

# Sign-posting the phase diagram of QCD

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- 1 Introduction
- 2 Fluctuations of conserved quantities
- 3 Comparing data and lattice

# Outline

- 1 Introduction
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# Heavy-ion physics

## Experimental observations

Many interesting new phenomena: jet quenching, elliptic flow, strange chemistry, fluctuations of conserved quantities ...

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## Theoretical underpinning

Does QCD describe this matter? Is there a new nonperturbative test of QCD?

# Predictions from QCD

- Lagrangian has free parameters: cutoff  $a$ , quark masses  $m_u \simeq m_d \ll \Lambda_{QCD}$ ,  $m_s \simeq \Lambda_{QCD}$ ,  $\dots$
- Compute enough quantities from QCD:  $m_\pi(a, m_{ud}, m_s, \dots)$ ,  $m_K(a, m_{ud}, m_s, \dots)$ ,  $f_K(a, m_{ud}, m_s, \dots)$ ,  $f_\pi(a, m_{ud}, m_s, \dots)$ ,  $m_\rho(a, m_{ud}, m_s, \dots)$ ,  $m_p(a, m_{ud}, m_s, \dots)$ ,  $T_c(a, m_{ud}, m_s, \dots)$ ,  $T_E(a, m_{ud}, m_s, \dots)$ ,  $\mu_E(a, m_{ud}, m_s, \dots)$
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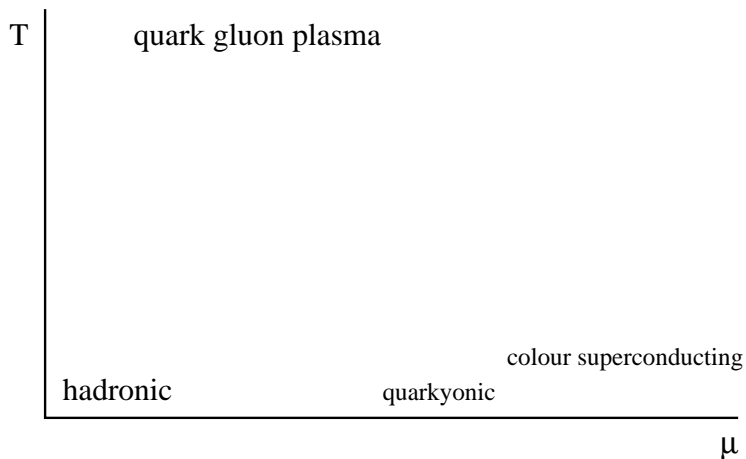
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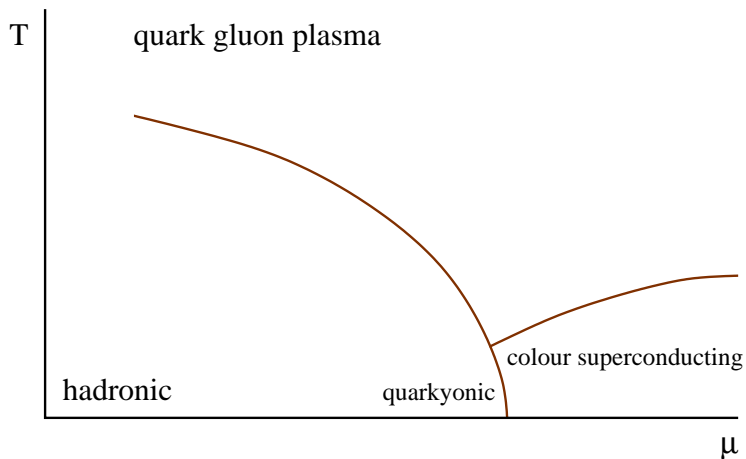
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- Most universal part of the solution: Moore's law

