Multi-parton scattering: an introduction to the theory

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Why is multi-parton scattering interesting?

- Could give important corrections to multi-jet $+ \gamma$ and other final states at the LHC. Study it as background to interesting processes.
- May tie into the study of generalized parton densities. There are very few experimental constraints as yet on these objects.
- Extremely important in heavy-ion collisions. The process of thermalization is due to multi-parton interactions. Study the hard and semi-hard limit of these processes in order to constrain the physics of heavy-ion collisions.
- Could give new non-perturbative information on correlations of flavours and colours within a hadron. Completely new area of study: not available directly by any other means.
- Experimentally challenging to separate these from the more standard processes. sections.

Processes studied

double Drell-Yan	Goebel+ (1980), Halzen+ (1987), Kom+ (2011)
multiple W/Z production	Godbole+ (1990), Eboli+ (1998), del Fabbro+ (2000, 2005), Kulesza+ (2000), Maina (2009, 2011), Gaunt+ (2010), Berger+ (2011)
$\overline{c}c\overline{c}c$	Kom+ (2011), Baranov+ (2011), Novoselov (2011)
multi-jets	Humpert (1983, 1985), Amettler+ (1986), Mangano (1989), del Fabbro+ (2002), Domdey+ (2009), Berger+ (2009)
photons + jets	Drees+ (1996)

Double scattering

Think of a beam falling on a fixed target. The flux of particles is $F = Nv/BA$, where N is the number of particles in a bunch, B the bunch length, A the transverse area of the beam, and v the mean velocity of the bunch. The time-scale of the problem is the bunch crossing time: B/v.

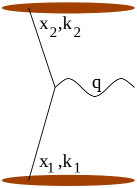
If the probability of scattering, p is sufficiently small, then the rate at which particles are scattered is Npv/B. As a result, the cross section for a single scattering is

$$\sigma = \frac{Npv}{FB} = pA.$$

The double scattering cross section is (dimensionally obvious):

$$\sigma_2 = \frac{Np^2v}{2FB} = \frac{\sigma^2}{2A}.$$

Drell-Yan W production

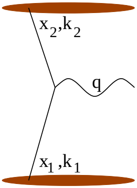


Leading order single W production

$$\frac{d\sigma}{d^4q} = \frac{d\hat{\sigma}}{d^4q}\int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \times f(x_1,\mathbf{k}_1)f(x_2,\mathbf{k}_2)\delta^2(q_T-\mathbf{k}_1-\mathbf{k}_2)$$

x_i are Bjorken variables, and $\mathbf{k}_i$ the quark transverse momenta.

Drell-Yan W production



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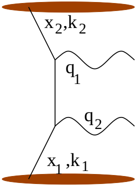
Dimensional analysis: The product of pdfs has to contribute mass dimension -2 to the result. But the pdfs depend on the hard scale only as $\ln Q^2$. So each must give a power of a soft scale Λ which characterizes the width in $\mathbf{k}_i$; maybe $Q_{\rm sat}$ or Λ_{QCD}. One must have

$$\frac{d\sigma}{d^4q} \simeq \frac{1}{Q^4\Lambda^2}$$

WW production through single parton scattering

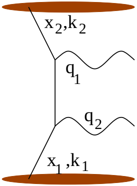


Leading order WW production

$$\frac{d\sigma}{d^4q_1d^4q_2} = \frac{d\hat{\sigma}}{d^4q_1d^4q_2}\int d^2\mathbf{k}_1 d^2\mathbf{k}_2$$
$$\times f(x_1,\mathbf{k}_1)f(x_2,\mathbf{k}_2)\delta^2(q_T^1+q_T^2-\mathbf{k}_1-\mathbf{k}_2)$$

x_1 and x_2 are longitudinal momentum fraction carried by quarks, $\mathbf{k}_1$ and $\mathbf{k}_2$ their transverse momenta, and q_T^1 and q_T^2 are the transverse momenta of the W.

WW production through single parton scattering



Leading order WW production

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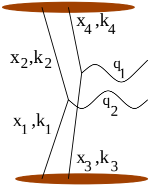
x_1 and x_2 are longitudinal momentum fraction carried by quarks, $\mathbf{k}_1$ and $\mathbf{k}_2$ their transverse momenta, and q_T^1 and q_T^2 are the transverse momenta of the W.

Dimensional analysis: Counting as before, we have

$$\frac{d\sigma}{d^4q_1d^4q_2} \simeq \frac{1}{Q^8\Lambda^2}$$

where Λ is the soft scale. The contribution to inclusive single W production is $\mathcal{O}(1/Q^4\Lambda^2)$ as expected, this being the NLO contribution to that process.

