 054006.

Diagonal fourth-order QNS



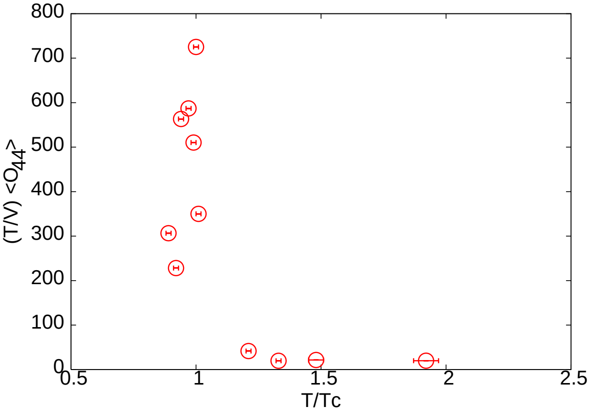
Non-zero for $T > T_c$. Has contribution from $\langle O_4 \rangle$, which has non-vanishing value for the ideal gas.

The operator O_4



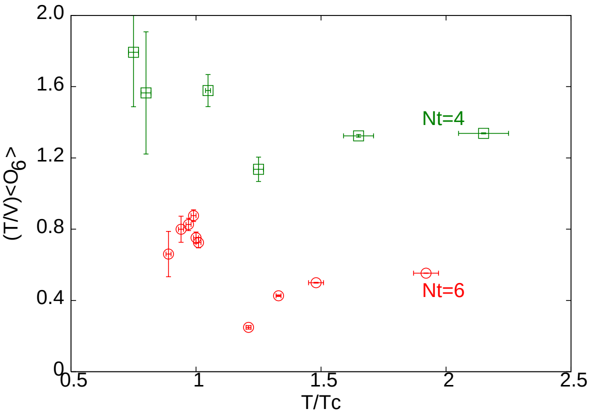
Rapid cross over from a small value in the hadronic phase to a non-vanishing value for the ideal gas.

"Susceptibility" of O_4: $\langle O_{44} \rangle_c$— 8th order QNS



This quantity peaks at the same coupling as χ_L and $\langle O_{22} \rangle_c$. Within the precision of our measurement there is no dependence of the cross over coupling on these observables.

The operator O_6— 6th order QNS



The operator expectation value $\langle O_6 \rangle$ has structure below T_c and hence its "susceptibility" cannot be used to probe the cross over coupling. Similar observation for $\langle O_8 \rangle$.

Outline

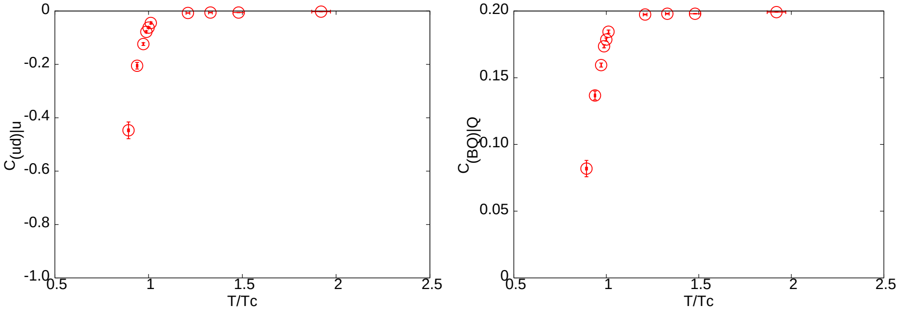
Linkage between quantum numbers

- Linkage between quantum numbers F and G is

$$C_{(FG)|F} = \frac{\chi_{FG}}{\chi_F} = C_{(GF)|F}.$$

  Measures amount of G excited per unit fluctuation in F. The quantities $C_{(FG)|F}$ and $C_{(FG)|G}$ not necessarily equal.
- $C_{(ud)|u} = -2/3$ at low temperature, since the pion is the lightest excitation, and at high temperature it vanishes. For $N_f = 2$ one has $C_{(ud)|u} = C_{(ud)|d}$.
- $C_{(BQ)|Q} = 0$ at low temperatures (pion is the lightest particle) and $1/5$ at high temperature.
- $C_{(BQ)|B} = 1/2$ at all temperatures by definition.

Results



Quick cross over from hadron gas behaviour to quark gas behaviour. Rounding seen close to T_c. Finite-size effects need to be investigated: $LT = 4$. Earlier computation with $N_t = 4$ saw less rounding with $LT = 6$.