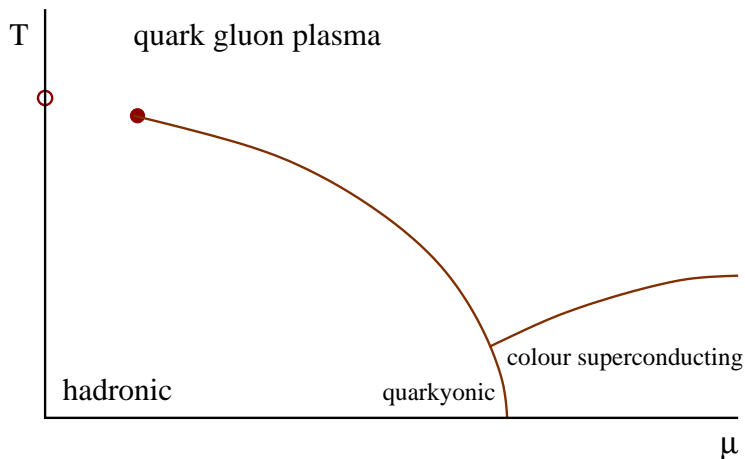
# The phase diagram of QCD



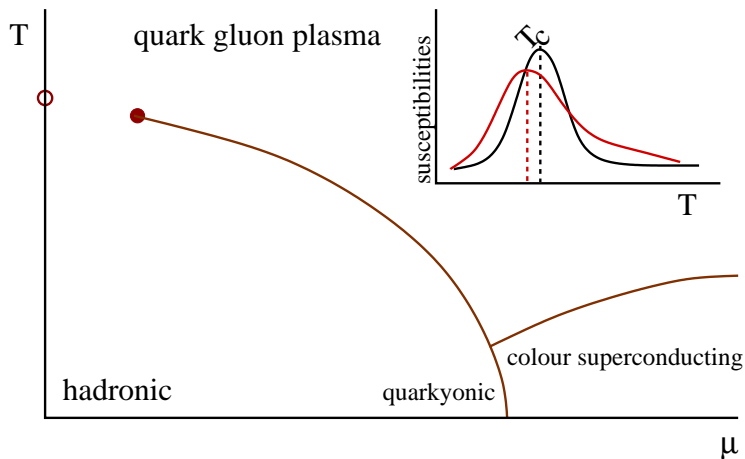
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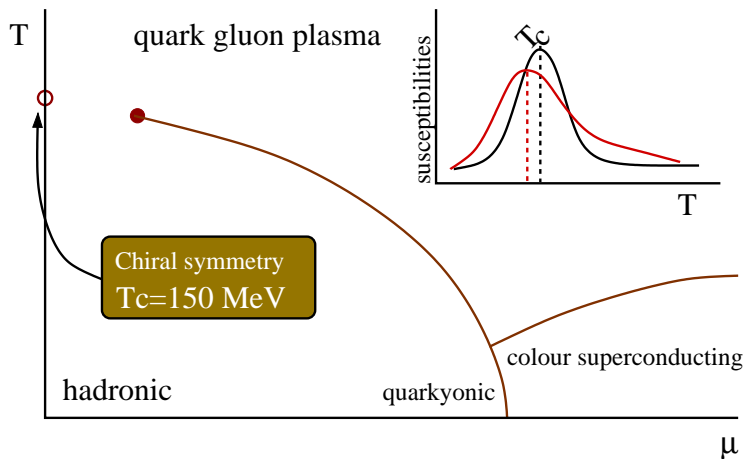
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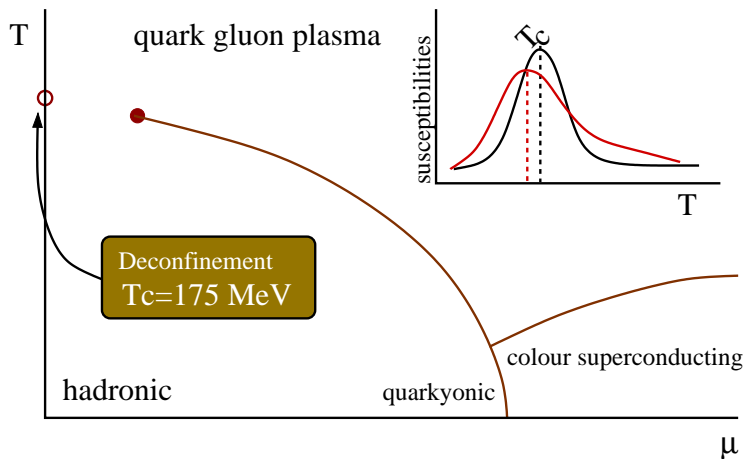
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Y. Aoki *et al.*, Phys. Lett. B 643 (2006) 46

