Scale for the phase diagram of QCD

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Introduction

T_c and the phase diagram

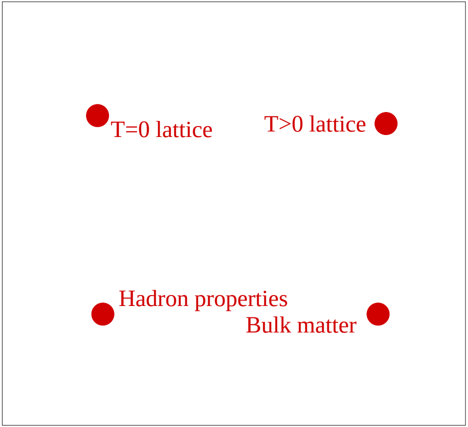
Fluctuations of conserved quantities

- Lattice
- Experiment

Comparing data and lattice

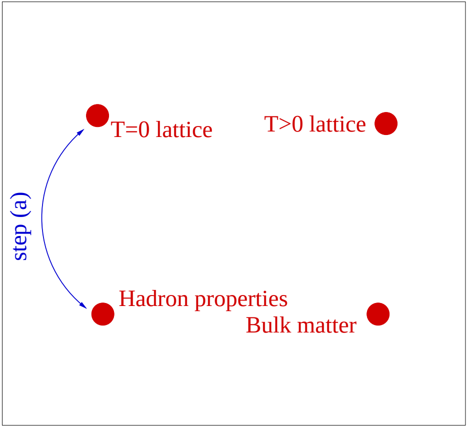
Outline

Non-perturbative tests of QCD



arxiv:1105.3934

Non-perturbative tests of QCD



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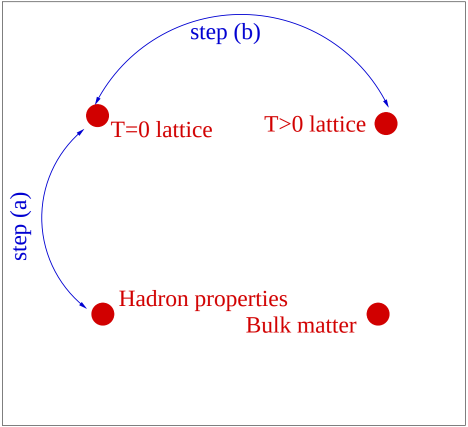

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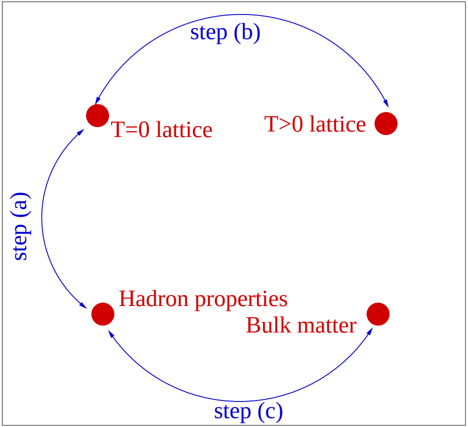

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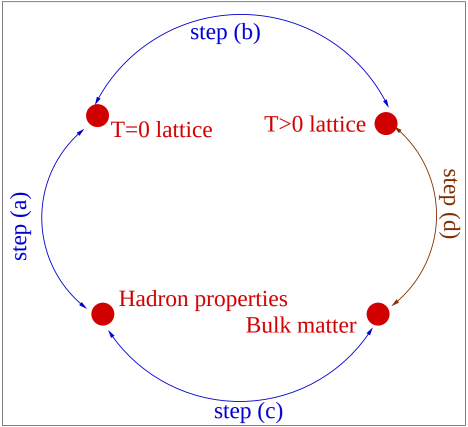

