How close is QCD to its conformal cousins?

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TIFR

ICTS discussion meeting on
Applied String Theory
TIFR, Mumbai
March 22, 2011

Outline

Why study gluoN_c plasmas?

1. Because they may model the kind of problems which we would like to solve for QCD. The $N_c \to \infty$, $g^2 \to 0$ ($\lambda = g^2 N_c$ fixed) limit is the toyland in parts of which the AdS/CFT correspondence can be applied. The approach to $N_c = 3$ from other parts of this toyland may be tested by computations with small N_c.

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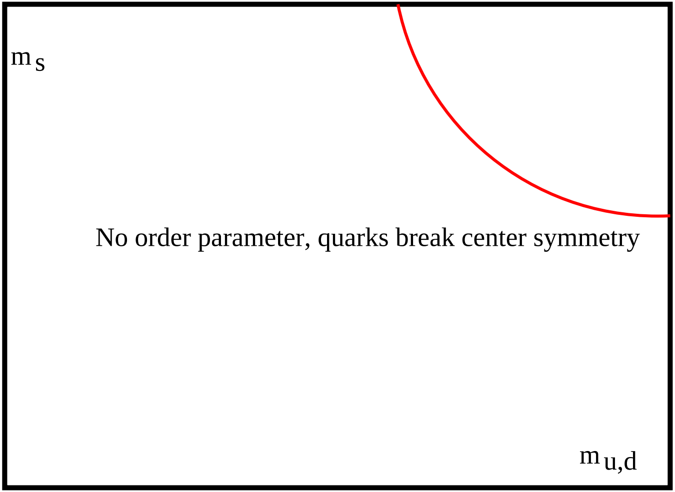
3. Because we can take the continuum limit for $3 \leq N_c \leq 10$ pure gauge theory and obtain precision results in thermodynamics. Also answer other interesting questions about the large N_c limit.

Flag diagram of QCD



Flag diagram is a projection of the phase diagram. Columbia 1990

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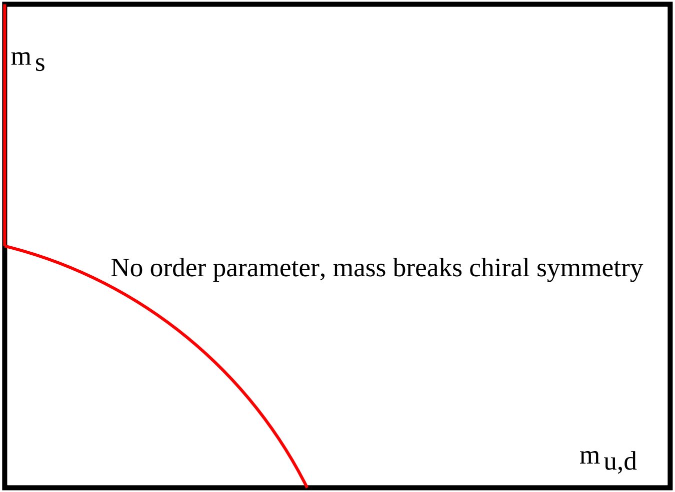

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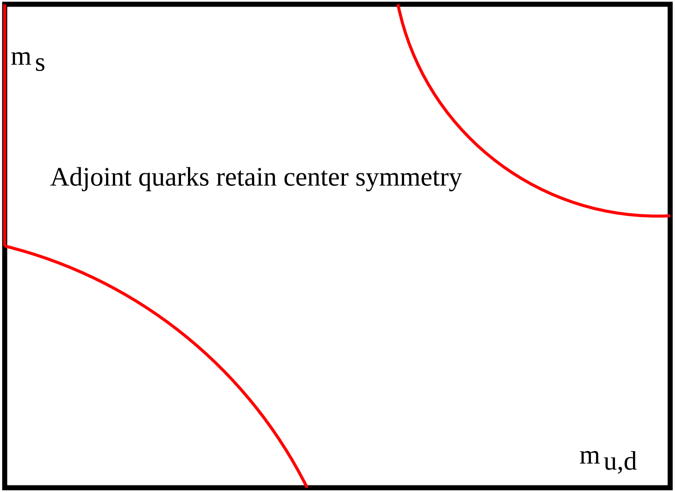

