The critical end point of QCD

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Ensembles

- **Event ensembles at colliders**: created from the full recorded data set through various selection criteria. Values of conserved quantities can fluctuate from one event to another
- **Thermodynamic ensembles**: defined by letting certain conserved quantities fluctuate (canonical) while keeping others fixed (micro-canonical).
- **Mapping between the two**: The system is a small part of a big (ion-ion) collider event, the heat-bath is the remainder. Is the event a thermostat? Probably yes, but need experimental check.

Is a collider event a thermostat?

- **Energy average over events**: m_T stands in for energy, and has an exponential distribution whose slope defines temperature. Hadron abundances in final state give a second measurement of temperature. Inclusive jets at LHC could measure temperature. Measurements of temperature must agree before we can start doing thermodynamics.
- **Other conserved quantities**: If there are long range spatial gradients of densities of conserved charges, then not a thermostat. No evidence for such spatial gradients. Diffusion time could be much smaller than the lifetime of fireball. (Evidence from lattice, toy models such as SYM using AdS/CFT)

Thermodynamic fluctuations

$$P(Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q}\right), \quad \text{so} \quad \langle \Delta Q^2 \rangle = VT\chi_Q$$

Bias free experimental measurement of χ_Q possible.
Asakawa, Heinz and Muller, Phys.Rev.Lett. 85, 2072, 2000; Jeon and Koch, Phys.Rev.Lett. 85, 2076, 2000.

$$P(B,Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q} - \frac{B^2}{2VT\chi_B} - \frac{QB}{VT\chi_{BQ}}\right)$$

Variances determined by diagonal susceptibilities, covariances by off-diagonal susceptibilities.

Susceptibilities

If $P(T, \mu, \cdots)$ is the pressure, then a susceptibility is

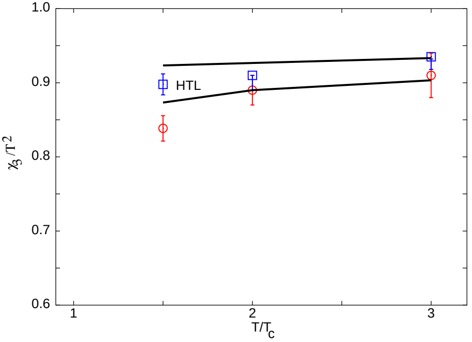
$$\chi = \frac{\partial n}{\partial \mu}, \qquad \text{where} \qquad n = \frac{\partial P}{\partial \mu}$$

Gottlieb, Liu, Toussaint, Renken, Sugar, Phys.Rev.Lett. 59, 2247, 1987.
For each conserved charge one chemical potential: μ_B, μ_Q, μ_S *etc.*. Two kinds of susceptibilities

$$\chi_i = \frac{\partial^2 P}{\partial \mu_i^2} \qquad \chi_{ij} = \frac{\partial^2 P}{\partial \mu_i \partial \mu_j}$$

diagonal and off-diagonal. Higher derivatives are called non-linear susceptibilities. Gavai and SG, Phys.Rev. D68, 034506, 2003.

Diagonal susceptibilities



In the high temperature phase of QCD the diagonal susceptibilities (but not the off-diagonal) agree with a resummed weak coupling computation.
Blaizot, Iancu, Rebhan, Phys.Lett. B523, 143, 2001
Gavai and SG Phys.Rev. D67, 034501, 2003