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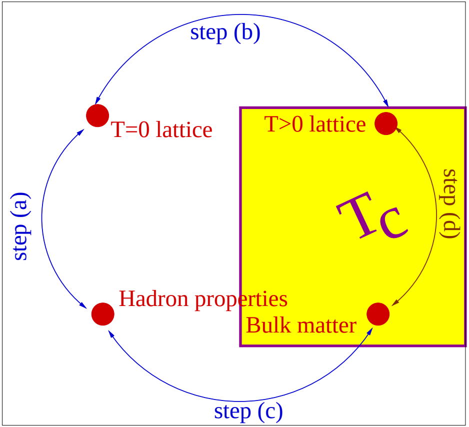

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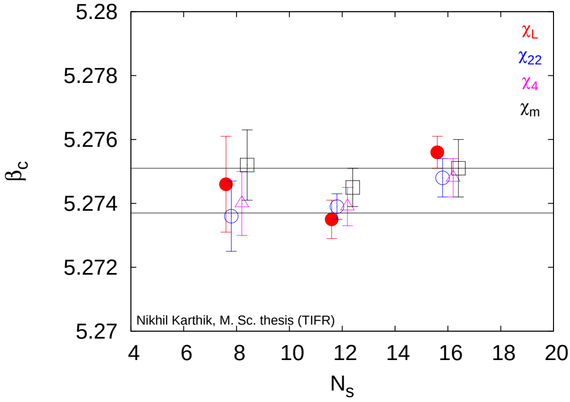
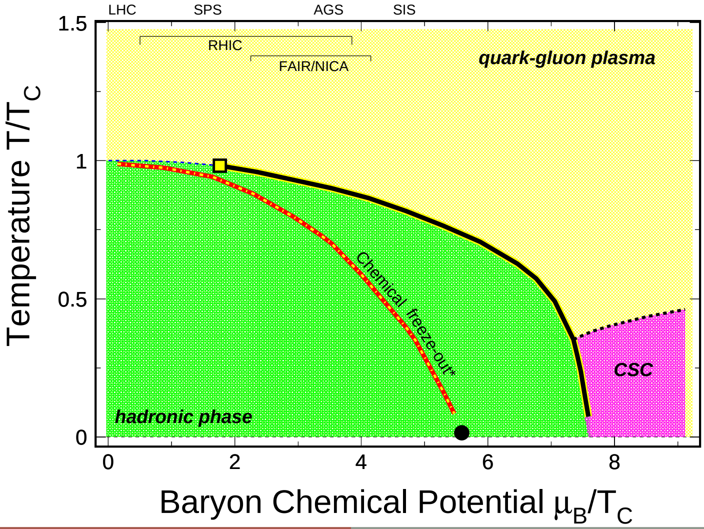
Outline

Defining T_c

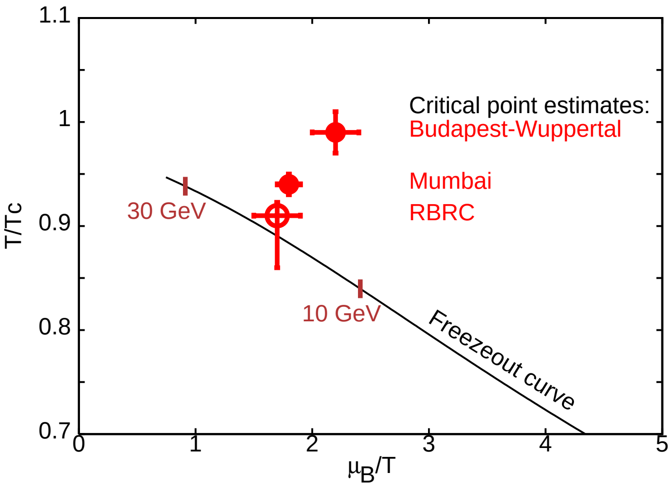


No phase transition at $\mu_B = 0$ in QCD. Pisarski Wilczek (1984), Brown *et al.*(1990), Fodor *et al.*(2005) T_c defined by the position of the maximum of a susceptibility; for definiteness, χ_L.