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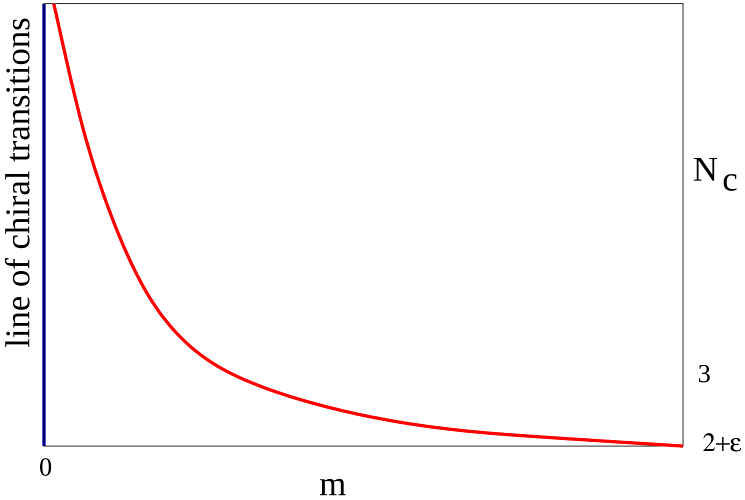
Flag diagram of gluoN_c plasmas



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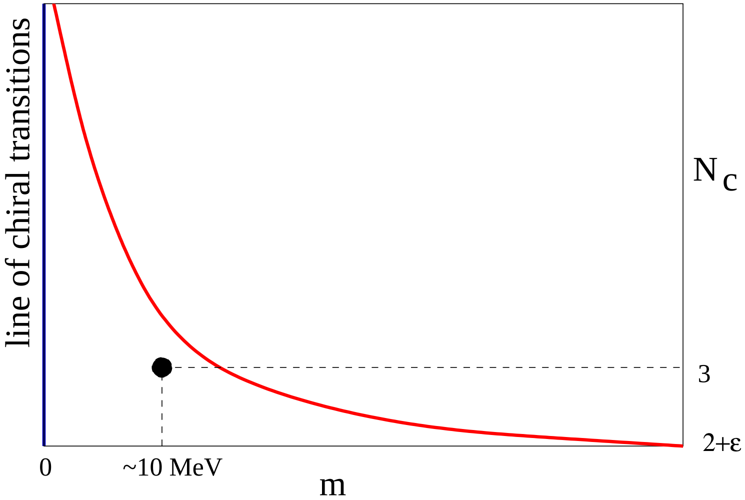

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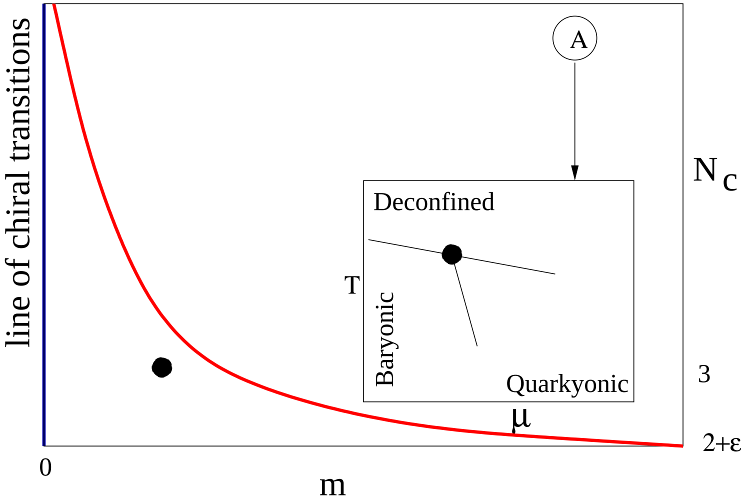

