The critical end point of QCD: lattice and experiment

Sourendu Gupta

ILGTI: TIFR

CPOD 2009
BNL, USA
June 10, 2009

1. On lattice

2. In experiment

Outline

1. On lattice
2. In experiment

The method

Taylor expansion of the pressure in μ_B

$$P(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n)}(T)\mu_B^n$$

has Taylor coefficients that need to be evaluated only at $\mu_B = 0$ where there is no sign problem. The baryon number susceptibility (second derivative of P) has a related Taylor expansion

$$\chi_B(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n+2)}(T)\mu_B^n.$$

χ_B diverges at the critical point. Series expansion can show signs of divergence. If all the coefficients are positive, then the divergence is at real μ_B.

The method is perfectly general and can be applied to any theory. (Gavai, SG, hep-lat/0303013).

The implementation

- Our implementation is in $N_f = 2$ QCD using staggered quarks.
- Light quark bare masses are tuned to give $m_\pi = 230$ MeV.
- Currently our results from two cutoffs, $\Lambda = 1/a \simeq 800$ MeV ($N_t = 4$: Gavai, SG, hep-lat/0412035) and 1200 MeV ($N_t = 6$: Gavai, SG, arxiv:0806.2233).
- Temperature scale setting performed by measuring the renormalized coupling in three different renormalization schemes. At these cutoffs different schemes give slightly different scales: 1% error estimated from this source.
- Lattice sizes of 4–6 fm per side near T_c: several pion Compton wavelengths, several thermal wavelengths.
- Simulation algorithm is R-algorithm. MD time step has been changed by factor of 10 without any change in results.

Remaining issues

- Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects: next.
- What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks (Gavai, SG, hep-lat/0510044).
- What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in Gavai, SG, Ray, nucl-th/0312010.
- What happens in the continuum limit? Estimate of μ_B^E may increase somewhat: current results.
- What if a different estimator of the critical point is used: must agree, at least in the large volume limit (later figure).
- Can the phase diagram be more complicated? Yes, we only find the nearest critical point to $\mu_B = 0$.

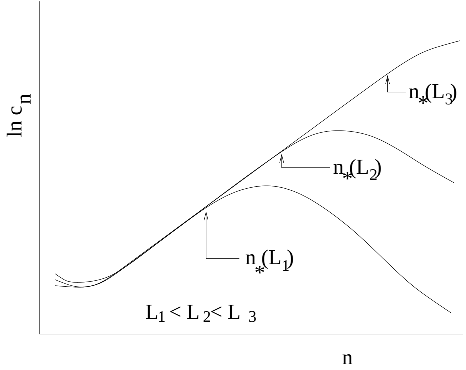
Finite size effects

- At critical point correlation length becomes infinite, appropriate susceptibilities diverge and free energy becomes singular ... in the infinite volume limit (van Hove's theorem).
- No numerical computation ever performed on infinite volumes.
- Deduce the existence of a critical point through extrapolations: finite size scaling (FSS) well developed for direct simulations.
- Example: peak of susceptibility scales as power of volume. Smaller effect: position of peak shifts from its infinite volume position by a different power of volume—

$$\chi_{max}(L) \propto L^p, \qquad T_c(L) = T_c - a/L^q, \qquad (p, q > 0).$$

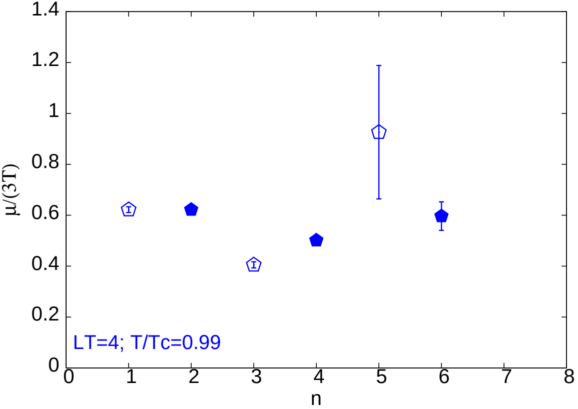
- FSS for series expansions: large and small effects.

Series expansions



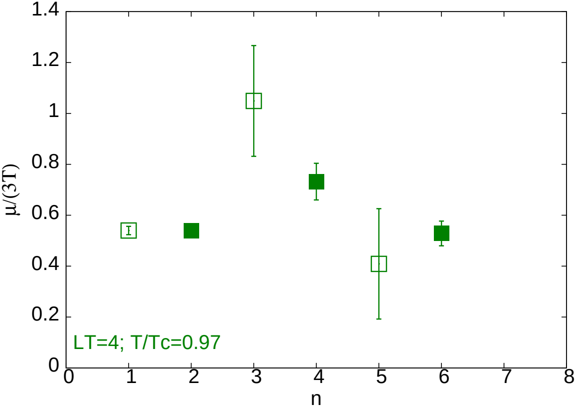
For the divergent quantity: $\chi_B(T, \mu_B) = \sum_n c_n \mu_B^n$, where $c_n = \chi^{(n+2)}(T)/n!$, the leading finite volume effects in the series coefficients.

