

# The gluon $N_c$ plasma and its 't Hooft limit (1, 2, 3, $\dots$ , $\infty$ )

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ILGTI: TIFR

Extreme QCD 2010  
Physikzentrum Bad Honnef, Germany  
June 21, 2010

RG scaling and 't Hooft scaling

Latent heat

Equation of state

Summary

## Limiting procedures

- ▶ 't Hooft scaling limit: take  $N_c \rightarrow \infty$  and  $g^2 \rightarrow 0$  keeping  $\lambda = g^2 N_c$  fixed. Correlation functions can be computed non-perturbatively but diagrammatically by resumming a well-defined class of diagrams. Simple caricature of hadron physics in this limit.

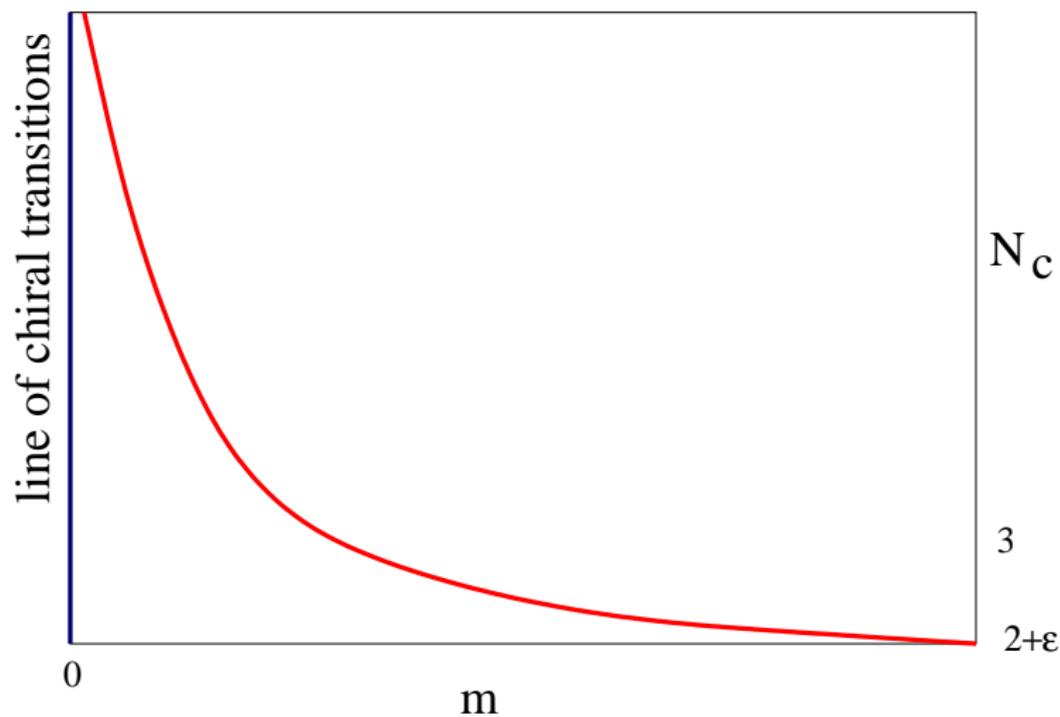
't Hooft, Coleman; tested by Teper and collaborators

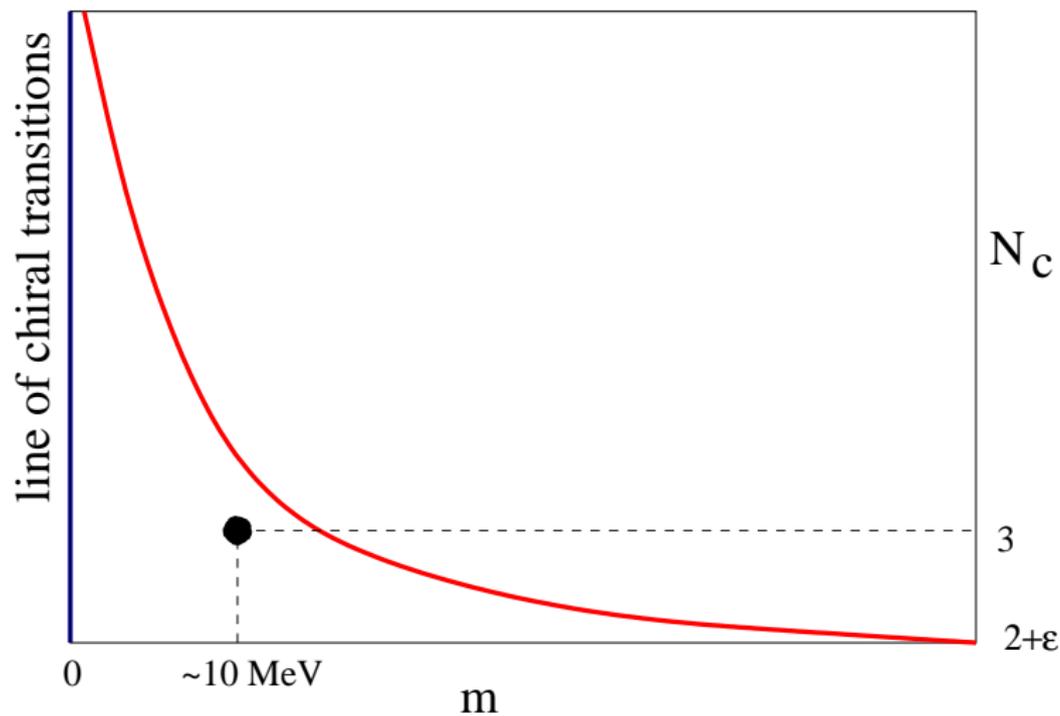
Extended to many other classes of theories. AdS/CFT correspondence works in the 't Hooft limit of  $N_c \rightarrow \infty$ .