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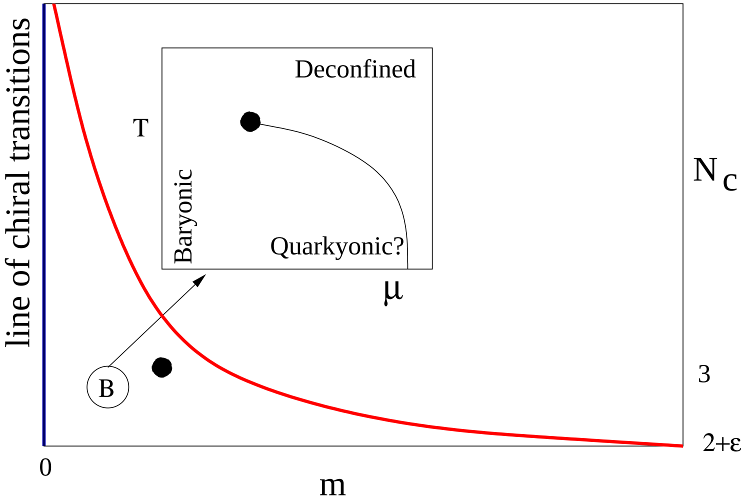


Outline

Cutoff and continuum theories

The cutoff theory on the lattice

The lattice spacing is a. Tune the lattice spacing by tuning the bare coupling, g_B^2. At finite temperature: $N_t a = 1/T$, *i.e.*, extent of Euclidean time direction is equal to $1/T$. High temperature expansions can be performed at strong coupling.

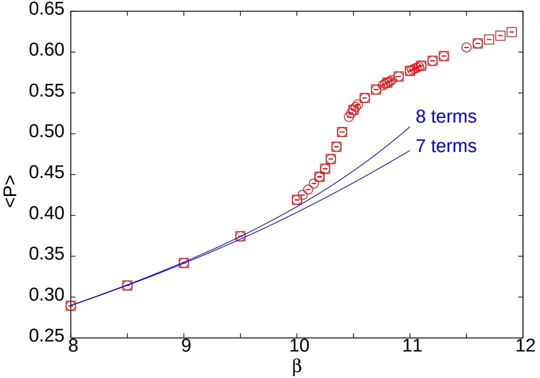
Strategy for taking the continuum limit

Fix N_t, vary g_B^2 to find where the deconfinement transition takes place. This gives $a_c(N_t)$. Then make a $T = 0$ measurement with a_c to find $\alpha_s(a_c)$. If a_c is small enough then 2-loop RG can be used to give

$$a_c \Lambda_{\overline{MS}} = \frac{N_t \Lambda_{\overline{MS}}}{T_c} = RG(\alpha_s).$$

SG, 1999

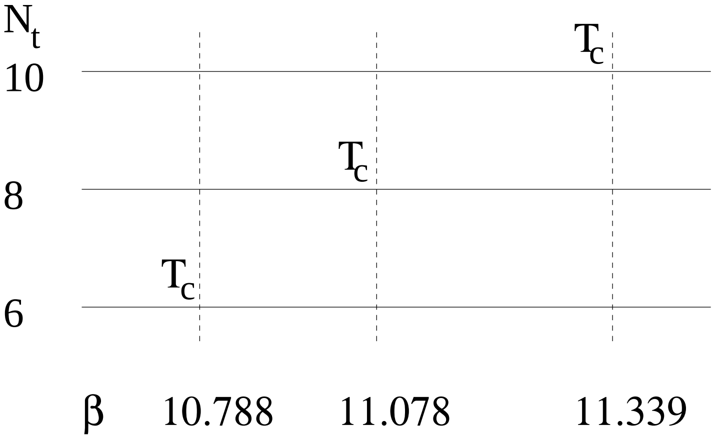
Unphysical strong–weak coupling crossover



Wilson action. Earlier generation of simulations limited by this bulk transition. Solution now: move to smaller lattice spacing; work only in the sector which can be continued to the continuum.

Gavai and Datta, Majumdar

Precision test of RG



β_c determined to one part in 10^4; α_s determined at same lattice spacing. 2-loop RG works with precision of about one part in 10^3 for $a \leq 1/(8T_c)$. For larger a: non-perturbative RG.