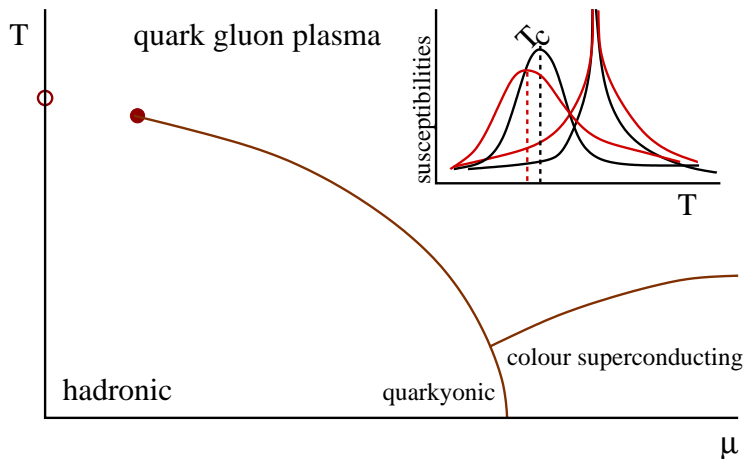


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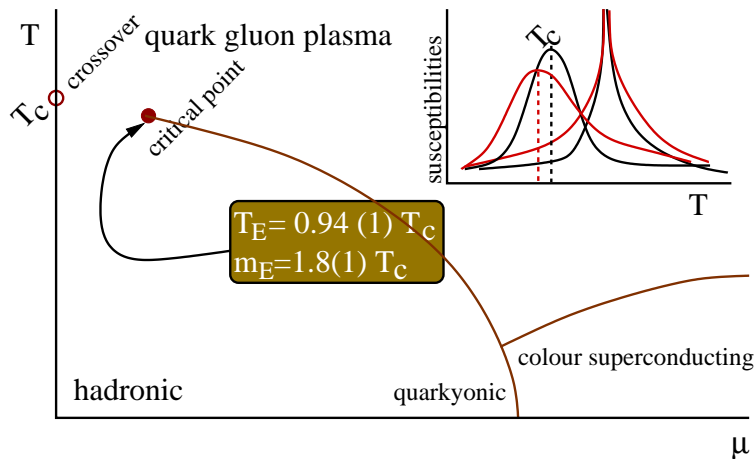
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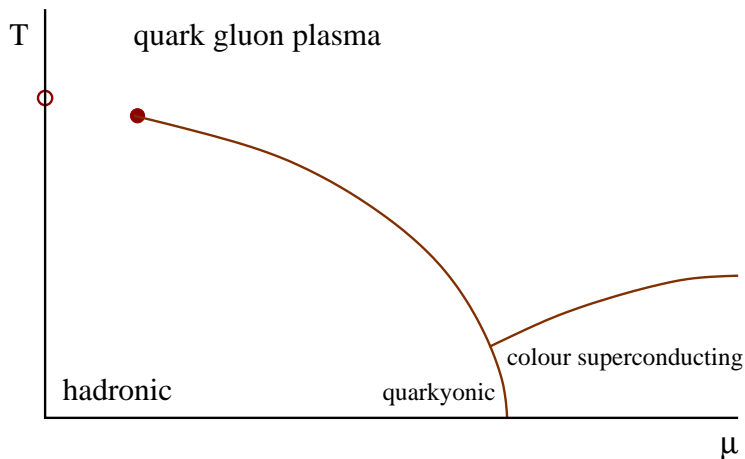
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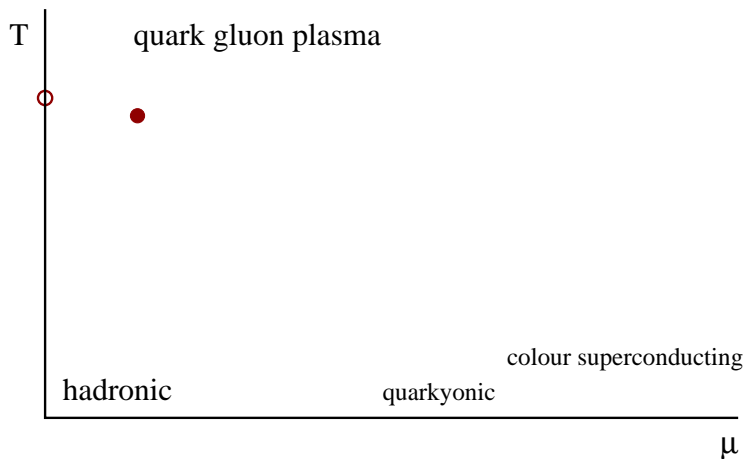
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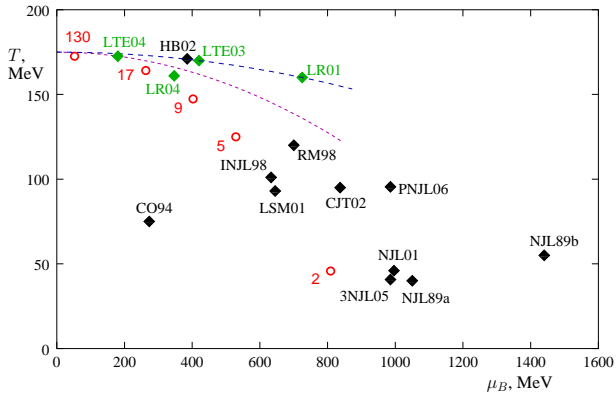
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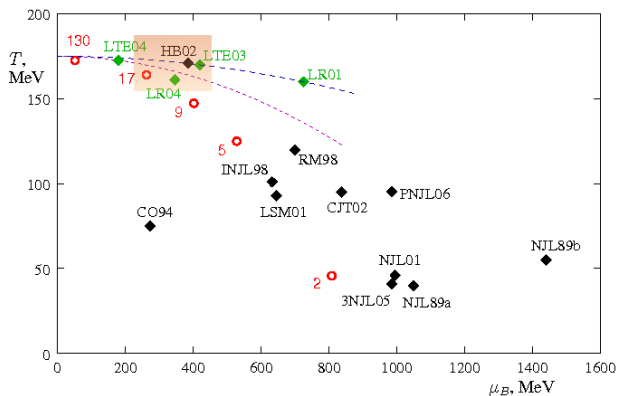
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# Improvement over time



