The critical end point of QCD: lattice and experiment

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Outline

The sign problem

QCD is solved on (super) computers by evaluating the path integral

$$Z = \int \prod_{\mathbf{x}\nu} dU_{\mathbf{x}\nu} \exp[-S_E(U_{\mathbf{x}\nu})], \quad U_{\mathbf{x}\nu} = \exp\left[i\int_{\mathbf{x}}^{\mathbf{x}+a\hat{\nu}} d\mathbf{y} A_\nu(\mathbf{y})\right].$$

$\mathbf{x}$: spacetime point, ν: directions. Continuum limit: number of points goes to infinity, therefore infinite dimensional integral. Lattice cutoff, $a \to 0$ using QCD beta function. But even at small a, many points; need to use Monte Carlo techniques.

Works when S_E real. Gauge part always real. Quark part? At zero μ_B one finds $\det D$ real. Therefore Monte Carlo works. At finite μ_B additional term $\mu\gamma_0$ like complex gauge field. Hence $\exp[-S_E]$ complex. Monte Carlo fails: **fermion sign problem**.

Sign problems everywhere: QCD at finite μ, Chern-Simmons theory, high temperature superconductors, many nano-systems $\cdots$. First solution in QCD.

The method

Taylor expansion of the pressure in μ_B

$$P(T, \mu_B) = \sum_n \frac{1}{n!} \chi^{(n)}(T) \mu_B^n$$

has Taylor coefficients that need to be evaluated only at $\mu_B = 0$ where there is no sign problem. The baryon number susceptibility (second derivative of P) has a related Taylor expansion

$$\chi_B(T, \mu_B) = \sum_n \frac{1}{n!} \chi^{(n+2)}(T) \mu_B^n.$$

χ_B diverges at the critical point. Series expansion can show signs of divergence. If all the coefficients are positive, then the divergence is at real μ_B.

The method is perfectly general and can be applied to any theory. (Gavai, SG, hep-lat/0303013).

The implementation

- Our implementation is in $N_f = 2$ QCD using staggered quarks.
- Light quark bare masses are tuned to give $m_\pi = 230$ MeV.
- Currently our results from two cutoffs, $\Lambda = 1/a \simeq 800$ MeV ($N_t = 4$: Gavai, SG, hep-lat/0412035) and 1200 MeV ($N_t = 6$: Gavai, SG, arxiv:0806.2233).
- Temperature scale setting performed by measuring the renormalized coupling in three different renormalization schemes. At these cutoffs different schemes give slightly different scales: 1% error estimated from this source.
- Lattice sizes of 4–6 fm per side near T_c: several pion Compton wavelengths, several thermal wavelengths.
- Simulation algorithm is R-algorithm. MD time step has been changed by factor of 10 without any change in results.

Remaining issues

- Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects: next.
- What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks (Gavai, SG, hep-lat/0510044).
- What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in Gavai, SG, Ray, nucl-th/0312010.
- What happens in the continuum limit? Estimate of μ_B^E may increase somewhat: current results.
- What if a different estimator of the critical point is used: must agree, at least in the large volume limit (later figure).
- Can the phase diagram be more complicated? Yes, we only find the nearest critical point to $\mu_B = 0$.

Critical end point

- Multiple criteria agree:
    - Small window in T where all the coefficients are positive.
    - Stability of radius of convergence with order and estimator
    - Finite size effects follow correct trend; more planned for the future.
    - Pinching of the radius of convergence with T.
- This gives $T^E/T_c = 0.94 \pm 0.01$ and μ_B^E/T^E as below

N_t	$V = (4/T)^3$	$V \to \infty$
4	1.3 ± 0.3	1.1 ± 0.1
6	1.8 ± 0.1	?

- Very naively: extrapolate to $V \to \infty$ by same factor, extrapolate to $a \to 0$ as a^2 (staggered quarks), then $\mu_B^E \simeq 325$ MeV. Somewhat lower at $m_\pi = 140$ MeV. Many assumptions, many caveats. May be in the range **μ_B^E=250–400 MeV with T^E=165–175 MeV**.

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Gaussian Fluctuations

Fluctuations are Gaussian

At any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B \rangle.$$

Suggestion by Stephanov, Rajagopal, Shuryak: measure the susceptibility by examining the Gaussian. Bias-free measurement possible: Asakawa, Heinz, Muller; Jeon, Koch.

Why Gaussian?

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem** (CLT).

Is the current RHIC point non-critical?

Answer

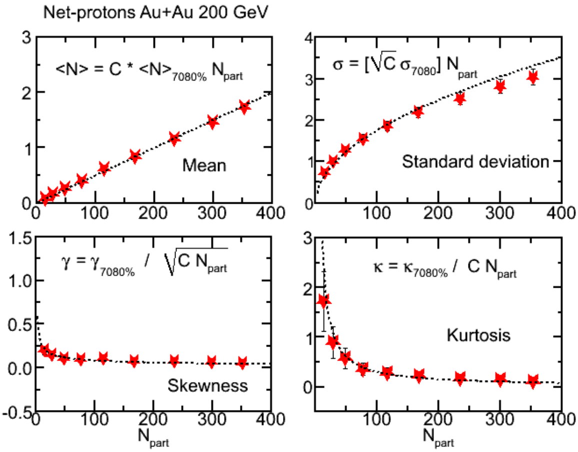
Check whether CLT holds.
Recall the scaling of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Caveat

Make sure that the nature of the physical system does not change while changing the volume. Perhaps best accomplished by changing rapidity acceptance while keeping centrality fixed. Alternative tried by STAR is to change the number of participants.