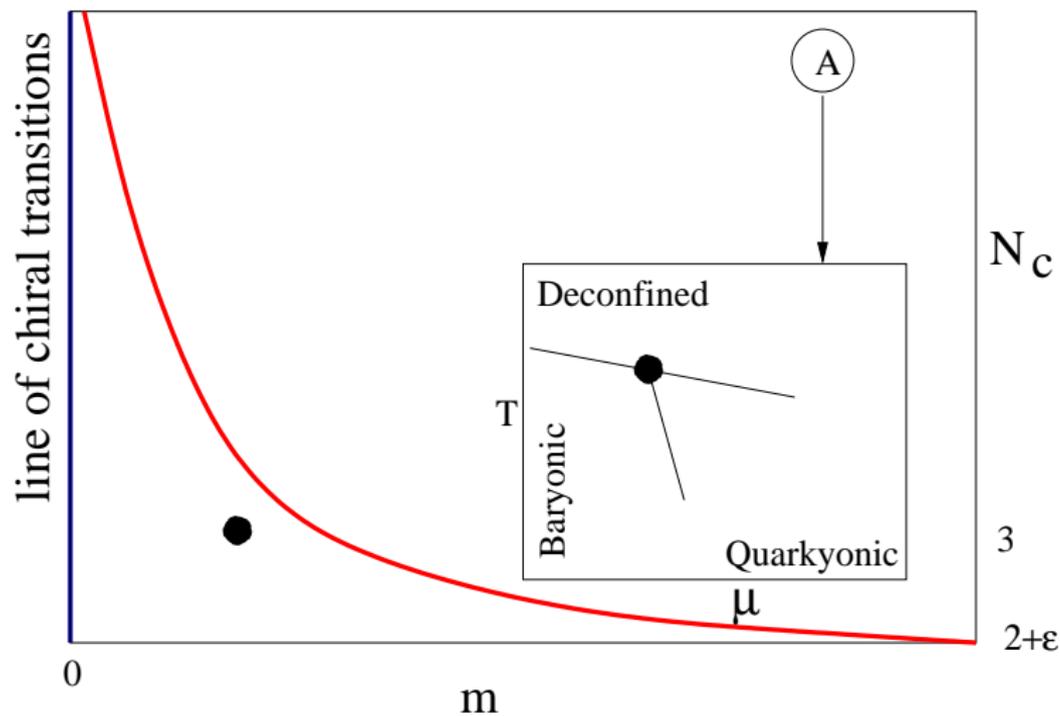
- ▶ Strong scaling limit: work at (fixed) long distance scales and take  $N_c \rightarrow \infty$ . Non-perturbative content of the theory may be explored on the lattice and the limit taken.

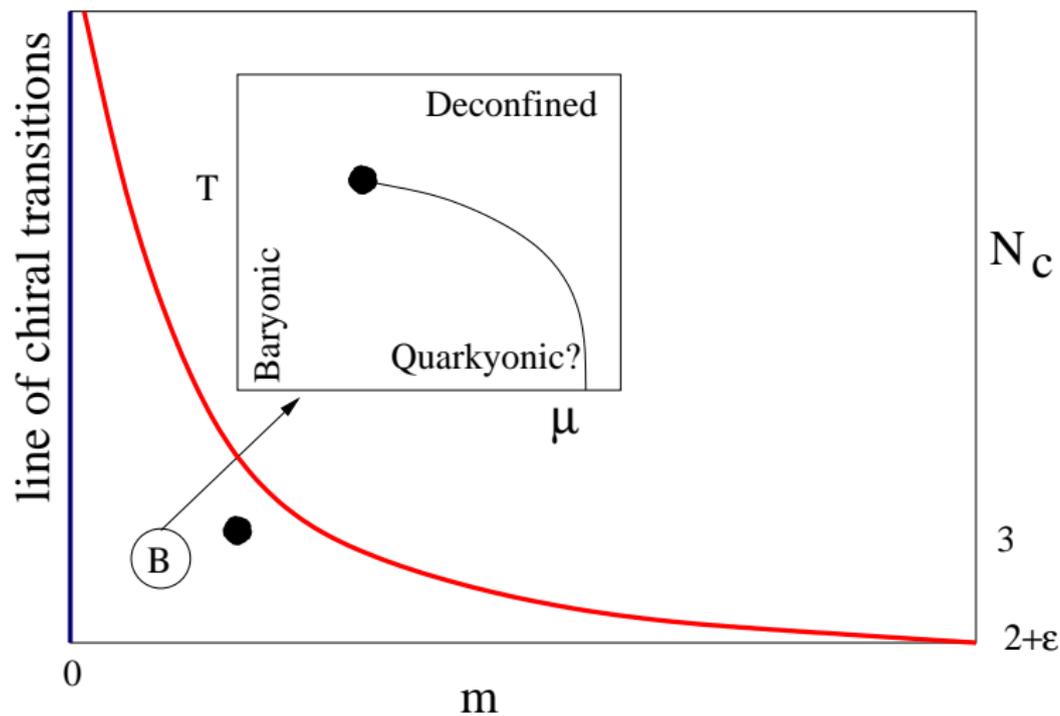
Teper, Lucini and collaborators

Corrections organized in power of  $1/N_c$

Flag diagram of gluo $N_c$  plasmas

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# Outline

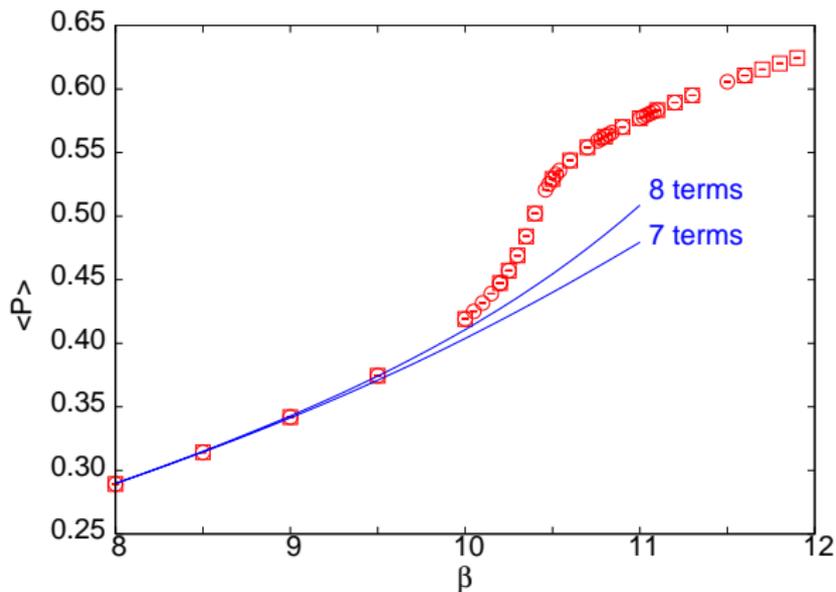
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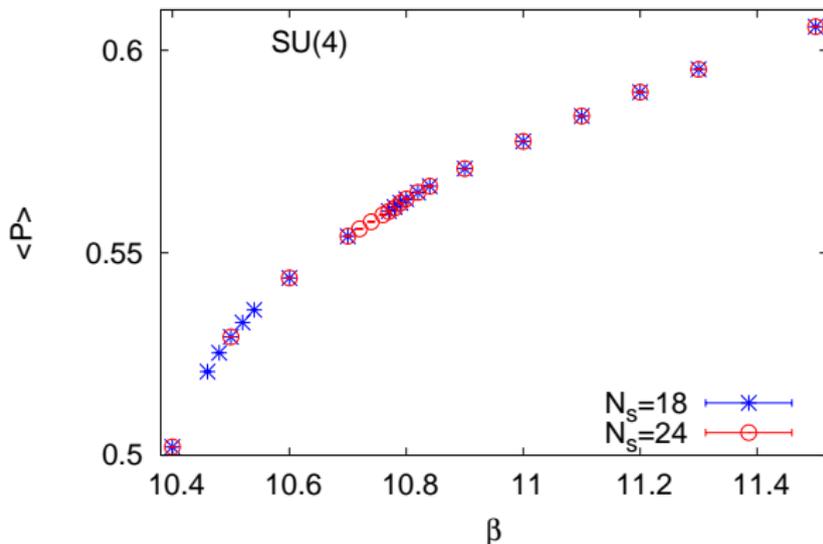
# Unphysical strong–weak coupling crossover