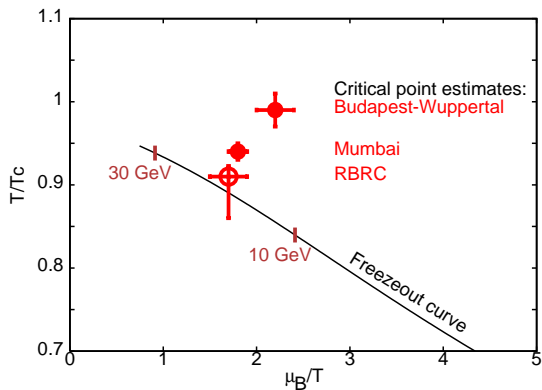
Compilations: Stephanov, Lattice 2006

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Compilations: SG, Quark Matter 2011



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# Fluctuations of conserved quantities

## Observations

In a single heavy-ion collision, each conserved quantity ( $B$ ,  $Q$ ,  $S$ ) is exactly constant when the full fireball is observed. In a small part of the fireball they fluctuate: from part to part and event to event.

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## Thermodynamics

If  $\xi^3 \ll V_{obs} \ll V_{fireball}$ , then fluctuations can be explained in the grand canonical ensemble: energy and  $B$ ,  $Q$ ,  $S$  allowed to fluctuate in one part by exchange with rest of fireball (diffusion: transport).

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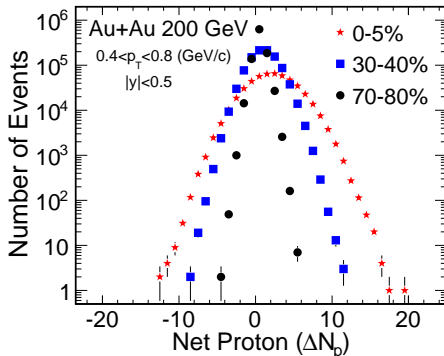
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## Comparison

Is the observed volume small compared to the volume of the fireball? Are observations in agreement with QCD thermodynamics?