WW production through double parton scattering

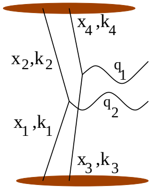


z: transverse separation of the two partons

Leading order WW production in DPS:

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \frac{d\hat{\sigma}}{d^4q_1} \frac{d\hat{\sigma}}{d^4q_2}$$
$$\times \int d^2\mathbf{z} d^2\mathbf{k}_1 d^2\mathbf{k}_2 d^2\mathbf{k}_3 d^2\mathbf{k}_4$$
$$\delta^2(\mathbf{k}_1 + \mathbf{k}_2 - q_T^2)\delta^2(\mathbf{k}_3 + \mathbf{k}_4 - q_T^1)$$
$$\times F(x_1, x_3, \mathbf{k}_1, \mathbf{k}_3, \mathbf{z}) F(x_2, x_4, \mathbf{k}_2, \mathbf{k}_4, \mathbf{z})$$

WW production through double parton scattering



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Leading order WW production in DPS:

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \frac{d\hat\sigma}{d^4q_1}\,\frac{d\hat\sigma}{d^4q_2}$$
$$\times \int d^2\mathbf{z} d^2\mathbf{k}_1 d^2\mathbf{k}_2 d^2\mathbf{k}_3 d^2\mathbf{k}_4$$
$$\delta^2(\mathbf{k}_1+\mathbf{k}_2-q_T^2)\delta^2(\mathbf{k}_3+\mathbf{k}_4-q_T^1)$$
$$\times F(x_1,x_3,\mathbf{k}_1,\mathbf{k}_3,\mathbf{z})F(x_2,x_4,\mathbf{k}_2,\mathbf{k}_4,\mathbf{z})$$

Dimensional analysis: Since the F's cannot depend on Q^2, we have

$$\frac{d\sigma}{d^4q_1 d^4q_2} \simeq \frac{1}{Q^8\Lambda^2}$$

exactly as in the single parton scattering!

Inclusive cross sections

Single parton scattering production of inclusive W at LO is

$$\sigma = \hat{\sigma} \int q_L dq_L d^2\mathbf{k}_1 d^2\mathbf{k}_2 f(x_1, \mathbf{k}_1) f(x_2, \mathbf{k}_2) \propto \frac{1}{\Lambda^2}.$$

Leading order W production in double parton scattering is

$$\begin{aligned} \sigma_2 = \hat{\sigma}^2 \int & q_L^1 q_L^2 dq_L^1 dq_L^2 d^2\mathbf{z} d^2\mathbf{k}_1 d^2\mathbf{k}_2 d^2\mathbf{k}_3 d^2\mathbf{k}_4 \\ & \times F(x_1, x_3, \mathbf{k}_1, \mathbf{k}_3, \mathbf{z}) F(x_2, x_4, \mathbf{k}_2, \mathbf{k}_4, \mathbf{z}) \\ \propto \frac{1}{\Lambda^2} = \sigma^2 \Lambda^2 & \end{aligned}$$

So the area of the hadron should be taken to be around Λ^2. If this is taken to be roughly the total inelastic cross section, then $\Lambda^{-2} \simeq 20$ mb.

Is the DPS cross section of higher twist?

Inclusive cross sections

A simple argument shows that the inclusive cross section for double parton scattering can be written as

$$\sigma_2 \propto \sigma^2 \Lambda^2, \qquad \Lambda^2 = 1/\mathrm{area}.$$

The DPS cross section is not suppressed by powers of Q^2. cross section.

Sufficiently differential cross sections

A power counting of the differential cross sections for double and single parton scattering *for the same final state* shows that the dependence on Q^2 is the same. So the differential cross section also need not be higher twist. **Diehl+ (2011)**

Signals of double parton scattering

1. W-pair is back to back, but each can have large q_T.
2. Decay leptons/jets are boosted; implies rapidity gap.
3. W^+W^+ or W^-W^- pairs are not allowed.

1. Each W in the pair must have moderate q_T.
2. Rapidity difference of decay jets is gapless.
3. Use leptons and b to reconstruct like-sign W pairs.