STAR measurements



STAR Collaboration: QM 2009, Knoxville.

QCD interpretation of STAR analysis

Can we compare STAR's measurements of $\sigma_{70-80\%}$ and $\mathcal{K}_{70-80\%}$ with lattice QCD?

Two questions to be answered before this is feasible:

1. N_{part} is a proxy for the volume. In changing this is the physics unchanged? Do the fluctuations give initial information or near-freezeout information? Need to develop a complete theory of diffusion+hydro (Son and Stephanov, hep-ph/0401052, Bower and Gavin, hep-ph/0106010, Bhalerao and SG, 0901.4677).
2. Have all other sources of non-Gaussianity have been subtracted out? What about jetty fluctuations, for example? Need studies of systematics.

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3V\left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3V\left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3V\chi^{(4)}.$$

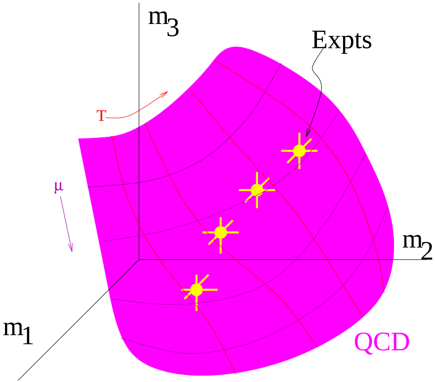
T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Control all backgrounds in the measurements of $[B^n]$. Then construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD (Table III of Gavai, SG, 2008).

Is there an internally consistent check that all backgrounds and systematic effects are removed and comparison with lattice QCD possible?

How to compare with QCD



As T and μ are varied, the QCD predictions will lie on a surface in the space of measurements (m_1, m_2, m_3). If the data lies on this surface then all non-thermal backgrounds are removed. Then a comparison with QCD and a measurement of T and μ is immediate. Similarly for Q and S.

Finding a critical point

Near the critical point $V/\xi^3 \simeq 1$: CLT breakdown, non-Gaussian behaviour. Critical scaling—

$$\chi^{(2)} \propto |\mu - \mu_B^E|^{-\gamma}, \quad \chi^{(4)} \propto |\mu - \mu_B^E|^{-\gamma-2} \qquad (\gamma > 0).$$

The Kurtosis diverges:

$$\mathcal{K} = -1 + \frac{\chi^{(4)}}{3[\chi^{(2)}]^2} \propto |\mu - \mu_c|^{\gamma-2};$$

(since $P = P_0 + p|\mu - \mu_c|^{-\gamma+2}$ is non-analytic but non-divergent). Fireball expansion rounds off the transition. Nevertheless, $\mathcal{K} \simeq \xi^4$ (Stephanov, 0809.3450). Along freezeout trajectory in an energy scan, the microscopic Kurtosis is non-monotonic. In experiment look for non-Gaussian E-to-E fluctuations.

One way to find the critical point

- Construct E-to-E distributions of B, Q and S. Since there are non-trivial linkages between them, comparison of the three distributions is important. Construct distributions in limited acceptance in order to simulate a grand canonical ensemble.
- Issues related to missed particles, in particular uncharged baryons and strange particles (neutrons and K^0). Require studies to see the effects of these.
- Observe the scaling of B, Q and S as a function of volume: if central limit theorem, then normal point. Otherwise close to critical point.
- Close to critical point the kurtosis does not scale with volume and may become very large due to critical exponent effects.
- Effect of hydrodynamic evolution needs to be included (next section).

Outline

Hydrodynamics and Transport coefficients

Every hydrodynamic equation is a combination of a conservation law and a **constitutive equation**. Simplest equations for diffusion:

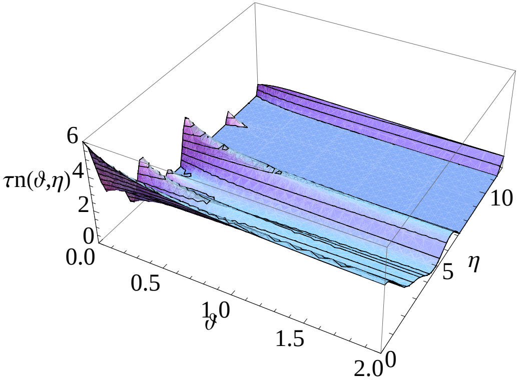
$$\frac{\partial n}{\partial t} = \nabla \cdot \mathbf{J}, \qquad \mathbf{J} = -\mathcal{D}\nabla \cdot n.$$

n conserved density, $\mathcal{D}$ diffusion coefficient: a transport coefficient.
This equation is not causal. Causal version obtained by introducing a memory kernel—

$$\tau_R \frac{\partial \mathbf{J}}{\partial t} + \mathbf{J} = -\mathcal{D}\nabla \cdot n.$$