Susceptibilities of conserved charges

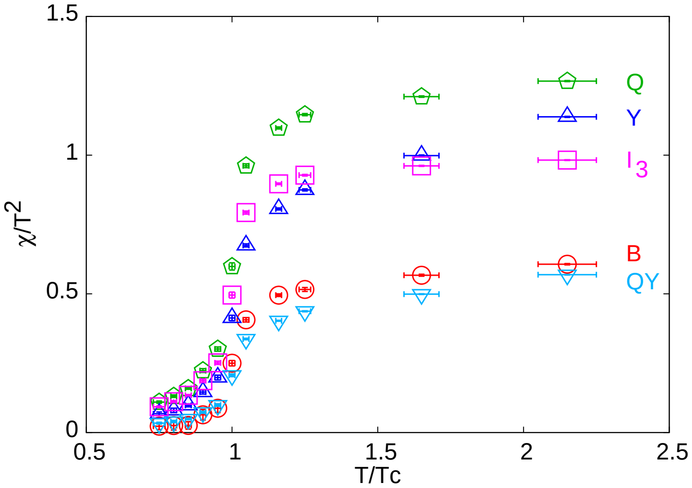


Figure: Since the fireball is in equilibrium only with respect to strong interactions, I_3 and Y are conserved.

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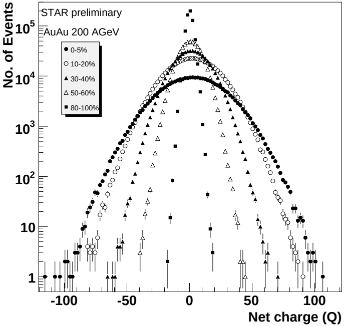

Distribution of net charge is Gaussian ($p_T < 1$ GeV, central rapidity slice).
Nayak (STAR) nucl-ex/0608021

What do fluctuations measure?

In thermodynamics

Fluctuations of energy measures specific heat (c_v), of volume measures compressibility (κ), of charge measures susceptibility (χ). Fluctuations are usually Gaussian.

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At colliders

Many sources of fluctuations: incomplete characterization of event, beam energy fluctuations, ion charge fluctuations, volume, detector noise: all are Gaussian. Hard to disentangle from thermodynamics without proper event characterization. (See, for example, centrality dependence of previous figure).

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- Problematic for studying thermodynamics: for thermodynamically large volumes, fluctuations are (almost) always a Gaussian, no matter what the dynamics is. Physical information is in the width of the Gaussian.
- Either find non-Gaussian thermodynamics or eliminate dependence on uncontrolled variables. (More later)

Exploring the phase diagram of QCD

Many dimensions

Phase diagram has $1 + 2N_f$ dimensions: T, N_f masses (one for each flavour of quarks) and N_f chemical potentials (one for each flavour). Masses cannot be tuned in the real world, so $1 + N_f$ dimensions.

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Experiment

By changing one parameter ($\sqrt{S}$) you explore only a line in the 4D phase space. By changing beam/target expand this to a thin strip. Clever ideas needed to investigate the full phase diagram.

The classic phase diagram

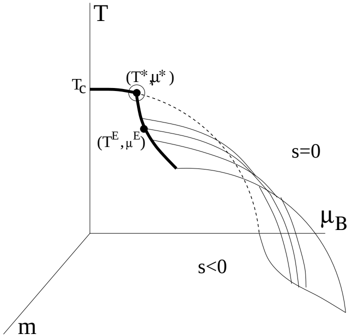

Figure: The tricritical point visible in a different universe where $m_\pi = 0$ (strictly). In our universe (where $m_\pi/m_\rho = 0.18$), the QCD critical end point is in the Ising universality class. A line of Ising critical points and a line of O(4) critical points meet at the tricritical point. Berges and Rajagopal Nucl. Phys. B 538, 215, 1999; Halasz, *et al.*, Phys. Rev. D 58, 096007, 1998.