Outline

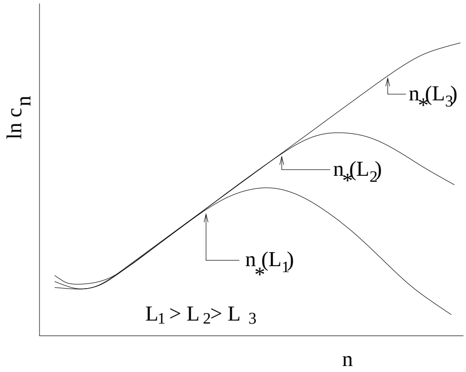
Finite size effects

- At critical point correlation length becomes infinite, appropriate susceptibilities diverge and free energy becomes singular ... in the infinite volume limit (van Hove's theorem).
- No numerical computation ever performed on infinite volumes.
- Deduce the existence of a critical point through extrapolations: finite size scaling (FSS) well developed for direct simulations.
- Example: peak of susceptibility scales as power of volume. Smaller effect: position of peak shifts from its infinite volume position by a different power of volume—

$$\chi_{max}(L) \propto L^p, \qquad T_c(L) = T_c - a/L^q, \qquad (p, q > 0).$$

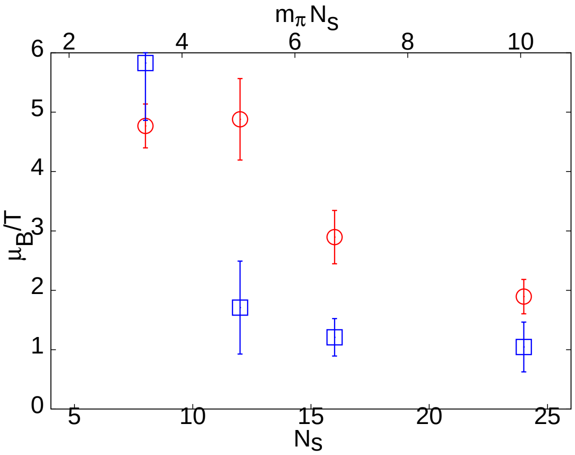
- FSS not well developed for series expansions; some aspects are known.

Series expansions



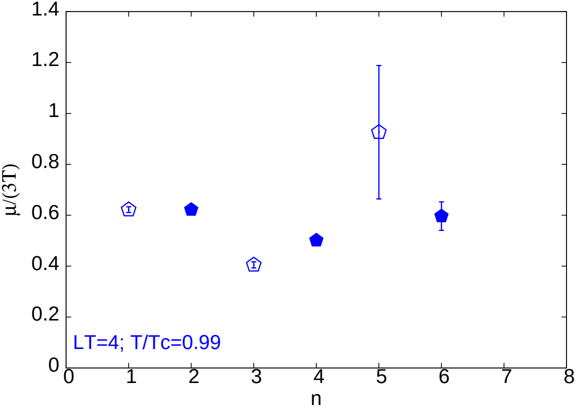
For a divergent quantity: $\chi(T,\mu_B) = \sum_n c_n(T)\mu_B^n$, the leading finite volume effects in the series coefficients.

$N_t = 4$



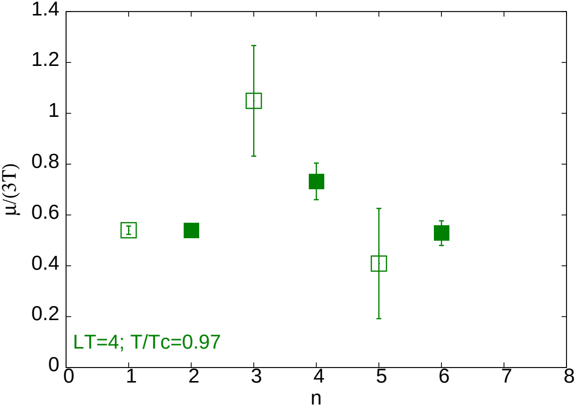
At fixed $T/T_c \simeq 0.95$. Circles: ratio of order 0 and 2; boxes: ratio of order 2 and 4. Gavai and SG, Phys. Rev. D 71, 2005, 114014.

$N_t = 6$: Radius of convergence



Filled symbols: $(\chi^{(0)}/\chi^{(n)})^{1/n}$. Open symbols: $\sqrt{\chi^{(n-1)}/\chi^{(n+1)}}$.

