Hydrodynamics at finite baryon density

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4. A simple extension may be required in relativistic regime: acauasality of hydro and diffusion handled by the second order (Kelly) formalism. Is it necessary, or is first order (Fick) diffusion enough?

Basic phenomenology

The diffusion advection equation

At top RHIC energy μ is small, and hence any number density is small. Entropy creation by diffusion is therefore small compared to the entropy in the fluid flow. Diffusion may be treated as the change in a tracer density in a given background flow. This approximation breaks down at lower $\sqrt{S}$ where μ is larger. Formalism is easily extended.

Bjorken attenuation

In this first work we assumed longitudinal flow. For ideal fluids the solution is that the initial density profile is stretched out by the background flow, but there is no change in shape. The equations give the solution $\tau n(\tau, \eta)$ is conserved (Bjorken attentuation).

Outline

Solving for Fick diffusion

The diffusion-advection equation is

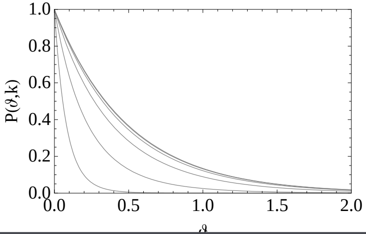
$$\partial_\tau n(k) + \frac{1}{\tau} n(k) + \frac{\mathcal{D}k^2}{\tau^2} n(k) = 0.$$

The solutions are

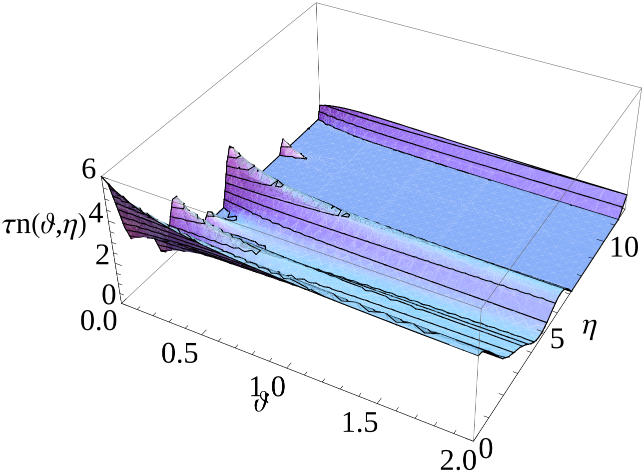
$$n(\tau, k) = n(\tau_0, k) \left(\frac{\tau_0}{\tau}\right) \exp\left[-\frac{\mathcal{D}k^2}{\tau_0}\left(1 - \frac{\tau_0}{\tau}\right)\right].$$

The $k = 0$ mode shows Bjorken attenuation. Typical behaviour for Fick diffusion: large k modes are suppressed quicker.

Power spectrum is defined to be $P(\vartheta, k) = |n(\vartheta, k)|^2$ where $\vartheta = \log(\tau/\tau_R)$.
τ_R is an arbitrary scale in Fick theory, but will get a physical interpretation in Kelly theory.



Fick diffusion: profile



The Peclet number

Compare the importance of flow and diffusion by comparing the 2nd and 3rd terms. Pick a scale in the density profile: $\lambda = \tau \sinh \Delta\eta = 1/k$. Then

$$\mathrm{Pe} = \frac{\lambda^2}{\tau \mathcal{D}} = \frac{\tau}{\mathcal{D}} \sinh^2 \Delta\eta,$$

For $\mathrm{Pe} \ll 1$ diffusion dominates over flow. For any given $\Delta\eta$, flow dominates at times larger than $\tau_{fl} \simeq \mathcal{D}/\sinh^2 \Delta\eta$.

Observational upper bound for $\mathcal{D}$ possible. At freezeout time τ_f, if there is no visible structure in the number density profile up to rapidity separation of $\Delta\eta$, then

$$\mathcal{D} \leq \tau_f \sinh^2 \Delta\eta.$$

Example: if $\Delta\eta = 0.3$ and $\tau_f = 10$ fm then $\mathcal{D} \simeq 1$ fm.