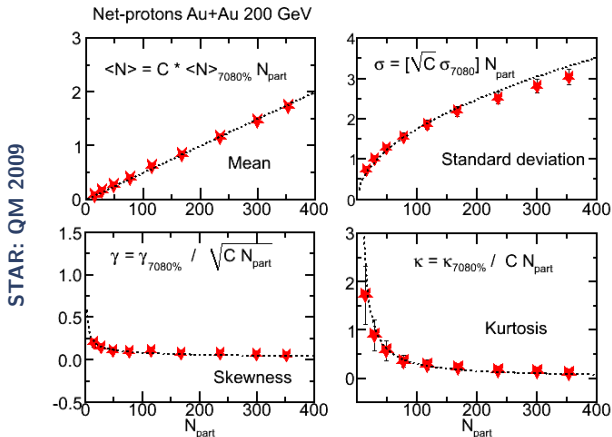
# Event-to-event fluctuations



STAR arxiv:1004.4959

Central rapidity slice taken.  $p_T$  of 400–800 MeV. Important to check dependence on impact parameter. Protons observed: isospin fluctuations small.

# Shape of distribution



Shape of distribution captured in cumulants  $[B^n]$ . Cumulants change with volume (proxy:  $N_{part}$ ), and tends to Gaussian.

# QCD predictions at finite $\mu_B$

Make a MacLaurin expansion of the (dimensionless) pressure:

$$\frac{1}{T^4} P(T, \mu) = \sum_{n=0}^{\infty} T^{n-4} \chi_B^{(n)}(T, 0) \frac{(\mu/T)^n}{n!},$$

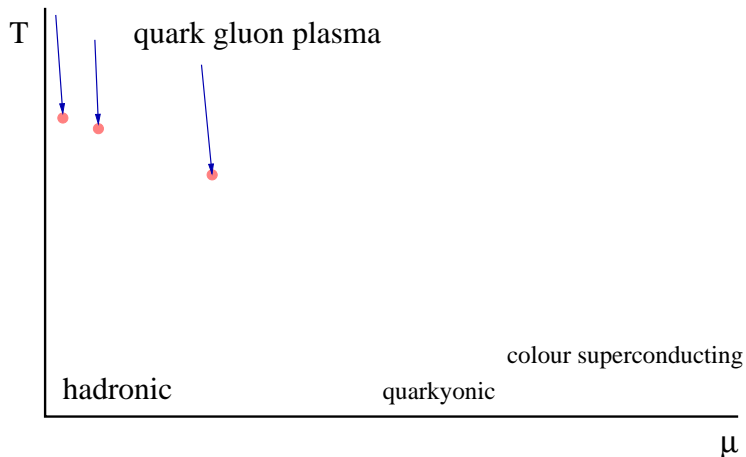
measure each NLS at  $\mu = 0$ , sum series expansion to find NLS at any  $\mu$ . Shape variables:  $[B^n] = (VT^3) T^{n-4} \chi_B^{(n)}(t, \mu)$ . Ratios of cumulants are state variables:

$$m_1 : \quad \frac{[B^3]}{[B^4]} = \frac{T \chi_B^{(3)}}{\chi_B^{(2)}} = S\sigma$$

$$m_2 : \quad \frac{[B^4]}{[B^2]} = \frac{T \chi_B^{(4)}}{\chi_B^{(2)}} = \kappa\sigma^2$$

$$m_3 : \quad \frac{[B^4]}{[B^3]} = \frac{T \chi_B^{(4)}}{\chi_B^{(3)}} = \frac{\kappa\sigma}{S}$$

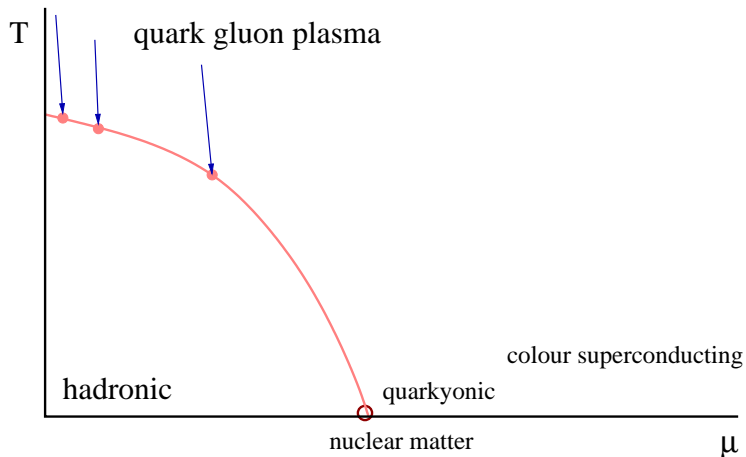
# The freezeout curve



Hadron gas models: Hagedorn, Braun-Munzinger, Stachel, Cleymans, Redlich, Becattini