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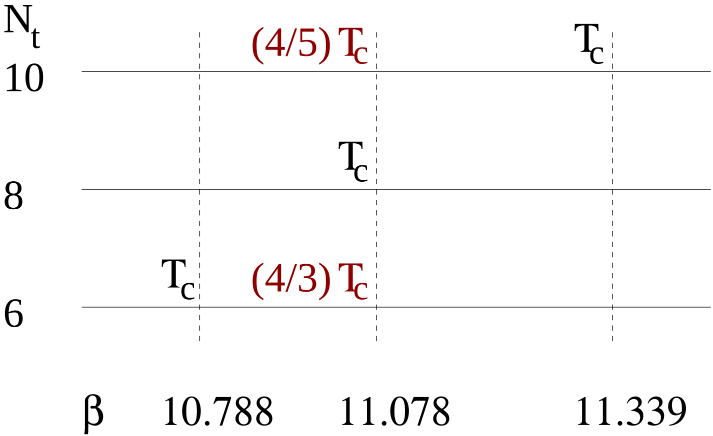

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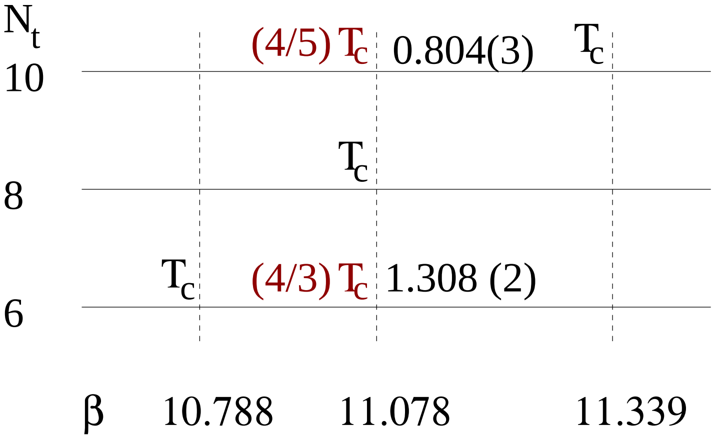

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$T_c(N_c)$

RG scaling yields measurement of $T_c/\Lambda_{\overline{MS}}$ at each N_c: *i.e.*, the phase boundary of pure gauge theory.
Convert to

$$\lambda_c(N_c) = N_c \alpha_S(2\pi T_c).$$

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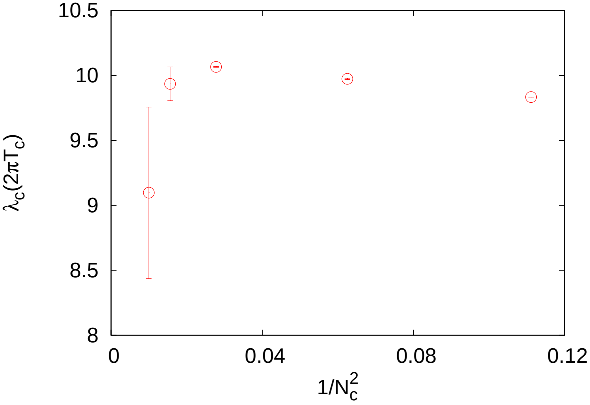
Alternate description of the phase boundary available: measure $T_c/\sqrt{\sigma}$ or T_c/M (σ: string tension, M: glueball mass). In terms of these, very small corrections in powers of $1/N_c^2$: call this the strong scaling procedure.

Results for the phase boundary



Phase boundary clearly nonlinear (even excluding $N_c = 8$ and 10).

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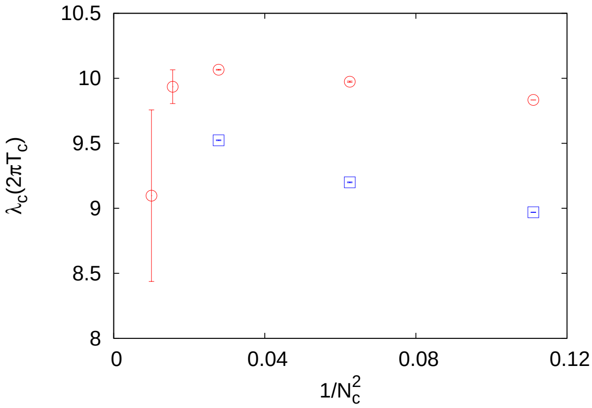

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Shape of the phase boundary

Best fit series

$$\lambda_c = \begin{cases} 9.8771(4) - \frac{14.2562(2)}{N_c^2} + \frac{54.7830(2)}{N_c^4} & \text{(non-perturbative)}, \\ 9.9904(6) + \frac{1.2081(3)}{N_c^2} - \frac{23.5709(3)}{N_c^4} & \text{(2-loop)}. \end{cases}$$

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Improving the series

Need more terms in the series; impossible with same data. Try a simple Padé resummation:

$$\lambda_c(N_c) = \lambda_c(\infty) + \frac{\gamma/N_c^2}{1 + N_*^2/N_c^2}$$

Best fit has $N_* \simeq 4$.