$N_t = 6$: Radius of convergence



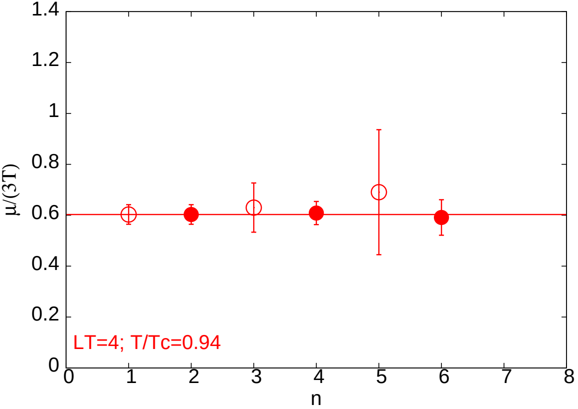
Filled symbols: $(c_0/c_n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

$N_t = 6$: Radius of convergence



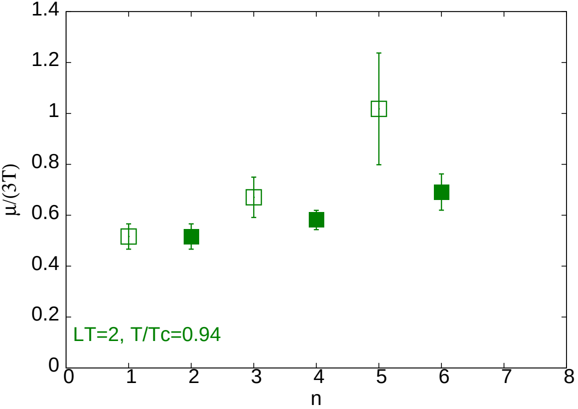
Filled symbols: $(c_0/c_n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

$N_t = 6$: Radius of convergence



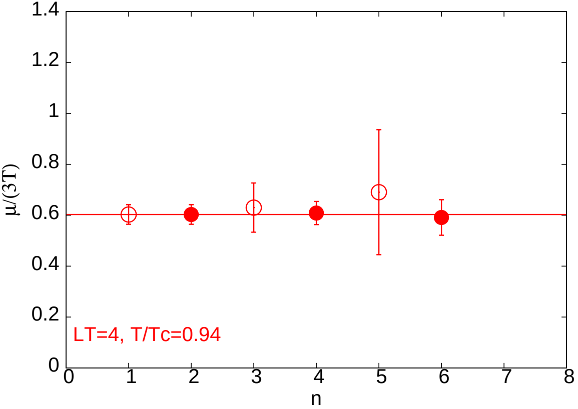
Filled symbols: $(c_0/c_n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

$N_t = 6$: Finite size scaling



Filled symbols: $(c_0/c_n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

$N_t = 6$: Finite size scaling



Filled symbols: $(c_0/c^n)^{1/n}$. Open symbols: $\sqrt{c_{n-1}/c_{n+1}}$.

Critical end point

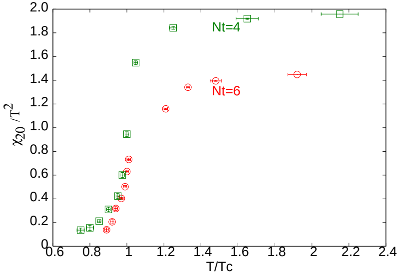
- Multiple criteria agree:
  - Small window in T where all the coefficients are positive.
  - Stability of radius of convergence with order and estimator
  - Finite size effects follow correct trend; more planned for the future.
  - Pinching of the radius of convergence with T.
- This gives $T^E/T_c = 0.94 \pm 0.01$ and μ_B^E/T^E as below

N_t	$V = (4/T)^3$	$V \to \infty$
4	1.3 ± 0.3	1.1 ± 0.1
6	1.8 ± 0.1	?

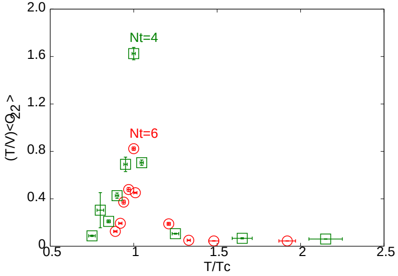
- Very naively: extrapolate to $V \to \infty$ by same factor, extrapolate to $a \to 0$ as a^2 (staggered quarks), then $\mu_B^E \simeq 325$ MeV. Somewhat lower at $m_\pi = 140$ MeV. Many assumptions, many caveats. May be in the range $\mu_B^E = 250$–**400 MeV with** $T^E = 165$–**175 MeV**.