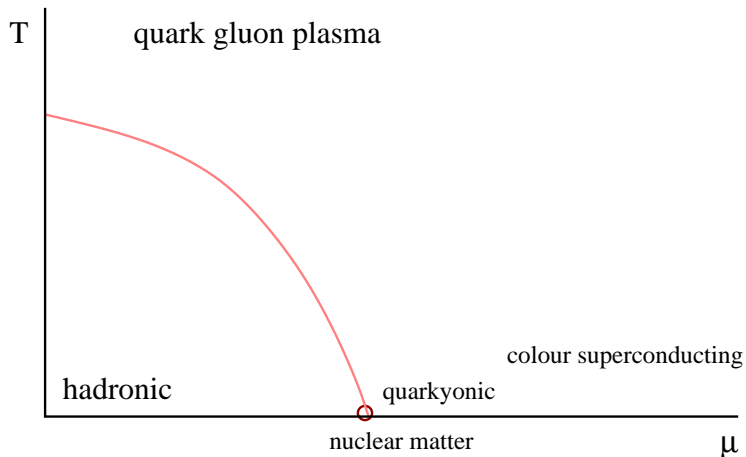


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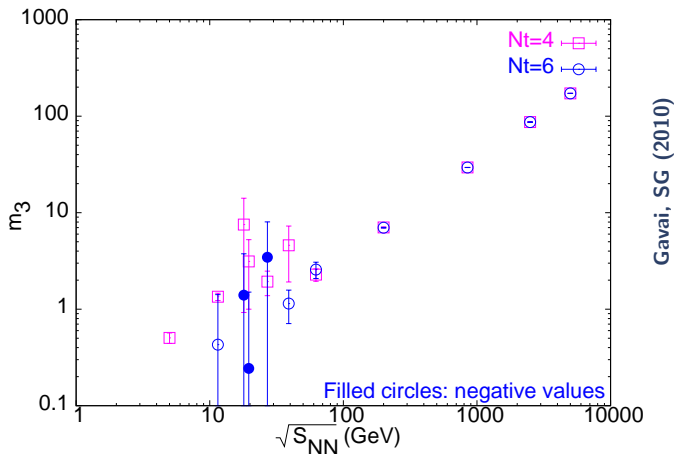
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# Predictions along the freezeout curve

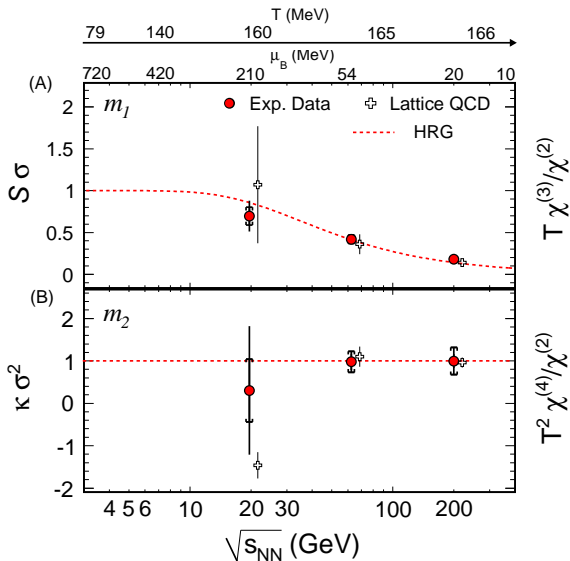


Lattice predictions along the freezeout curve of HRG models using  $T_c = 170$  MeV.