Outline

Phase diagrams

RG scaling and the continuum limit

The equation of state

Summary

The equation of state of gluoN_c plasma

Energy density and pressure both functions of T:

$$\frac{E}{T^4} = \frac{1}{VT^2} \left. \frac{\partial \log Z}{\partial T} \right|_V \qquad \frac{P}{T^4} = \frac{1}{T^3} \left. \frac{\partial \log Z}{\partial V} \right|_T .$$

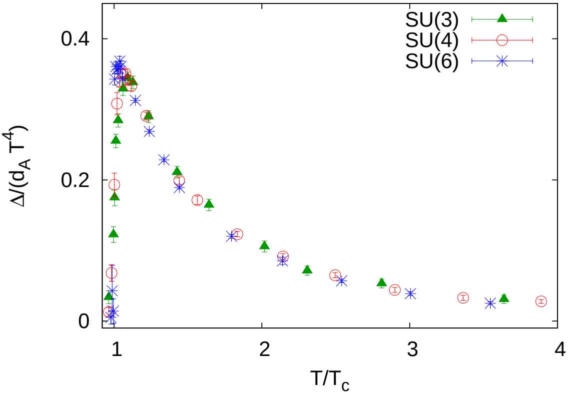
Derived quantities are

$$\frac{\Delta}{T^4} = \frac{E - 3P}{T^4} \qquad \frac{S}{T^3} = \frac{E + P}{T^4}.$$

These dimensionless quantities are functions of $T/\Lambda_{\overline{MS}}$. One can also obtain the speed of sound. A second derivative of the free energy give the specific heat c_V.

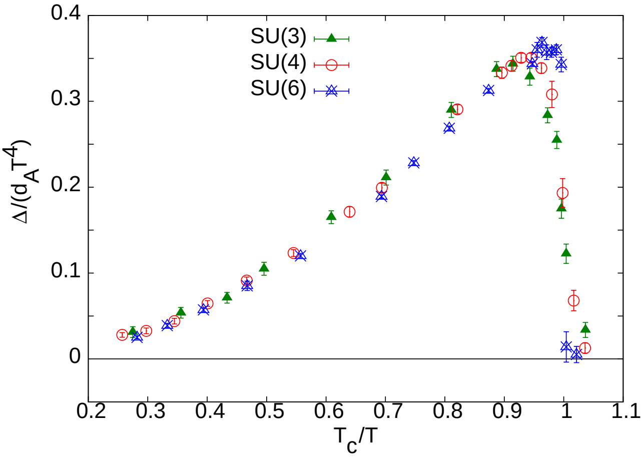
N_c dependence: scaling as N_c^0 below T_c, as $d_A = N_c^2 - 1$ above T_c.

Conformal symmetry breaking



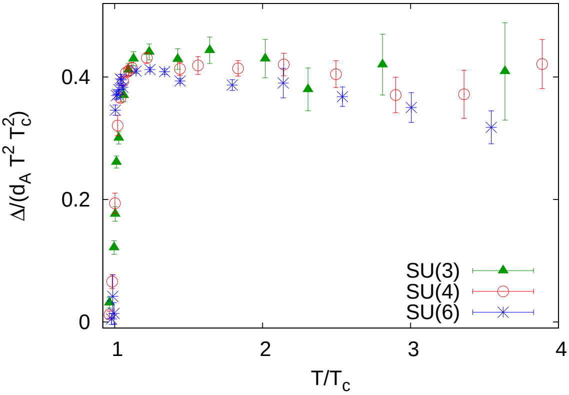
Good scaling of $\Delta/T^4 = (E - 3P)/T^4$ with N_c at fixed T/T_c (strong N_c scaling).
$\Delta^{1/4} \simeq T$ even at $T \simeq 2T_c$ for $N_c = 3$: no conformal symmetry

Detailed view near T_c



Peak shifts towards T_c with increasing N_c. Evidence for stronger finite size effects for $N_c = 3$? Measure correlation lengths in future.