Extrapolation to physical pion mass

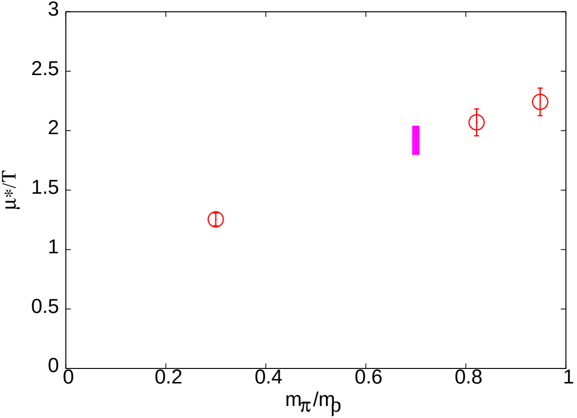

Figure: Gavai and Gupta, Nucl. Phys. A 785, 18, 2007. Bar is estimated from data in Allton *et al.*, Phys. Rev. D 71, 054508, 2005.

Estimating the critical point

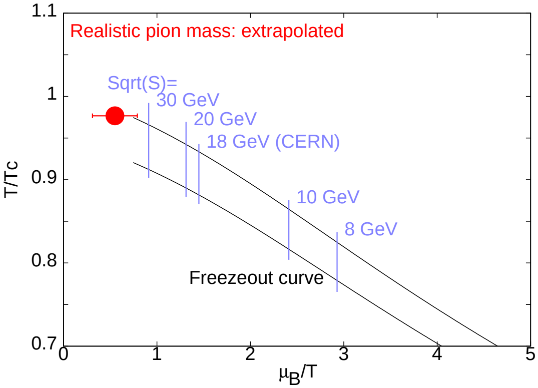

Figure: Statistics, quark mass, finite volumes, method, are now under control. Lattice spacing is the final frontier. In quenched computations this ratio increases with N_t (Gavai and Gupta, Phys. Rev. D 68, 034506, 2003). Continuum limit could be 25% larger, if quenched is an accurate guide.

Remark 1: The real-world tri-critical point

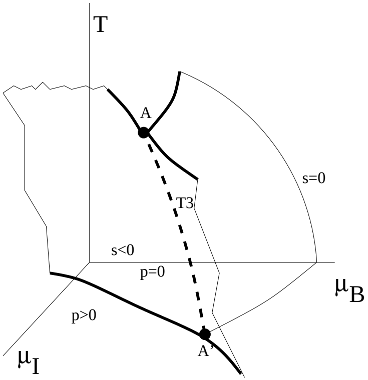

Figure: Surfaces of two phase coexistence; meet along a line of three phase coexistence, end point of which is a new tricritical point at large μ_I— **the observable tricritical point of QCD**. SG, arXiv:0712.0434.

Remark 2: The effect of the strage quark

- Fodor and Katz, JHEP, 0404, 050, 2004 indicated that when the quarks had physical mass then the strange quark does not make too much of a difference to the phase diagram. Calculations were made at the correct quark mass using reweighting methods.

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- Recently de Forcrand and Philipsen, JHEP, 0701, 077, 2007 claimed that for physical quark masses there are no phase transitions at all. Calculations were made at large quark masses using the imaginary chemical potential method.
- An investigation of the full phase diagram (SG, arXiv:0712.0434) reveals that the observations of deF+P can be accomodated in a less pessimistic phase diagram: one where the phase diagram is qualitatively as shown before. Further lattice computations for $N_f = 2 + 1$ are welcome.

Physics of the QCD critical point

Baryon susceptibility and other QNS diverge

At the QCD critical point P is non-analytic and hence χ_B diverges: $\chi(B) \simeq |\mu_B - \mu_B^E|^{-\delta_B}$. Lattice data is consistent with Ising-like values of δ_B. The divergence in χ_B also causes $\chi_{u,d,s}$ to diverge at the QCD critical point.

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The correlation between net flavour numbers diverges at the critical point. It is not yet clear whether other correlation lengths diverge.

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The fireball falls out of equilibrium

The relaxation time for chemistry diverges at the critical point. As a result, the fireball falls out of chemical equilibrium as it approaches the QCD critical point.