A conjectured phase diagram of QCD



Lattice estimates of the critical end point



Free-scale predictions from lattice

1. Tunable parameters are quark masses (m_q) and bare coupling. All computations in a finite box: box size must be large enough.
2. Since $m_\pi^2 \propto m_q$, so getting m_π correct requires tuning the bare quark mass.
3. Lattice spacing a is the inverse momentum cutoff. Predictions for hadron masses: $am =$ pure number. Then is (m'/m) same as in experiment? Yes, to some approximation.
4. Then lattice is predictive but has a free scale: put in one scale and get others. For example: if $am_{\rm std} = K$ and $am' = K'$ then $m' = (K'/K)m_{\rm std}$.

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QCD predictions at finite μ_B

Make a MacLaurin expansion of the (dimensionless) pressure:

$$\frac{1}{T^4}P(t,z) = \sum_{n=0}^{\infty} T^{n-4}\chi_B^{(n)}(t,0)\frac{z^n}{n!}, \qquad \text{where} \quad t = \frac{T}{T_c}, z = \frac{\mu_B}{T}.$$

and measure each NLS at $z = 0$. Gavai, SG (2003)
By resumming the series, construct the lattice predictions for:

$$T^{n-4}\chi_B^{(n)}(t,z) = \frac{1}{T^4}\frac{\partial^n P(t,z)}{\partial z^n}, \qquad \text{where} \quad t = \frac{T}{T_c}, z = \frac{\mu_B}{T}.$$

Series resummation needed since the series can diverge near a critical point: ie, any term of the series is as important as any other, and neglect of an infinite number of terms is not justified. Gavai, SG (2008)

Some remarks

NLS determine the shape of the distributions of fluctuations in the grand canonical ensemble.

1. In the thermodynamic ($V \to \infty$) limit the distribution is Gaussian. Only the mean, $[B]$ and variance ($\sigma^2 = [B^2]$) characterize the distribution. Skewness ($S = [B^3]/[B^2]^{3/2}$), Kurtosis ($\kappa = [B^4]/[B^2]^2$), etc, vanish.
2. Study of finite-volume effects gives more information about the theory than thermodynamic analysis.
3. Do not try to remove all Poisson fluctuations; QCD predicts some $\chi_B^{(2)}$.
4. Do not try to remove or correct for all correlations; QCD predicts some: $\chi_B^{(n)}$.

Ratios of cumulants

NLS determine the shape of the distributions of fluctuations in the grand canonical ensemble. Cumulants $[B^n]$ depend linearly on the volume:

$$[B^n] = \left(VT^3\right) T^{n-4} \chi_B^{(n)}(t, z).$$

Ratios of cumulants are state variables independent of the volume: well-determined functions on the phase diagram.

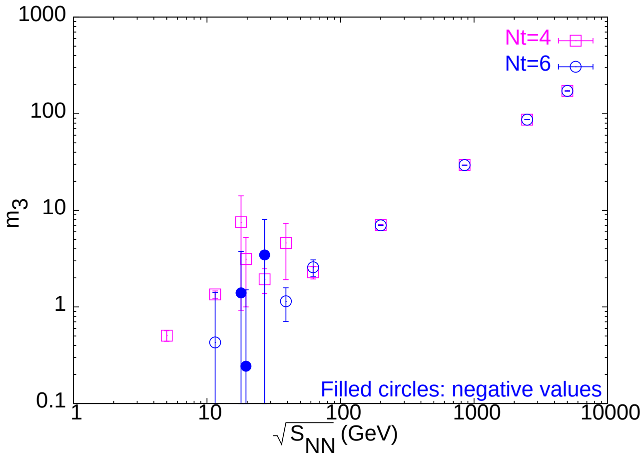
$$m_1 : \qquad \frac{[B^3]}{[B^4]} = \frac{T\chi_B^{(3)}}{\chi_B^{(2)}} = S\sigma$$

$$m_2 : \qquad \frac{[B^4]}{[B^2]} = \frac{T\chi_B^{(4)}}{\chi_B^{(2)}} = \kappa\sigma^2$$

$$m_3 : \qquad \frac{[B^4]}{[B^3]} = \frac{T\chi_B^{(4)}}{\chi_B^{(3)}} = \frac{\kappa\sigma}{S}$$