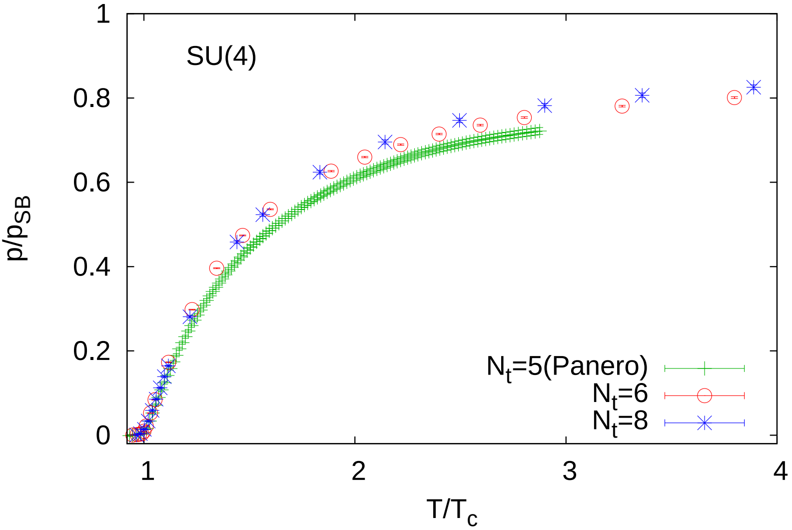
Evidence for mass?



$\Delta/T^2 \simeq$ constant is generic evidence for mass scales persisting in the high temperature phase.

Meisinger, Miller, Ogilvie, 2002; Pisarski, 2007

Cutoff dependence of pressure



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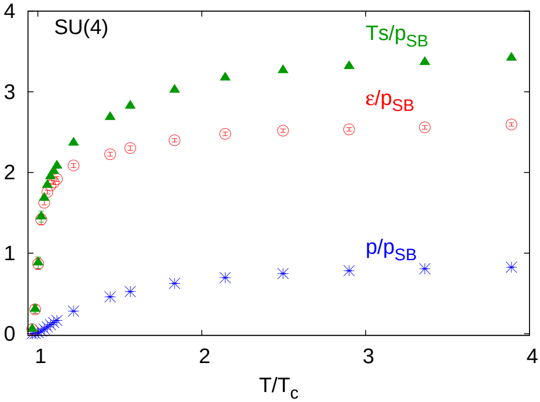

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Good scaling of EOS with N_c at fixed T/T_c (strong N_c scaling).

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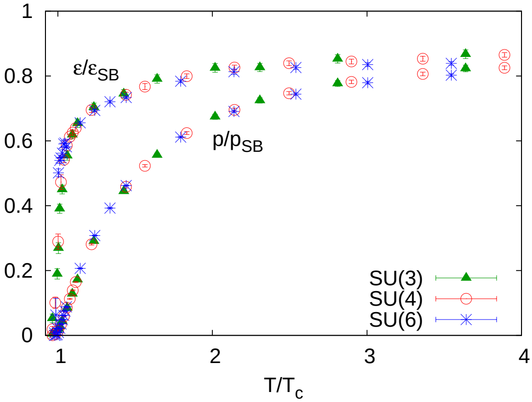

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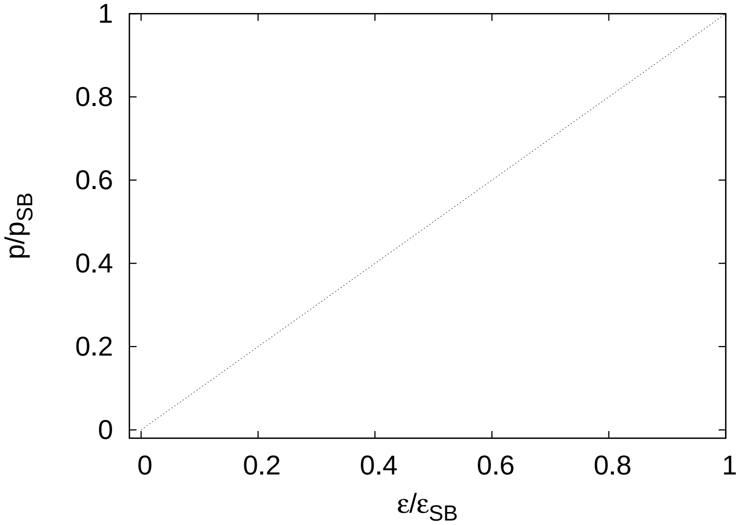
Conformal theory?