Wilson action. Earlier generation of simulations limited by this bulk transition; solution now: move to smaller lattice spacing.

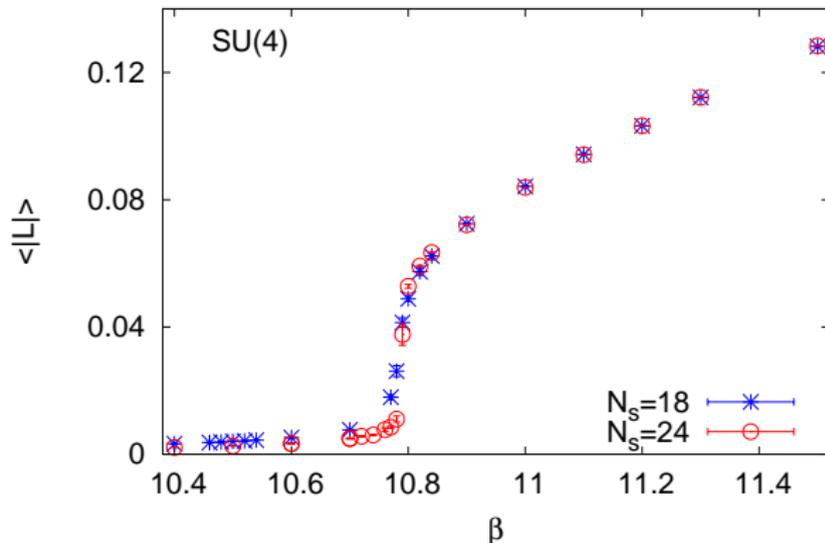
Teper and collaborators, Panero

... no longer a problem



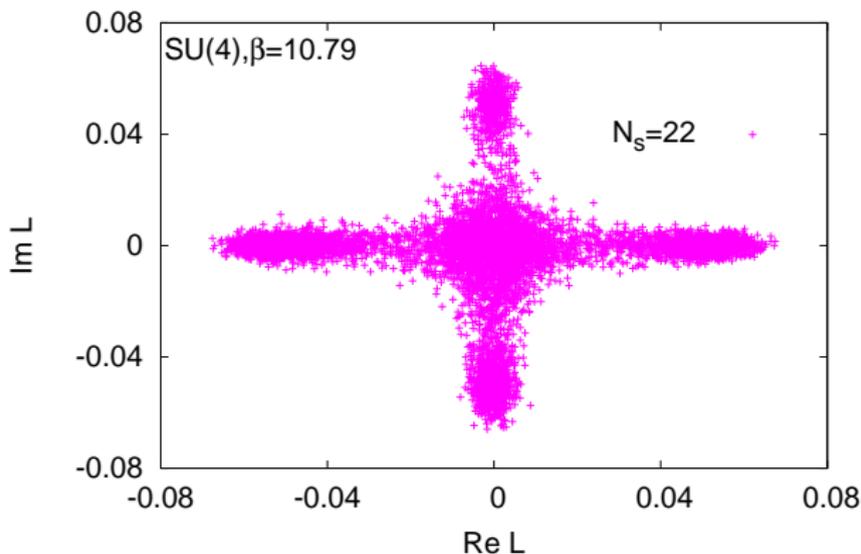
Clear 1st order transition signal in  $L$  but not in plaquette: therefore thermal transition, not bulk. Similarly for  $N_c = 6, 8$  and  $10$ . Computations for  $N_t = 6, 8, 10$  and (sometimes)  $12$ .