Outline

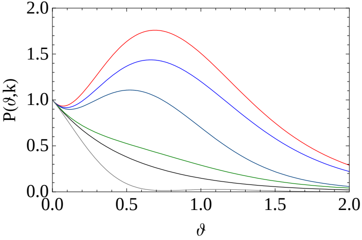
Kelly diffusion

To second order, the diffusion equations become

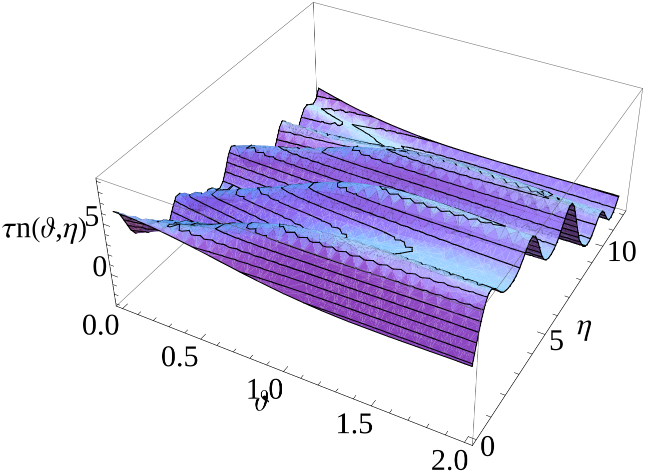
$$
\begin{aligned}
0 &= \partial_\tau n + \frac{1}{\tau} n + \frac{1}{\tau}\partial_\eta \nu, \\
0 &= \tau_R \partial_\tau \nu + \nu + \frac{\mathcal{D}}{\tau}\partial_\eta n.
\end{aligned}
$$

An f-sum rule gives the relation $\mathcal{D} = c_s^2 \tau_R$. Changing variables to $\vartheta = \log(\tau/\tau_R)$ and taking Fourier transforms simplifies the equations.

Transient growth can occur for certain sets of initial conditions. Analysis is technical, and given in the paper.



Kelly diffusion: profile

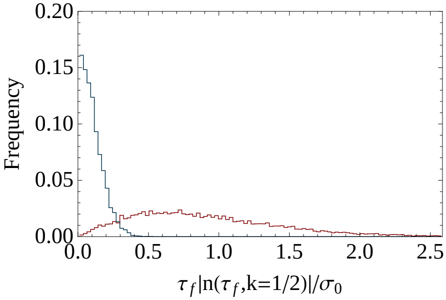


Outline

Power spectrum and flow coefficients

- For azimuthally symmetric flows, the only fluctuations are in the ϕ direction. Fourier coefficients of the flow are the v_k. Their study gives information on initial conditions and evolution of hydrodynamic flow velocities.
- For diffusion, it is interesting to consider the number density profile in the longitudinal direction. Fourier coefficients are the $\tau n(\tau, k)$. The real part is encoded into the power spectrum $P = |n|^2$. These give information on the initial conditions and diffusive evolution of number densities.
- Since functions on the circle are periodic, k of v_k are discrete (integer k). No periodicity in η direction, so k of $P(k)$ are continuous.
- Event-to-event fluctuations possible (already observed for $k = 0$!) so event averaged quantity not useful. Instead study histograms.

Diffusion: experimental test



The experimental observable corresponding to the power spectrum is

$$\overline{P}(\tau_f, k) = \left| \sum_{j=1}^{N_t} q_j \mathrm{e}^{-ik\eta_j} \right|^2 ,$$

where the sum is over tracks.

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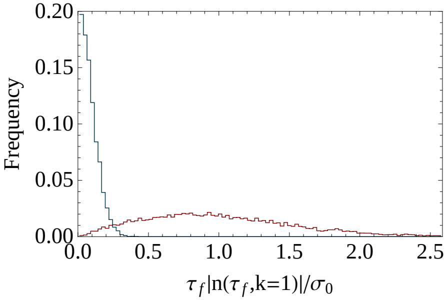

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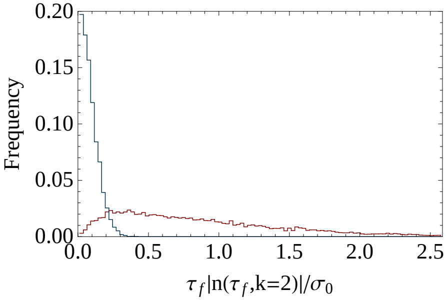

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