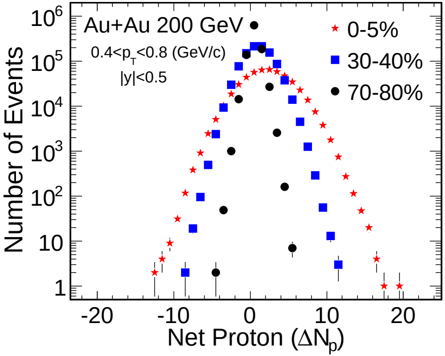
Compare lattice and experiment SG, 2009

Predictions along the freezeout curve



Lattice predictions along the freezeout curve of HRG models using $T_c = 170$ MeV. Gavai, SG (2010)

Event-by-Event fluctuations of protons



Central rapidity slice taken. p_T of 400–800 MeV. Important to check dependence on impact parameter. STAR 2010

Some observations

1. Cumulants of the distribution, $[B^n]$, measure the shape. Shape variables $\langle B \rangle$, $\sigma^2 = [B^2]$, skewness $S = [B^3]/\sigma^3$ and Kurtosis $\kappa = [B^4]/\sigma^4$ scale as CLT with change in V (proxy measure: N_{part}).
2. Experiments are blind to neutrons. Because isospin fluctuations are relatively small, the shape of the E/E neutron distribution is expected to be similar to E/E proton distributions. Can be neglected if this does not change $[B^n]$ by more than its errors. Assumption tested in event generators.

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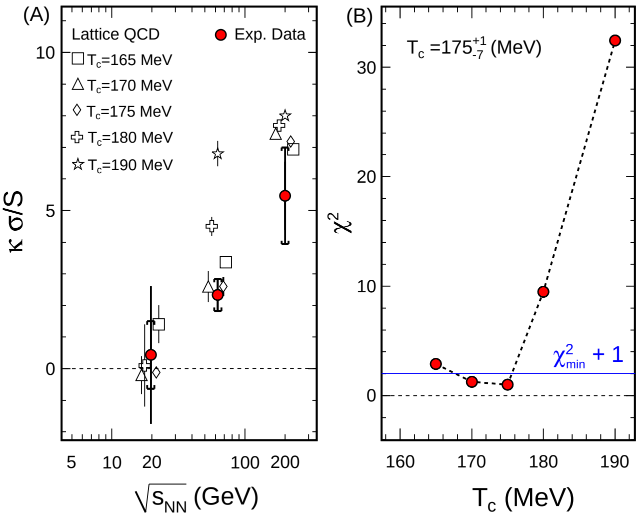
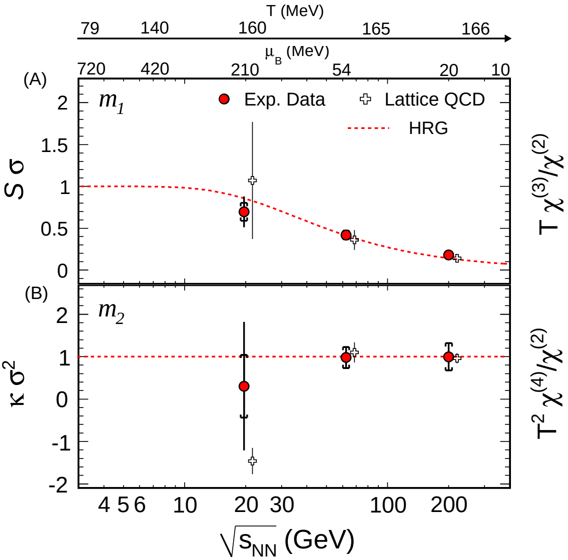
Tuning lattice scale to match data