... no longer a problem



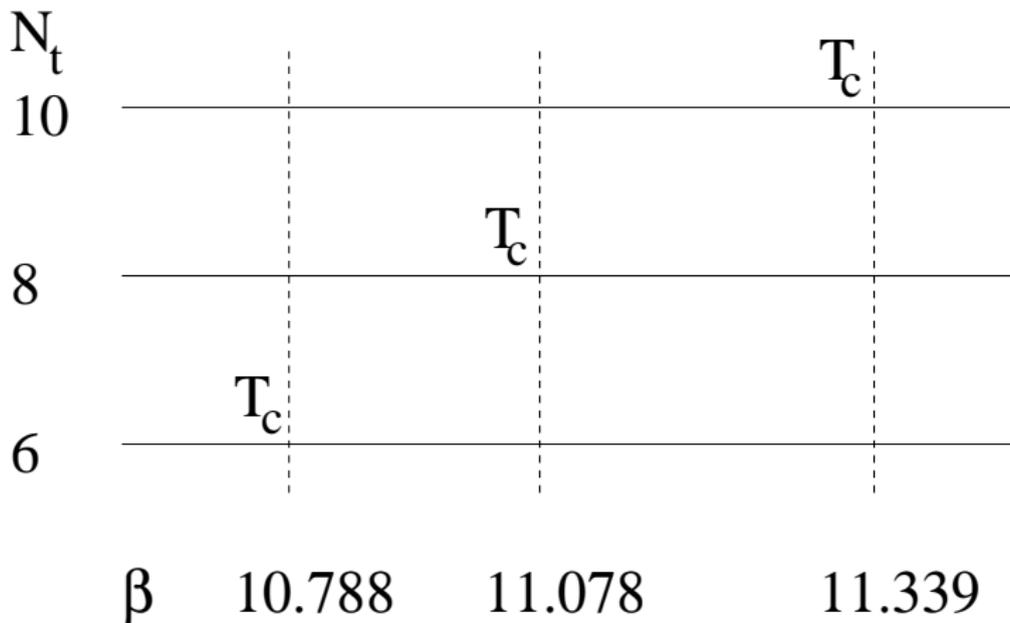
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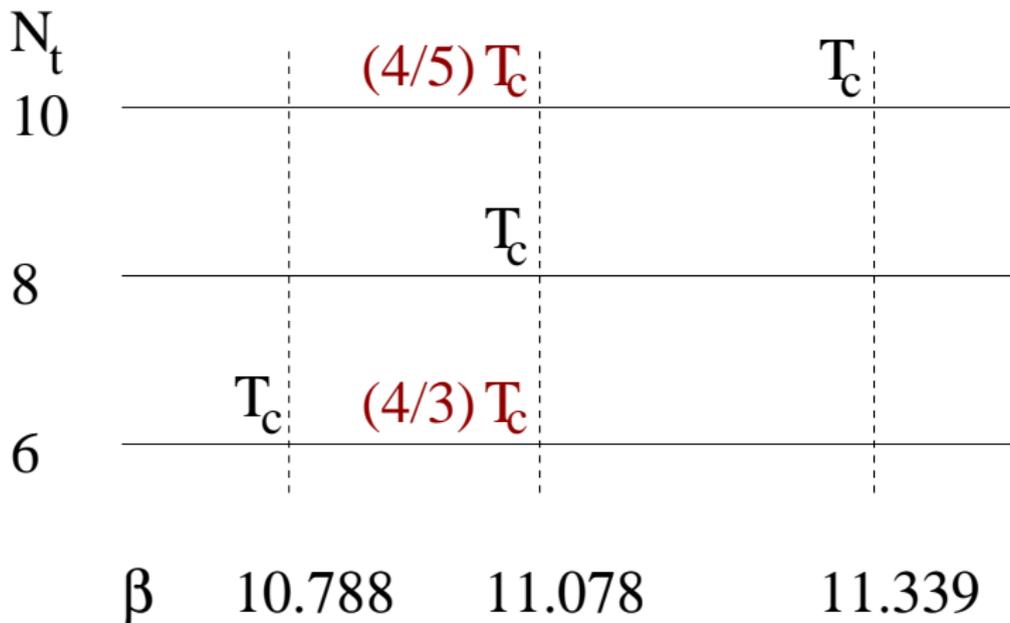
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## Precision test of RG



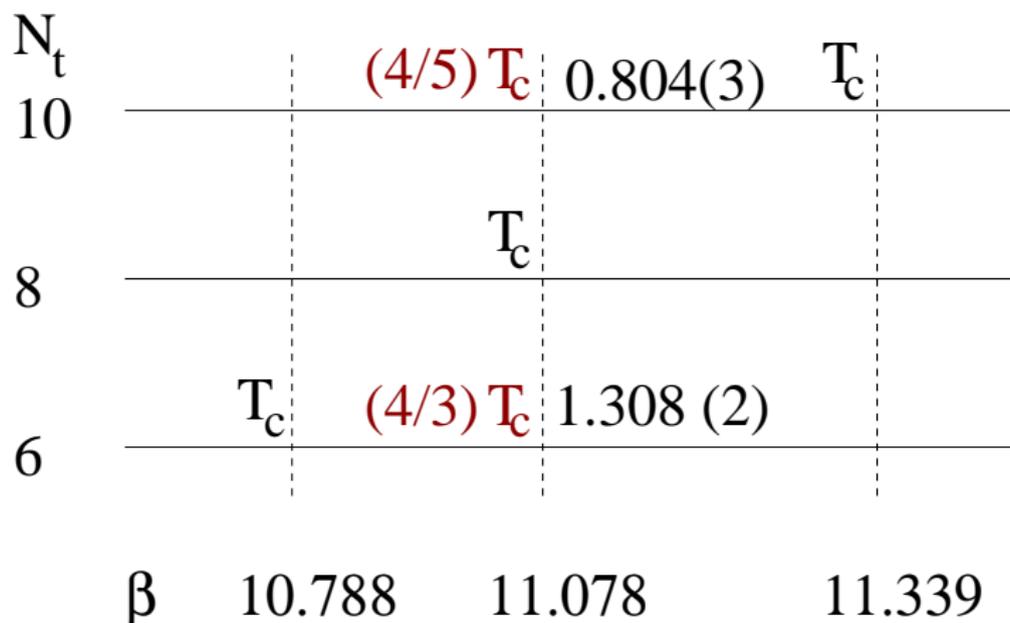
$\beta_c$  determined to one part in  $10^4$ . Strong coupling determined non-perturbatively. 2-loop RG works with precision of two parts in  $10^3$  for  $a \leq 1/(8T_c)$ . For larger  $a$ : non-perturbative RG.