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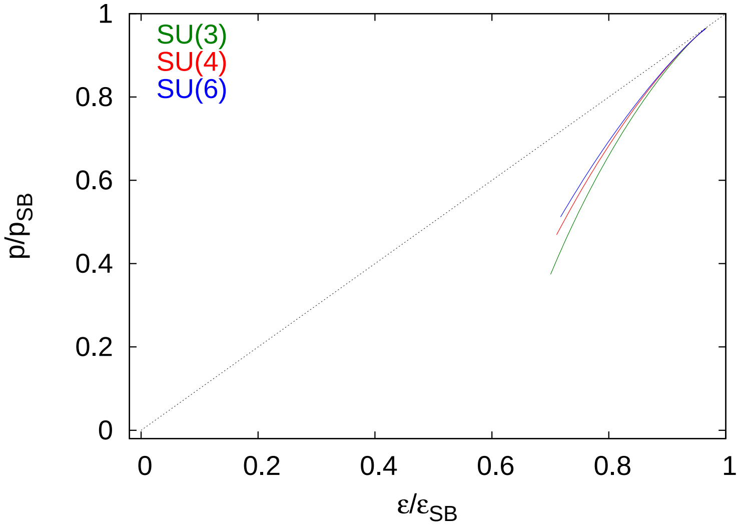

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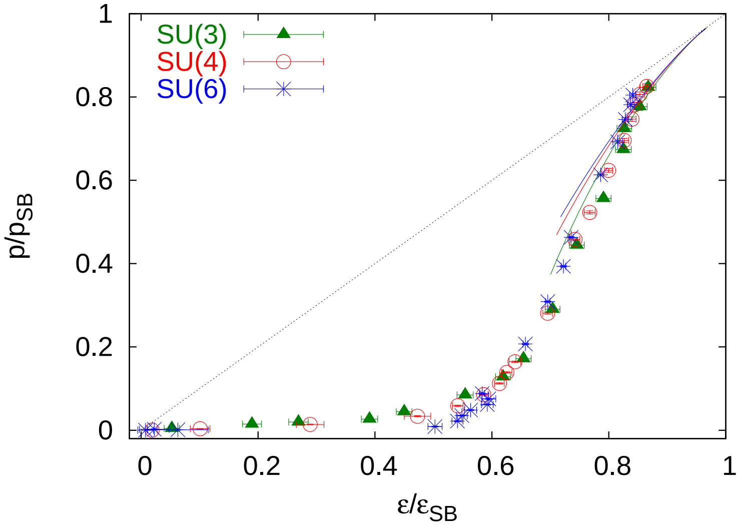

Outline

Main results

- Location of deconfining first order transition measured with high accuracy. Yields high precision test of RG scaling. Continuum limit can be taken from $a \leq 1/(8T_c)$ using 2-loop RG.
- Phase boundary, $T_c(N_c)$, determined accurately at finite N_c. Extrapolation to $N_c \to \infty$ hard in terms of $T_c/\Lambda_{\overline{MS}}$ ('t Hooft scaling) because series expansion in $1/N_c^2$ may have radius of convergence which excludes $N_c \leq 4$. In terms of $T_c/\sqrt{\sigma}$ may be simpler (strong scaling).
- EOS determined with good accuracy. Strong scaling works, *i.e.*, E/T^4, P/T^4, *etc.*, scale well from $N_c \simeq 3$ to $N_c = \infty$ at fixed T/T_c. Possible violation of strong scaling near T_c.
- EOS measured on lattice closer to weak-coupling theory than to conformal theory up to $T \simeq$ 2–3T_c. At $T = 2T_c$, $\Delta^{1/4} \simeq T$.