Checking the match



Conclusions

T_c

Comparison of lattice and data along the freezeout curve gives

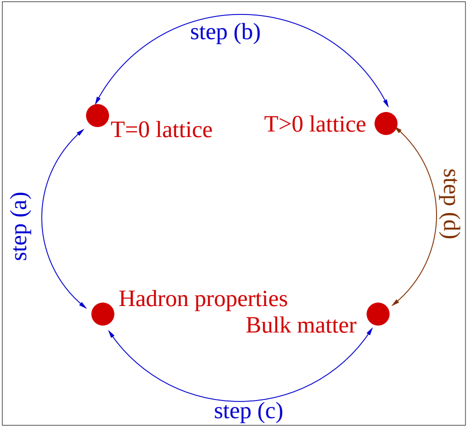
$$T_c = 175^{+1}_{-7}\ \mathrm{MeV}.$$

In agreement with other scale settings on the lattice. Indicates that non-perturbative phenomena in single hadron physics and strong interaction thermodynamics are mutually consistent through QCD.

Thermalization

1 parameter tuning makes thermodynamic predictions agree with data for 2 ratios at 3 energies. Indicates thermalization of the fireball at chemical freezeout. Departures from central limit theorem and thermalization in the energy scan could indicate a critical point.

Lattice scale setting logic



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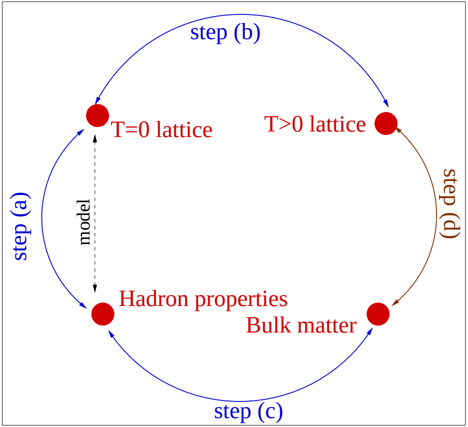

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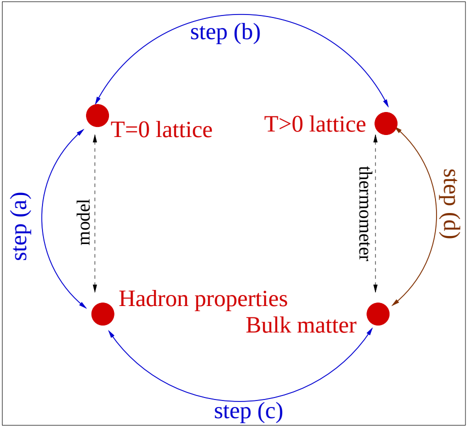

Universality and the QCD critical point

Near the critical end point one notionally decomposes the free energy into singular and regular parts

$$F(T,\mu) = F_s(T,\mu) + F_r(T,\mu), \qquad (T,\mu) \simeq (T^E,\mu^E).$$

F_s exhibits universal scaling behaviour: Ising model computation possible. Universality visible when $F_s \gg F_r$.
F_r not universal: different for nearest-neighbour Ising model, next-nearest couplings, QCD, etc. How wide is the QCD critical region?
Preliminary approximation from lattice: $|\mu - \mu^E| \simeq 80$ MeV Gavai and SG (2008).

Why thermodynamics and not dynamics?

Chemical species may diffuse on the expanding background of the fireball, so why neglect diffusion and expansion? Abdel Aziz, Gavin (2004), Bhalerao, SG (2009), Hirano, Monnai (2010)

First check whether the system size, ℓ, is large enough compared to the correlation length ξ: Knudsen's number $K = \xi/\ell$. If $K \ll 1$, ie, $\ell \gg \xi$ then central limit theorem will apply to microscopic fluctuations.

Next, compare the relative importance of diffusion and advection through Peclet's number:

$$\mathcal{W} = \frac{\ell^2}{t\mathcal{D}} = \frac{\ell v_{flow}}{\mathcal{D}} = \frac{\xi v_{flow}}{K\mathcal{D}} = \frac{v_{flow}}{Kc_s} = \frac{M}{K}.$$

After chemical freeze-out one expects $\mathcal{W} \gg 1$, so flow dominates: fluctuations are frozen in. Detailed work needed.