$N_t = 6$: Radius of convergence



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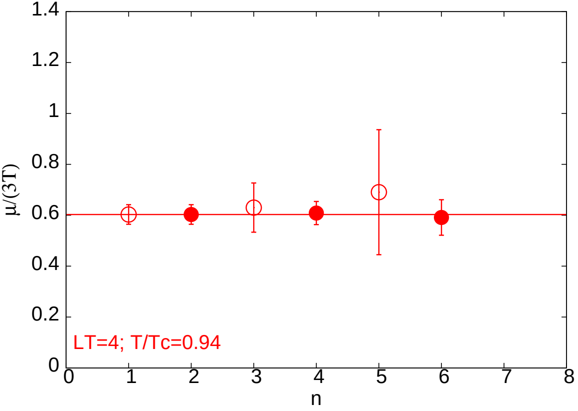

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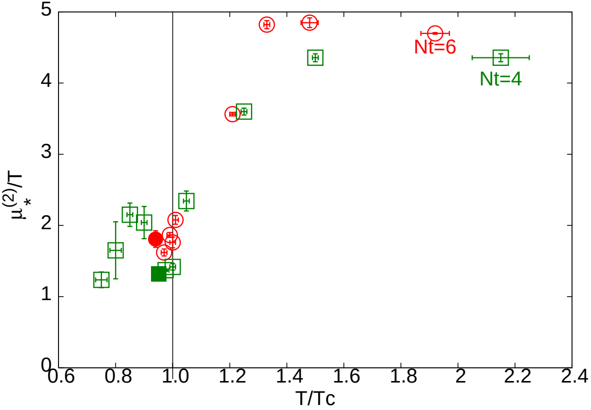
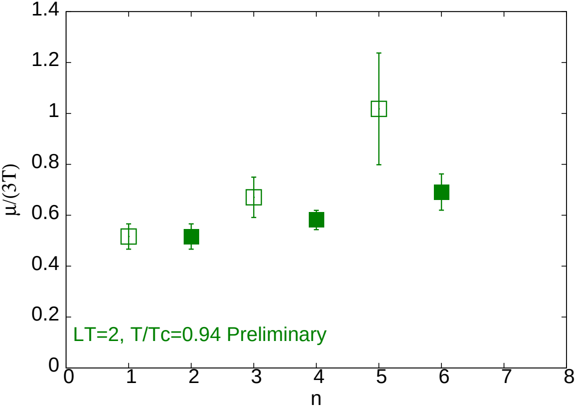
Radius of convergence



Lattice spacing dependence quantifies possible systematic errors.

$N_t = 6$: Finite size scaling



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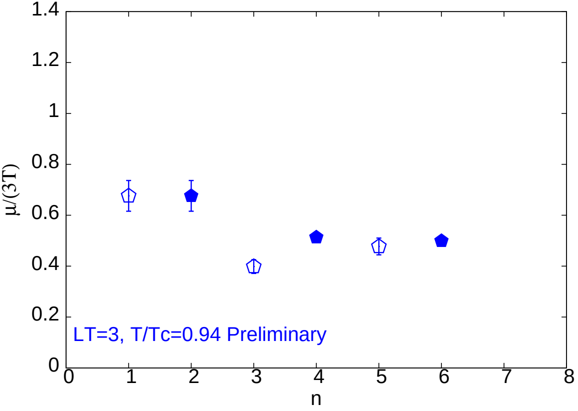

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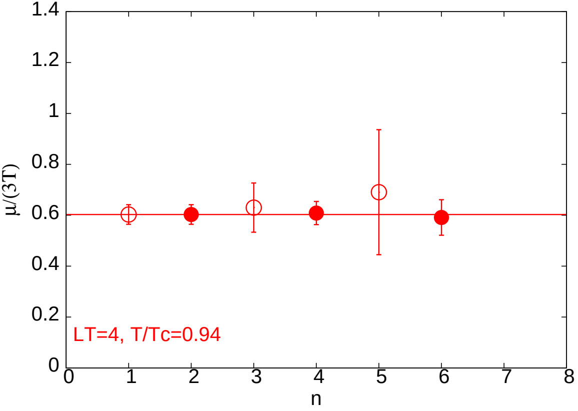


Filled symbols: $(\chi^{(0)}/\chi^{(n)})^{1/n}$. Open symbols: $\sqrt{\chi^{(n-1)}/\chi^{(n+1)}}$.

Critical end point

- Multiple criteria agree:
  - Stability of radius of convergence with order and estimator
  - Pinching of the radius of convergence with T.
  - Smallest T where all the coefficients are positive.
  - Finite size effects roughly correct; more planned for the future.
- This gives

$$\frac{\mathbf{T^E}}{\mathbf{T_c}} = \mathbf{0.94 \pm 0.01} \qquad \text{and} \qquad \frac{\mu_{\mathbf{B}}^{\mathbf{E}}}{\mathbf{T^E}} = \mathbf{1.8 \pm 0.1}$$

  with $Nf = 2$ when $m_\pi/m_\rho \simeq 0.3$ at a finite volume with $LT = 4$ and lattice cutoff of $a = 1/6T^E$.
- For a lattice cutoff of $a = 1/4T^E$ at the same renormalized quark mass and on the same volume we had found a similar value for T^E/T_c and $\mu_B^E/T^E = 1.3 \pm 0.3$. Extrapolation to $L \to \infty$ reduced this to 1.1 ± 0.1.