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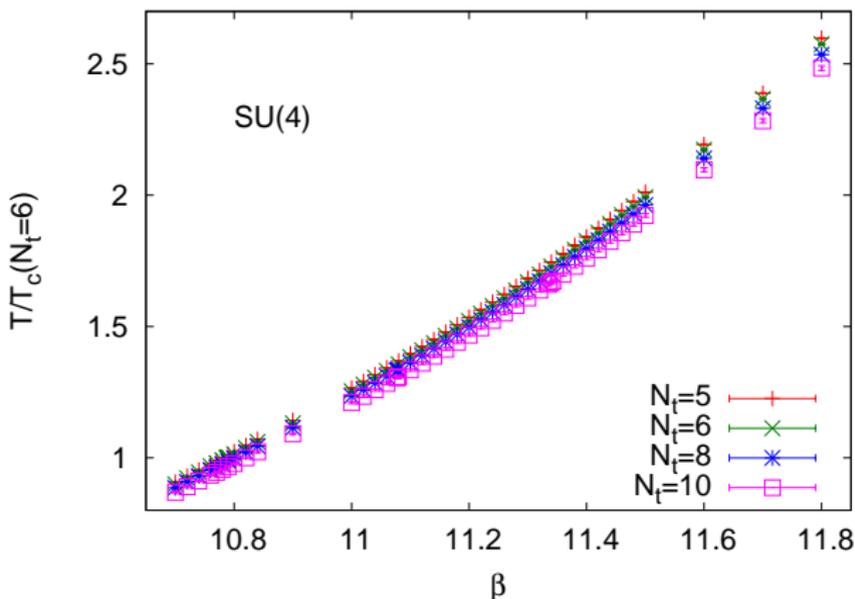
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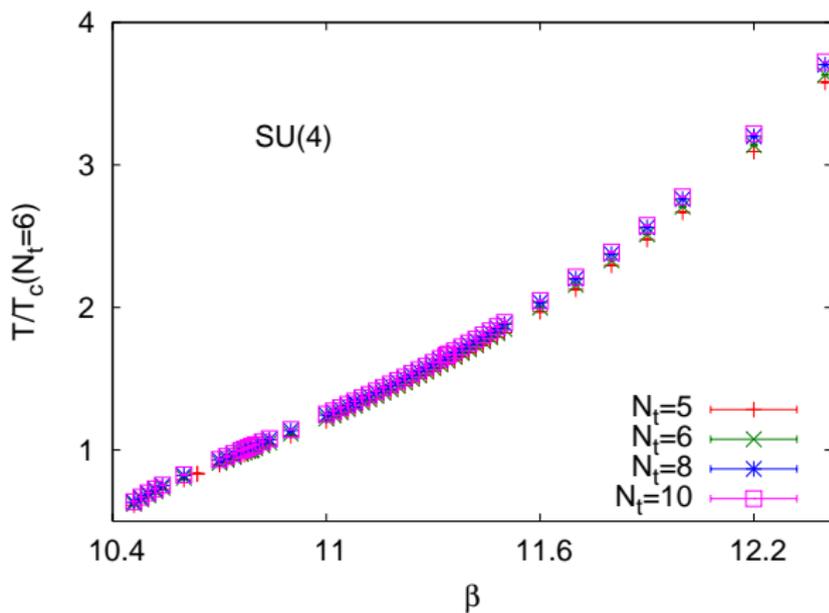
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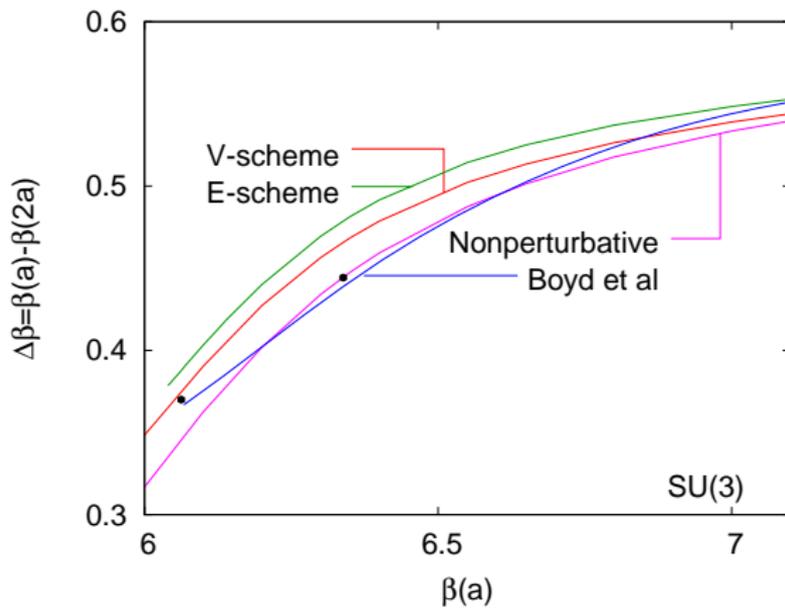
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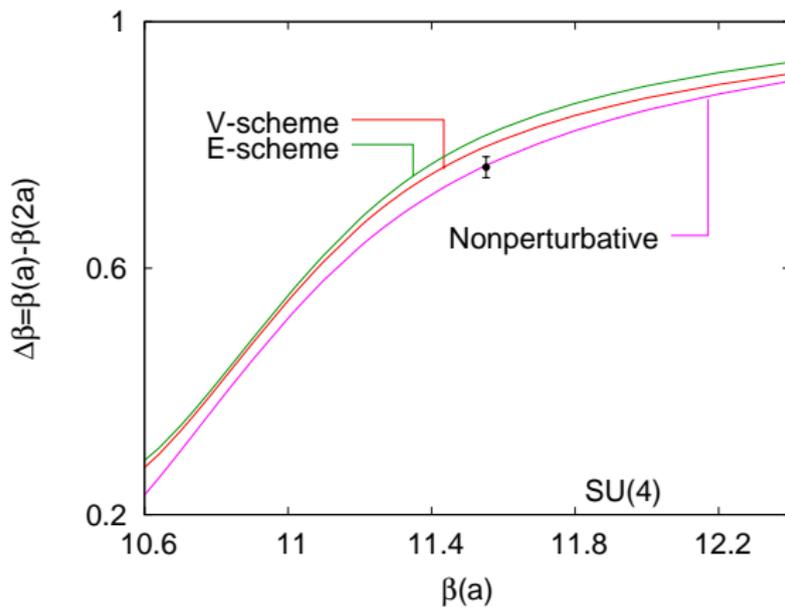


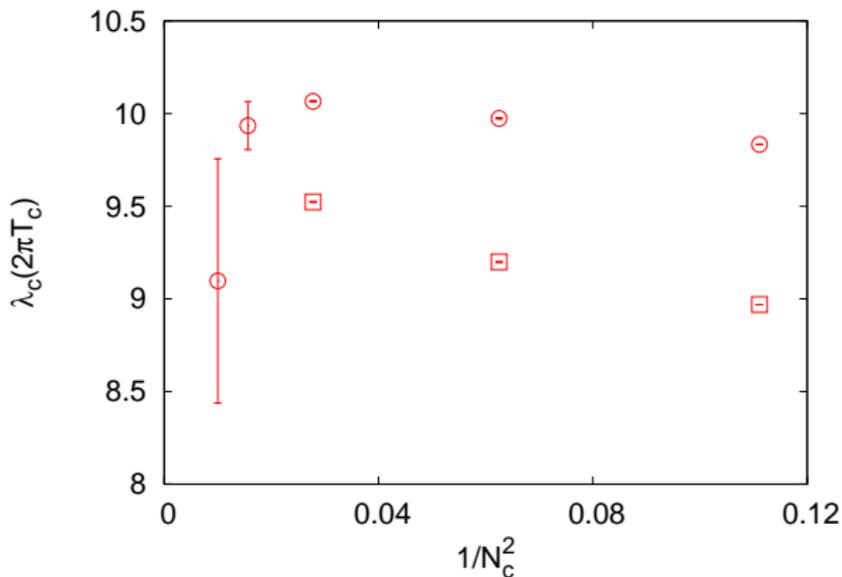
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## Step-scaling functions



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't Hooft scaling: fixed  $\lambda$ , fixed physics

$\lambda_c = g^2(T_c)N_c$ . Clearly non-linear (even ignoring  $N_t = 8$  and 10).