Diverging susceptibilities

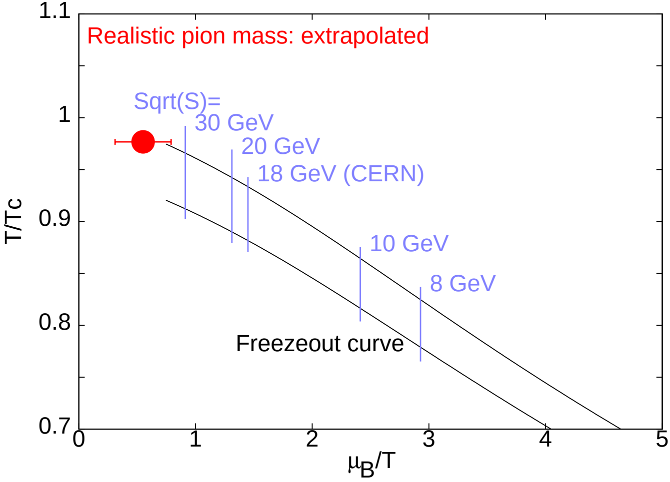


Figure: Find the width of the distribution of B (net baryon number) as a function of $\sqrt{S}$ and look for non-monotonic behaviour.

Remarks: Diverging susceptibilities

- Missing neutrons may impact this signal crucially. Is it better to study net strangeness (strange minus anti-strange)?
- This is a study of Gaussian fluctuations. Control over Poisson uncertainties in the specification of the state are important. Study the effect of slicing in centrality, rapidity, phase space volume.
- Covariances such as χ_{us} also diverge. Is it possible to have equal control over such covariances and study the ratio variance/covariance (Koch, Majumder, Randrup, Phys. Rev. Lett, 95, 182301, 2005; Gavai and Gupta, Phys. Rev, D 73, 014004, 2006)? At the critical point C_{BS} need not be quark-like!

Diverging correlation length

$$P(Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q}\right), \quad \text{so} \quad \langle \Delta Q^2 \rangle = VT\chi_Q$$

This is true only when $V^{1/3} \gg \xi$ When this condition is violated, the distribution of B (or u, d, s) need not be Gaussian. Look for Kurtosis, $K_{B,u,d,s}$. The function $K(\sqrt{S})$ should peak if one passes close to the critical point.

Caveats: missing neutrons are a problem; net charge distributions could be better controlled. Do Gaussian fluctuations in volume *etc.*, cancel out in study of K?

The fourth cumulant

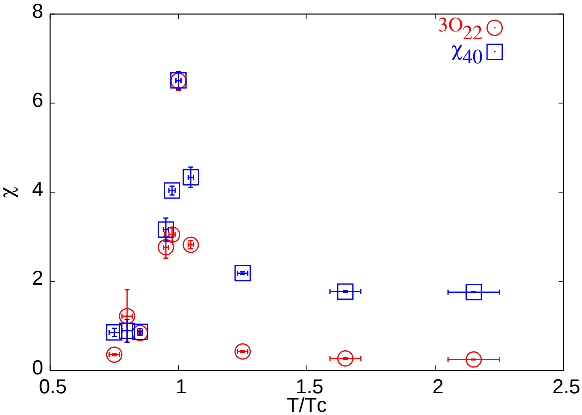


Figure: Measurements of K give the ratio of the fourth cumulant and the square of the second. The fourth cumulant is a strong function of the temperature. Gavai and Gupta, Phys. Rev. D 72, 054006, 2005