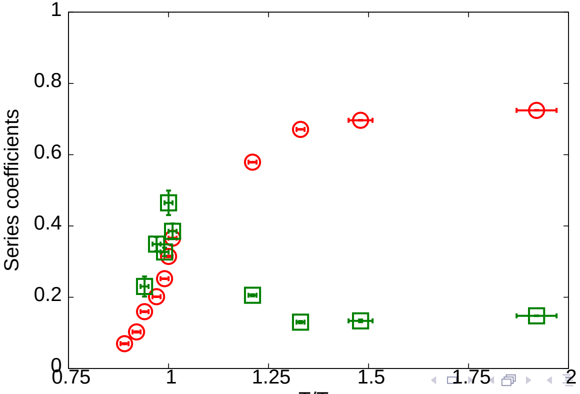
Outline

Fluctuations of Baryon number

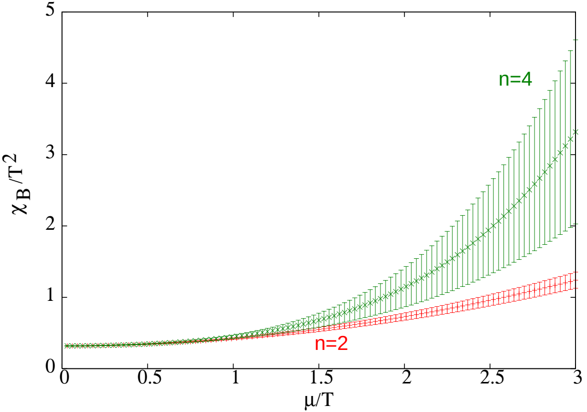
Suggestion by Stephanov, Rajagopal, Shuryak; Asakawa, Heinz, Muller; Jeon, Koch

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right).$$

Extrapolate χ_B to finite chemical potential: peak at T_c?

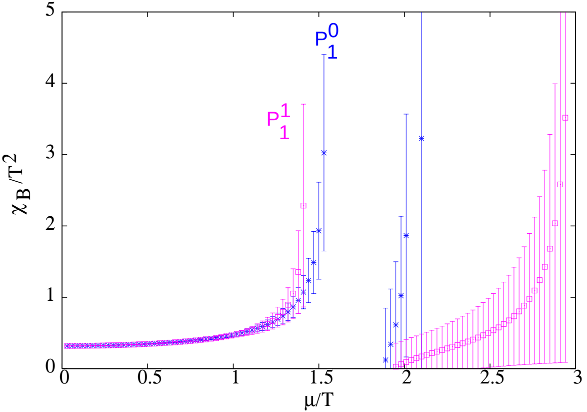


Sum the series



Summing the series never shows critical behaviour: sum is a polynomial and smoothly behaved. The sum peaks at T_c: incorrect (see SG, SEWM 2006).

Critical fluctuations



Use Padé approximants for the extrapolations: divergence at the critical end point (see Lombardo, Mumbai 2005). Error propagation requires care: see arXiv:0806.2233 [hep-lat].

Outline

Summary 1: finite temperature

1. Simulations of $N_f = 2$ QCD (staggered quarks, Wilson action) with renormalized quark mass $m_\pi \simeq 230$ MeV with $1/a \simeq 1200$ MeV and $LT = 2$, 3 and 4.
2. Finite temperature cross over located at $\beta_c = 5.425(5)$, consistent with previous computations at neighbouring masses. Consistent measurements obtained with χ_L, $(T/V)\langle O_{22}\rangle_c$ and $(T/V)\langle O_{44}\rangle_c$ within precision of this computation.
3. Cutoff artifacts seen in many QNS. Surprisingly, measurements are more well-behaved at smaller lattice spacing (see χ_{60} and χ_{80}, for example).

Summary 2: finite chemical potential

1. Very stable estimate of the critical end point: three criteria agree.

$$\frac{T^E}{T_c} = 0.94 \pm 0.01 \qquad \text{and} \qquad \frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$$

   with lattice cutoff of $a = 1/6T^E \simeq 1100$ MeV, compared to $\mu_B^E/T^E = 1.3 \pm 0.3$ at $a = 1/4T^E \simeq 750$ MeV on the same spatial volume.

2. Series extrapolation needs resummation: Padé approximants are one possible resummation.
3. Linkage between u and d quantum numbers disappears at $T \simeq T_c$ when $\mu_B = 0$. How abrupt? Requires finite size scaling.