## Very large corrections

Best-fit function:

$$\lambda_c = \begin{cases} 9.8771(4) - \frac{14.2562(2)}{N_c^2} + \frac{54.7830(2)}{N_c^4} & \text{(non-perturbative),} \\ 9.9904(6) + \frac{1.2081(3)}{N_c^2} - \frac{23.5709(3)}{N_c^4} & \text{(2-loop).} \end{cases}$$

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For  $N_c = 3$

$$\lambda_c = \begin{cases} 9.8771(4) - 1.584 + 0.676 & \text{(non-perturbative),} \\ 9.9904(6) + 0.134 - 0.291 & \text{(2-loop).} \end{cases}$$

$1/N_c^2$  and  $1/N_c^4$  corrections nearly equal to each other.

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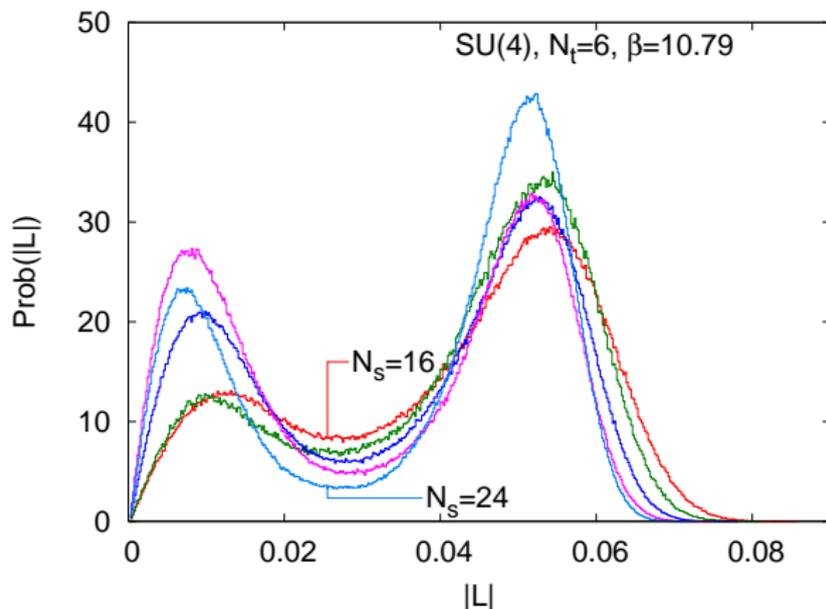
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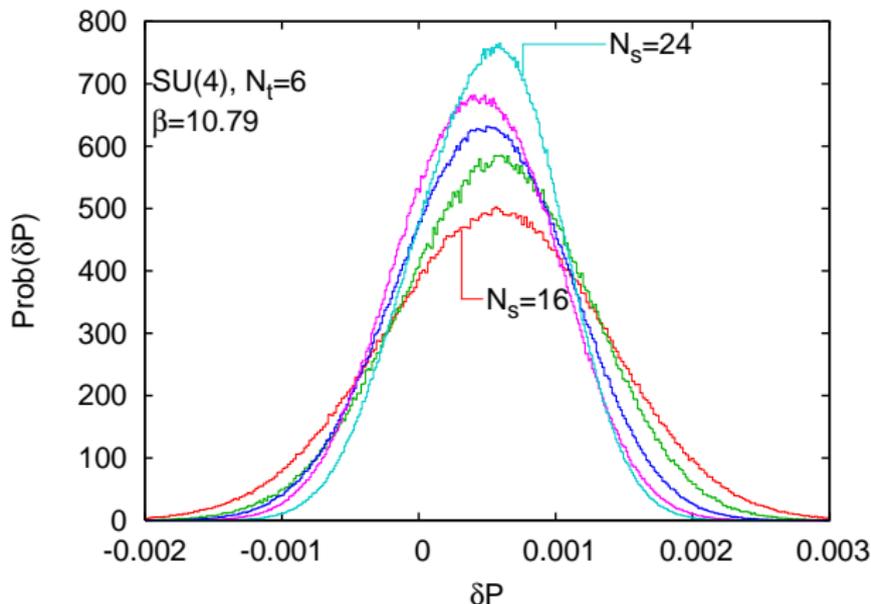
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## Multiple peaks?



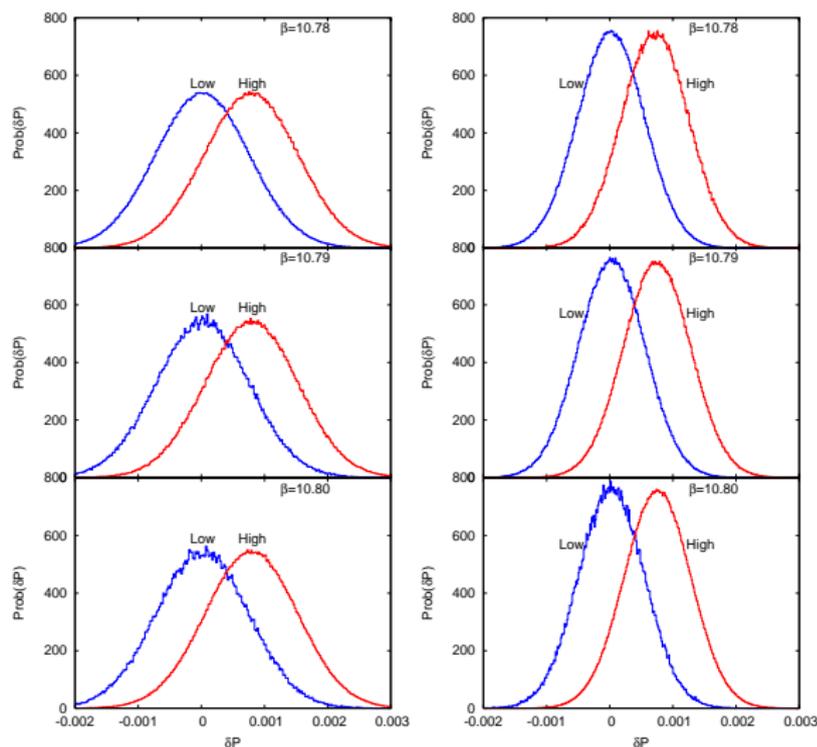
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## Isolate phases using order parameter

Stable against change in  $V$  and  $a$

Is  $N_c = 3$  special?