Diverging relaxation time

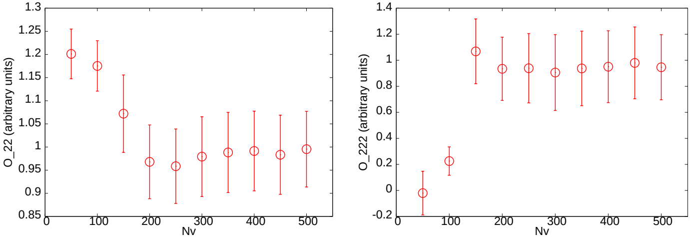
- Freezeout occurs when the relaxation time of a fluid exceeds typical time scales of the fluid flow. When the fireball cools sufficient close to the critical point, freezeout will occur, because the relaxation time grows: $\tau \propto |T - T^E|^{-z}$.
- The phenomenological observation that $E/N \simeq 1$ GeV at freezeout is obtained by studying average chemical abundances on freezeout away from a critical point. Is this violated at a critical point?
- Since the system is strongly correlated $(V^{1/3} \simeq \xi)$ there could be strong event-to-event fluctuations in chemical abundances.
- A study of critical fluid dynamics is called for. This is just beginning (Son and Stephanov, Phys. Rev. D 70, 056001, 2004; Koide, J. Phys. G 31, 1055, 2005; Ohnishi, Fukushima and Ohta, Nucl. Phys. A 748, 260, 2005; Nonaka and Asakawa, Phys. Rev, C 71, 044904, 2005; *etc.*).

Summary

- Critical point for $N_f = 2$ QCD known with good control of volume, quark masses, statistics, methods. Continuum extrapolation remains to be performed. Expected to increase the estimate of μ_B^E marginally. Within this uncertainty, the critical point is (luckily) within reach of an energy scan at RHIC.
- Strong signals possible for critical end point in experiments. These include non-monotonic behaviour with $\sqrt{S}$ of the following:
  1. Gaussian width of event-to-event distributions of conserved flavour quantum numbers (B, Q *etc.*)
  2. Non-Gaussian behaviour of these distributions, leading to non-vanishing Kurtosis and higher cumulants of these distributions
  3. Enhanced event-to-event fluctuations in the chemical composition at freezeout even in the commonest hadrons.

Statistics

Numerical estimates of traces are made by the usual noisy method, which involves the identity $I = \overline{|r\rangle\langle r|}$, where r is a vector of complex Gaussian random numbers. We need upto 500 vectors in the averaging.



Central value: measurement with exactly N_v vectors; bars: config-to-config variation. Statistics of vectors (N_v) is the big issue. Statistics of configs secondary.

Volume dependence

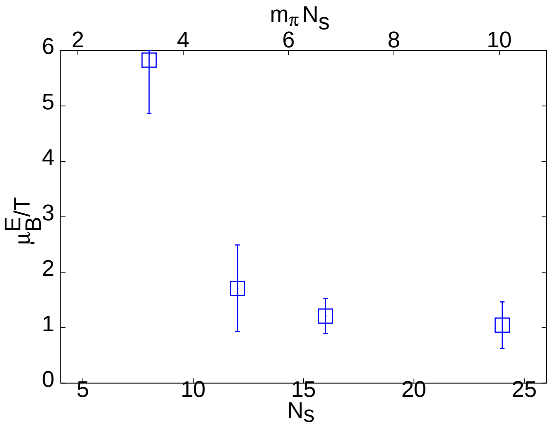


Figure: Estimates of the critical μ_B fall with volume. Early computations on small volumes found very high values for μ_B^E, compatible with the measurements shown here.

Methods

Rewighting

Fodor and Katz, JHEP, 0203, 014, 2002 [hep-lat/0106002]
BS1: Allton *et al.*, Phys. Rev. D 66, 074507, 2002 [hep-lat/0204010]
Exponentially hard in volume.

Imaginary chemical potential

de Forcrand Philipsen, Nucl. Phys. B 642, 290, 2002 [hep-lat/0205016]
D'Elia and Lombardo, Phys. Rev. D 67, 014505, 2003 [hep-lat/0209146]
Compute at several imaginary μ_B and extrapolate to real.

Taylor expansion

Gavai and Gupta, Phys. Rev. D 68, 034506, 2003 [hep-lat/0303013]
BS2: Allton *et al.*, Phys. Rev. D 71, 054508, 2005 [hep-lat/0501030]
Compute series coefficients directly at $\mu = 0$.