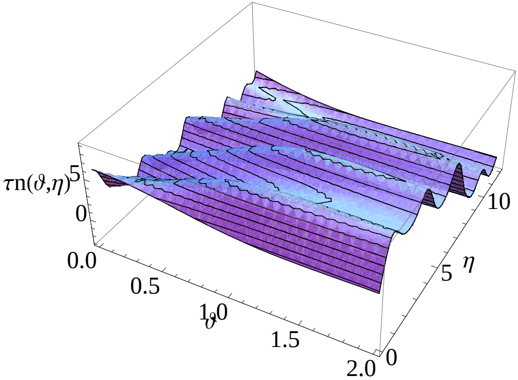
New transport coefficient: τ_R, has interpretation of a relaxation time. τ_R, c_s and $\mathcal{D}$ related in kinetic theory. Same relation arises in field theory as a f-sum rule. SG 2007; Bhalerao, SG, 2009.
Causal diffusion equation behaves like an underdamped harmonic oscillator. Long time limit same as usual diffusion equation: overdamped oscillator. Similar phenomena in hydrodynamics. Much harder to observe.

First order diffusion



Usual intuition: diffusion destroys structure, the sharpest structures are destroyed fastest.

Transient amplification of the profile



One draw from Gaussian random ensemble of initial conditions. Profile of initial n same as for the first order example before.

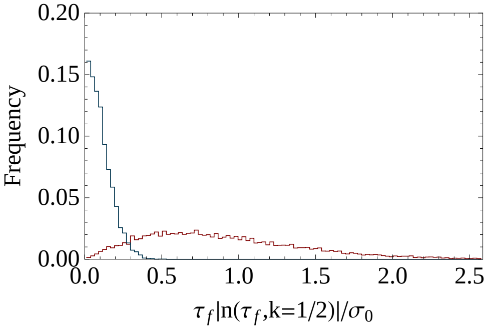
Main questions

- Only two parameters τ_R and $\mathcal{D}$. Much simpler than hydrodynamics, but similar physics.
- How large is τ_R? If $\tau_{\rm fo} \ll \tau_R$ only normal diffusion is seen. Experimental bounds?
- What is the value of $\mathcal{D}$? If measurable then first direct observation of a transport coefficient.
- Profiles not observable; too few particles.
- Convert to event by event variables.
  The power spectrum of the profile:

$$\overline{P}(\tau_f, k) = \left| \sum_{j=1}^{N_t} q_j \mathrm{e}^{-ik\eta_j} \right|^2,$$

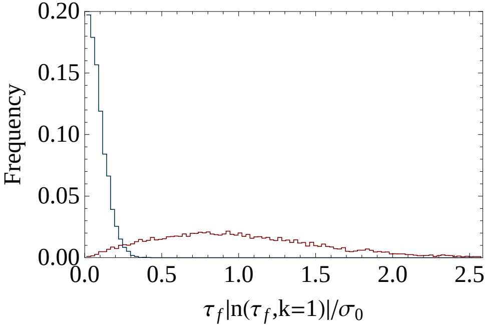
sum over tracks $j = 1, \cdots N_t$. Each event gives one value of $\overline{P}(\tau_f, k)$. Draw E-by-E histogram for each k. Can be done for $q = B$, Q and S.

Experimental signature



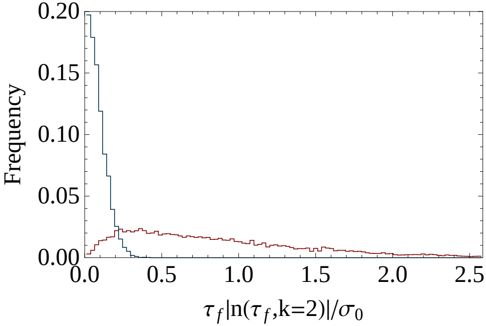
Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 1/2$.

Experimental signature



Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 1$.

Experimental signature



Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 2$.

Outline

Summary

1. Very naively: extrapolate to $V \to \infty$ by same factor, extrapolate to $a \to 0$ as a^2 (staggered quarks), then $\mu_B^E \simeq 325$ MeV. Somewhat lower at $m_\pi = 140$ MeV. Many assumptions, many caveats. May be in the range **μ_B^E=250–400 MeV with T^E=165–175 MeV**.
2. Observe the scaling of B, Q and S as a function of volume: if central limit theorem, then normal point. Otherwise close to critical point; then the kurtosis does not scale with volume and may become very large due to critical exponent effects.
3. Transport effects can be controlled. Consistent theory of hydrodynamics and diffusion can be constructed. The transport coefficients can be constrained by measuring the power spectrum of the number densities.

Thank you for Patnitop