$N_t$	$N_s$	$\Delta E/T_c^4$	$\Delta E/\Delta_{\max}$
4	16	2.06(1)(3)	
	24	1.93(1)(3)	
	32	1.90(2)(2)	
6	16	1.79(2)(4)	0.65(2)
	32	1.54(2)(5)	
	48	1.44(4)(3)	

$N_c \geq 4$  results stable for  $N_s/N_t \simeq 3$ . But large finite size effect for  $N_c = 3$ .

Scaling with  $N_c$ 

Latent heat depends on  $N_c$  even after scaling by number of gluons:

$T_A = N_c^2 - 1$ . Best fit result—

$$\frac{\Delta\epsilon}{T_A T_c^4} = 0.388(3) - \frac{1.61(4)}{N_c^2}.$$

$N_c = 3$  may have larger finite volume effects than larger  $N_c$ .

For  $N_c = 2$

$$\frac{\Delta\epsilon}{T_A T_c^4} = 0.388(3) - 0.40(1) = 0!$$

Good news? Bad news?

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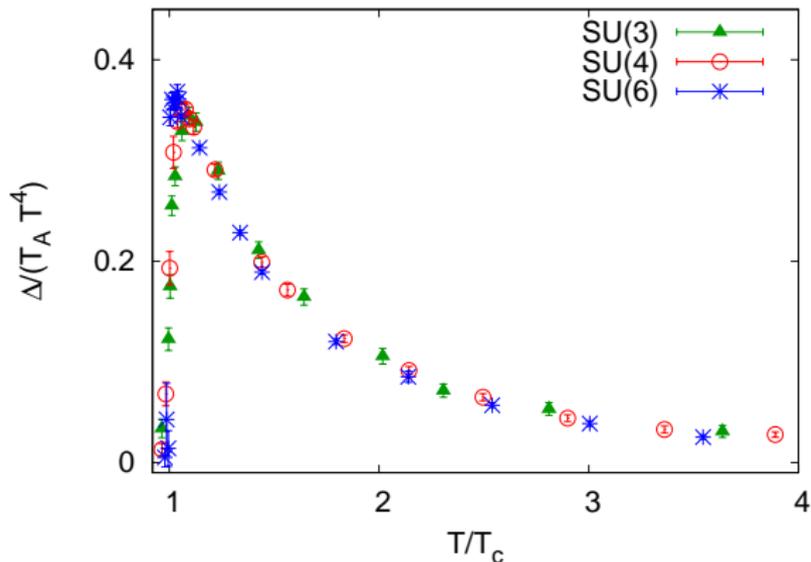
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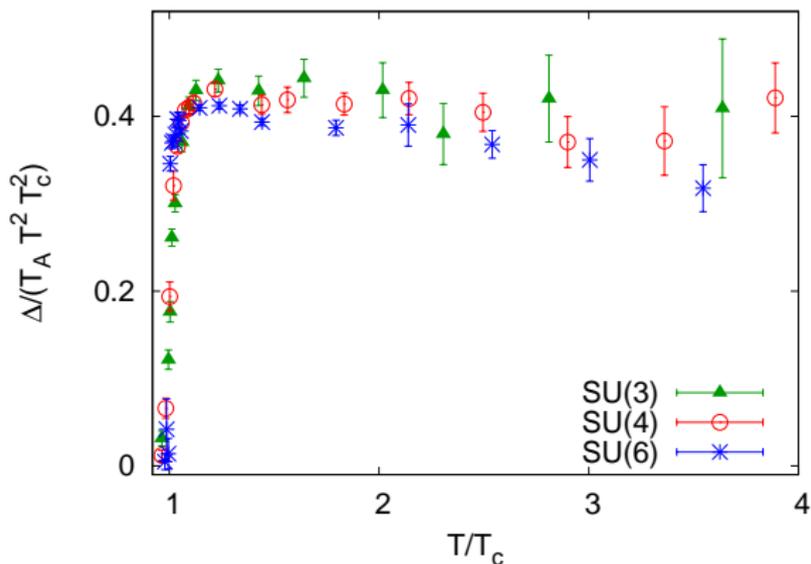
# Conformal symmetry breaking



Good scaling of  $\Delta/T^4 = (E - 3P)/T^4$  with  $N_c$  at fixed  $T/T_c$  (strong  $N_c$  scaling).

$\Delta^{1/4} \simeq T$  even at  $T \simeq 2T_c$  for  $N_c = 3$ : conformal symmetry

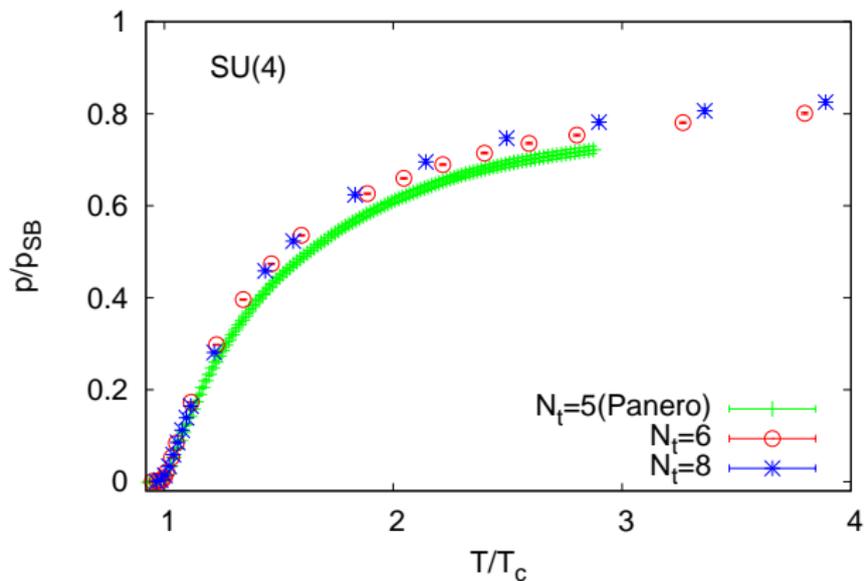
## Evidence for mass?