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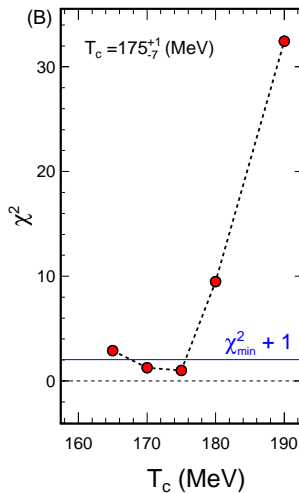
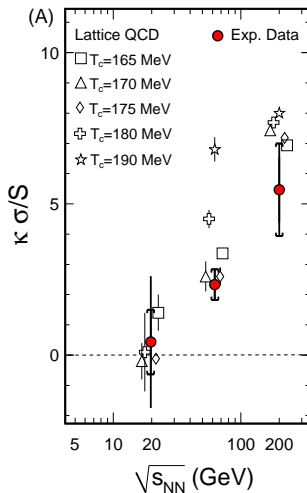
# Checking the match



$$T \chi^{(3)}/\chi^{(2)} \quad T^2 \chi^{(4)}/\chi^{(2)}$$

STAR arXiv:1004.4959

# Tuning lattice scale to match data



Science, 332 (2011) 1525

# Conclusions

## Thermalization

1 parameter tuning makes thermodynamic predictions agree with data for 2 ratios at 3 energies. Indicates thermalization of the fireball at freezeout.

## $T_c$

Comparison of lattice and data along the freezeout curve gives

$$T_c = 175^{+1}_{-7} \text{ MeV},$$

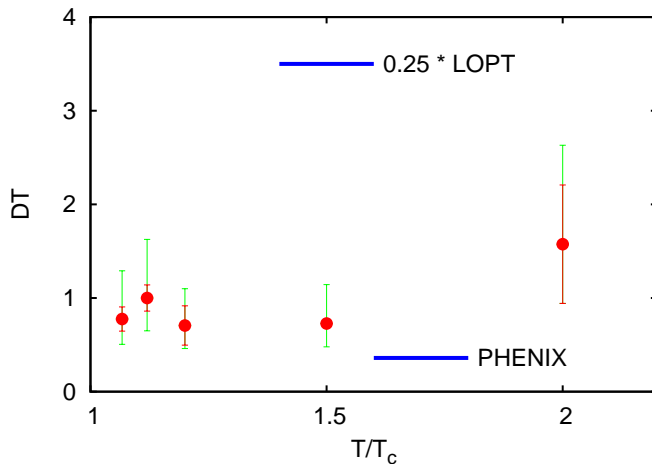
in agreement with other scale settings on the lattice. Indicates that non-perturbative phenomena in single hadron physics and strong interaction thermodynamics are mutually consistent through QCD.

# Systematics

- ❶ How important are isospin fluctuations?  
STAR 2010, Kitazawa and Asakawa 2011
- ❷ Is diffusion quantitatively under control?  
Banerjee, Datta, Gava, Majumdar 2011
- ❸ Are volume fluctuations important?  
STAR 2010
- ❹ How accurately is the freezeout curve known?
- ❺ Do chemistry and fluctuations freeze out at the same time?
- ❻ How important are finite lattice spacing artifacts?  
Redlich and Karsch 2011, Gava and SG 2010
- ❼ How good is the series expansion in  $\mu$ ?  
York and Moore 2011
- ❽ How good is the resummation of the series?
- ❾ How sensitive are the results to  $m_{ud}$  and  $m_s$ ?  
Gava and SG 2008

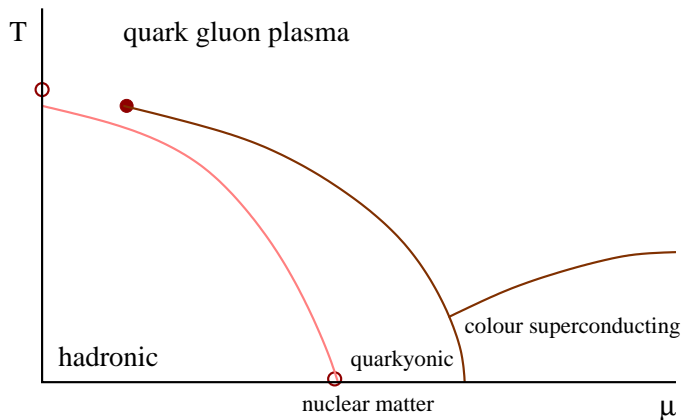


# First lattice results on heavy-quark diffusion



Banerjee, Datta, Gavai, Majumdar 2011

# Search for the critical point



Near a critical point system departs from equilibrium. If the critical point lies near the freezeout curve, then (1) Gaussian statistics will fail and (2) QCD predictions will not agree with data.