Series summation and resummation

Does this work:



$+\mu^2\times$

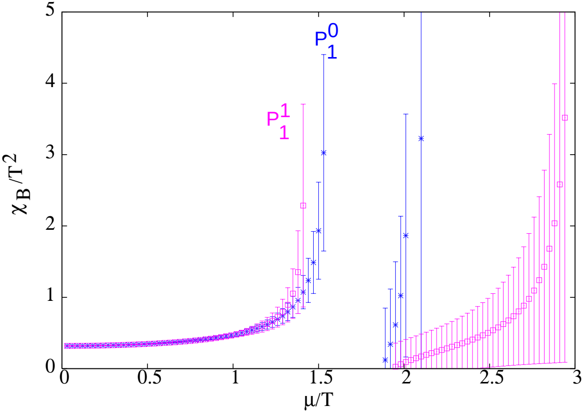


Peak at T_c, not T^E. Incorrect!

$$\chi_B(T,\mu_B) = \sum_n \frac{1}{n!}\chi^{(n+2)}(T)\mu_B^n.$$

At critical point, χ_B diverges, all terms equally important; cannot truncate the series. Have to resum the whole series: Pade approximants.

Critical fluctuations



Use Padé approximants for the extrapolations: divergence only at the critical end point. Error propagation requires care: see arXiv:0806.2233 [hep-lat].

Outline

1. On lattice
2. In experiment

Gaussian Fluctuations

Fluctuations are Gaussian

At any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B\rangle.$$

Suggestion by Stephanov, Rajagopal, Shuryak: measure the susceptibility by examining the Gaussian. Bias-free measurement possible: Asakawa, Heinz, Muller; Jeon, Koch.

Why Gaussian?

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem** (CLT).

Is the current RHIC point non-critical?

Answer

Check whether CLT holds.

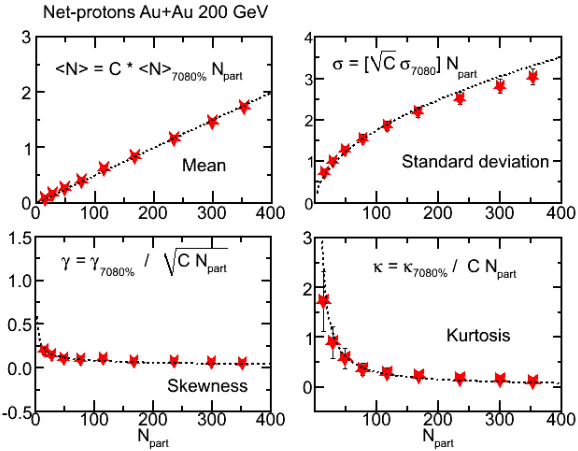
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Caveat

Make sure that the nature of the physical system does not change while changing the volume. Perhaps best accomplished by changing rapidity acceptance while keeping centrality fixed. Alternative tried by STAR is to change the number of participants.

STAR measurements



STAR Collaboration: QM 2009, Knoxville.

QCD interpretation of STAR analysis

Can we compare STAR's measurements of $\sigma_{70-80\%}$ and $\mathcal{K}_{70-80\%}$ with lattice QCD?

Two questions to be answered before this is feasible:

1. N_{part} is a proxy for the volume. In changing this is the physics unchanged? Do the fluctuations give initial information or near-freezeout information? Need to develop a complete theory of diffusion+hydro (Son and Stephanov, hep-ph/0401052, Bower and Gavin, hep-ph/0106010, Bhalerao and SG, 0901.4677).
2. Have all other sources of non-Gaussianity have been subtracted out? What about jetty fluctuations, for example? Need studies of systematics.

Finding a critical point

Near the critical point $V/\xi^3 \simeq 1$: CLT breakdown, non-Gaussian behaviour. Critical scaling—

$$\chi^{(2)} \propto |\mu - \mu_B^E|^{-\gamma}, \quad \chi^{(4)} \propto |\mu - \mu_B^E|^{-\gamma-2} \qquad (\gamma > 0).$$

The Kurtosis diverges:

$$\mathcal{K} = -1 + \frac{\chi^{(4)}}{3[\chi^{(2)}]^2} \propto |\mu - \mu_c|^{\gamma-2};$$

(since $P = P_0 + p|\mu - \mu_c|^{-\gamma+2}$ is non-analytic but non-divergent). Fireball expansion rounds off the transition. Nevertheless, $\mathcal{K} \simeq \xi^4$ (Stephanov, 0809.3450). Along freezeout trajectory in an energy scan, the microscopic Kurtosis is non-monotonic. In experiment look for non-Gaussian E-to-E fluctuations.

One way to find the critical point

- Construct E-to-E distributions of B, Q and S. Since there are non-trivial linkages between them, comparison of the three distributions is important. Construct distributions in limited acceptance in order to simulate a grand canonical ensemble.
- Issues related to missed particles, in particular uncharged baryons and strange particles (neutrons and K^0). Require studies to see the effects of these.
- Observe the scaling of B, Q and S as a function of volume: if central limit theorem, then normal point. Otherwise close to critical point.
- Close to critical point the kurtosis does not scale with volume and may become very large due to critical exponent effects.
- Effect of hydrodynamic evolution still to be understood properly.