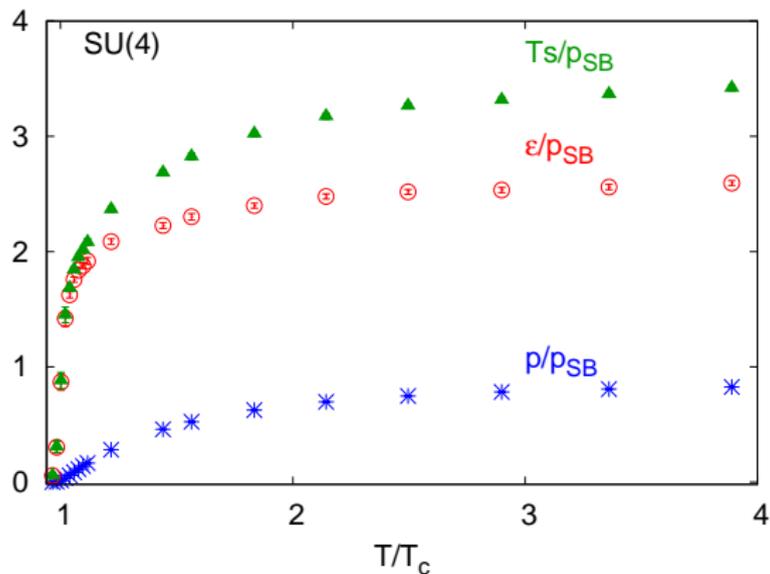
$\Delta/T^2 \simeq \text{constant}$  is generic evidence for mass scales persisting in the high temperature phase.

Meisinger, Miller, Ogilvie, 2002; Pisarski, 2007

## Cutoff dependence of pressure



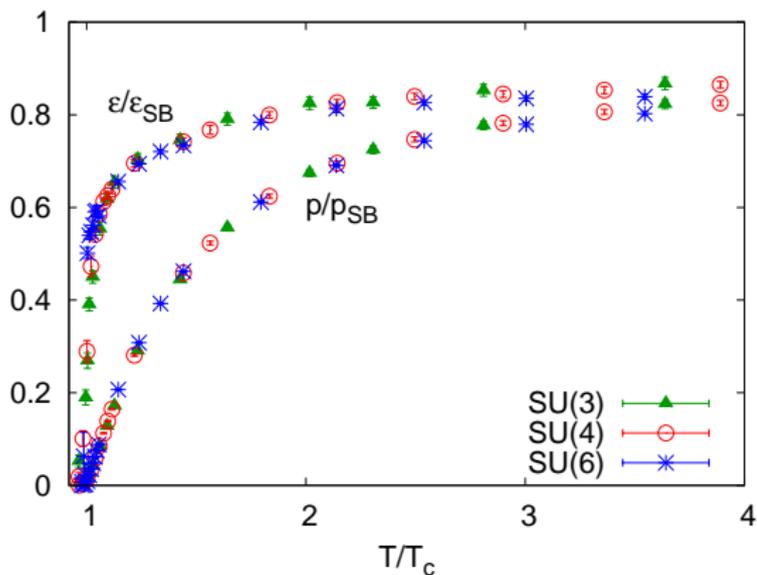
## Main results



Far from ideal gas.

Good scaling of EOS with  $N_c$  at fixed  $T/T_c$  (strong  $N_c$  scaling).

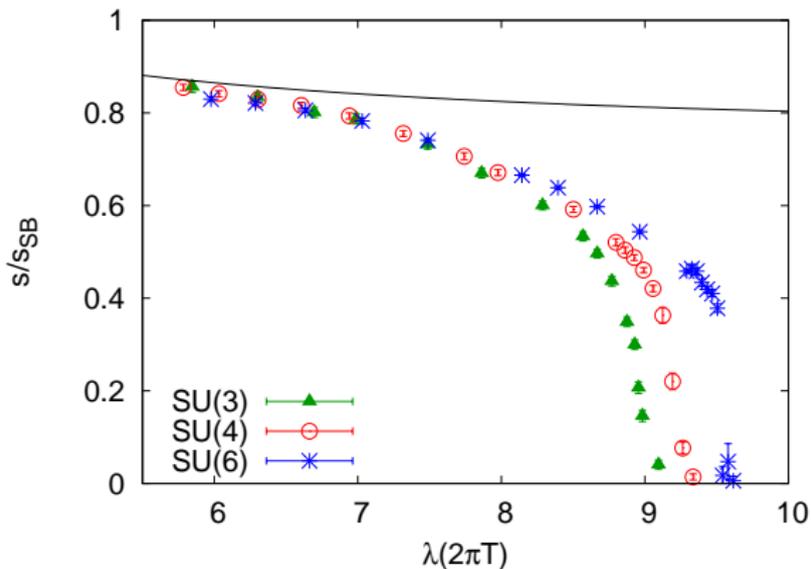
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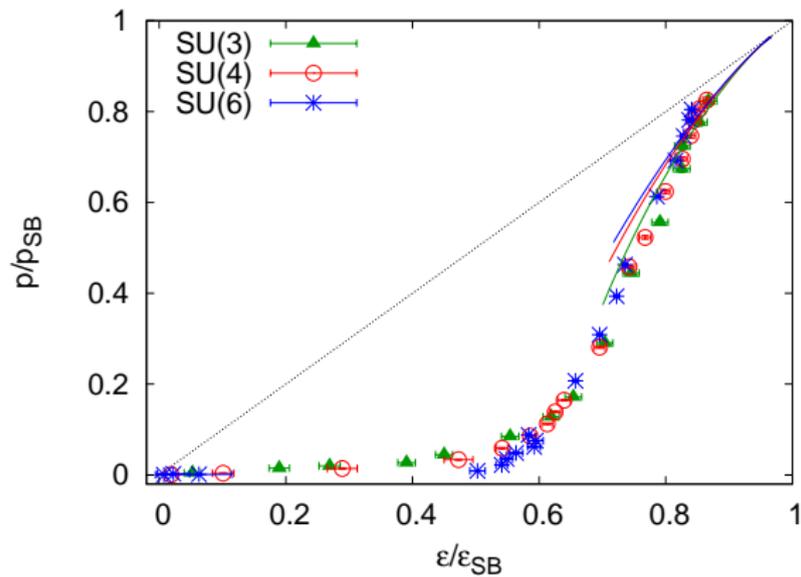
## 't Hooft scaling



't Hooft scaling fails for  $T \leq 2T_c$ .

$\mathcal{N} = 4$  SYM does not describe pure gauge theory.

## Conformal theory?



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## Main results

- ▶ Location of deconfining first order transition measured with high accuracy. Yields high precision test of RG scaling and 't Hooft scaling. 't Hooft coupling at  $T_c$  has large  $1/N_c$  corrections near  $N_c = 3$ . Signs of breakdown of the 't Hooft procedure.
- ▶ New method developed for determination of latent heat in gluo $N_c$  plasmas. Find

$$\frac{\Delta\epsilon}{T_A T_c^4} = 0.388(3) - \frac{1.61(4)}{N_c^2}.$$

- ▶ Strong  $N_c$  scaling works very well for EOS. Conformal symmetry strongly broken up to  $T \simeq 3T_c$ . Resummed weak coupling theory works much better in description of lattice data.