Kinematic signals already noted in Godbole+ (1990). Flavour signals already noted in Diehl+ (2011), Manohar+ (2012)

A simple model: effective cross sections

In the absence of experimental constraints, it is ok to start with the approximation

$$F(x,y,\mathbf{k},\mathbf{k}',\mathbf{z}) = f(x,\mathbf{k})f(x,\mathbf{k}')G(\mathbf{z}),$$

where the f's are known from experiment, and $G(\mathbf{z})$ is a form factor. If we define

$$\sigma_{\rm eff} = 1/\int d^2\mathbf{z}G(\mathbf{z})^2,$$

then any DPS cross section becomes $(2-\delta ij)\sigma_i\sigma_j/(2\sigma_{\rm eff})$. CDF and D0 obtain this factor from γ+3 jets final states:

$$\begin{aligned}\sigma_{\rm eff}^{\rm D0} &= 16.4\pm 0.3\pm 2.3\ {\rm mb},\\ \sigma_{\rm eff}^{\rm CDF} &= 14.5\pm 1.7\ ^{+1.7}_{-2.3}\ {\rm mb}.\end{aligned}$$

LHCb quotes $\sigma_{\rm eff}$ which is 2–3 times larger based on measurements of $2J/\psi$ and 4D final states, but agrees for $2J/\psi$+2D final state.

D0 0912.5104, CDF *Phys. Rev.* D56 (1997) 3811, LHCb 1205.0975

Note added: the like-sign W pair cross section

The SPS inclusive W production cross section $\sigma_W \simeq 10^2$ nb. Sanity check: $M_w^2 \sigma_W = \mathcal{O}(1)$. This implies that

$$r = \frac{\sigma_W}{\sigma_{\rm eff}} = 10^{-5}.$$

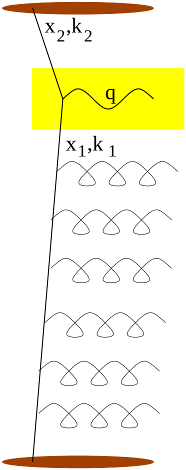
Then the DPS cross section is

$$\sigma_2 = \frac{1}{2}\,\frac{\sigma_W^2}{\sigma_{\rm eff}} = \frac{1}{2} r\sigma_W \simeq \frac{1}{2}\ \mathrm{pb}.$$

With an integrated luminosity of 10^2 fb^{-1}, this would give about 5000 events. The number of SPS events with like-sign W pairs and nothing else is zero. The 3PS cross section is smaller by a factor of r. So the expected number of like-sign W triplets is less than 1 with the same luminosity.

Thanks to Ankita Mehta and Kajari Majumdar for inputs

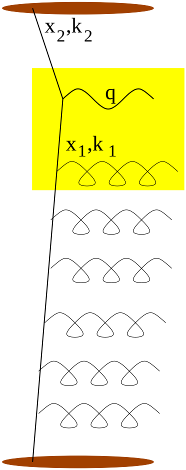
Towards a theory of double parton densities



Parton distributions evolve via splitting functions. Factorization scale: selected by what is included into the hard process.

By changing the factorization scale, one connects W production and W+jet production.

Towards a theory of double parton densities



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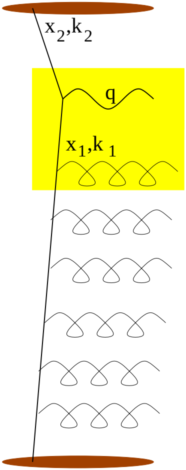
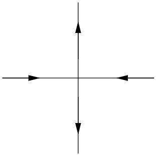


Parton distributions evolve via splitting functions. Factorization scale: selected by what is included into the hard process.

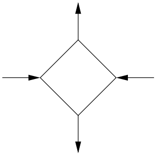
By changing the factorization scale, one connects W production and W+jet production.

If a second parton is included above the factorization scale from both processes, then one could mix single parton and double parton processes.

Simple estimate



SPS



DPS

Double parton scattering loop =

$$\int \frac{d^4k \times \text{spin} - \text{colour dependent factors}}{k^2(k+q_1)^2(k+q_1-q_2)^2(k+q_4)^2}$$

$$\propto \frac{1}{Q^2}$$

Modified by the factors in the numerator. Could become as small as $\log Q^2$ if there are enough terms in Q in the numerator.

Maybe there are really no double parton distributions?

Gaunt+ 2011, 2012

A complete solution

If $F(x, x', \mathbf{k}, \mathbf{k}', \mathbf{y}) \simeq 1/y^2$, then the integral

$$\int d^2\mathbf{y} F(x, x', \mathbf{k}, \mathbf{k}', \mathbf{y})$$

is logarithmically divergent. So the single parton distribution obtained by summing over all states of one of the partons needs to be regularized. This regularization can be performed. **Diehl+ (2011)**

Even so, integral such as the following appear in cross sections

$$\int d^2\mathbf{y} F(x_1, x_3, \mathbf{k}_1, \mathbf{k}_3, \mathbf{y}) F(x_2, x_4, \mathbf{k}_2, \mathbf{k}_4, \mathbf{y}).$$

This has a quadratic divergence. This can be absorbed into a renormalization of the composite operator. **Manohar+ (2012)**

Can the lattice help?

Leading Mellin moment of double parton distribution is given by

$$M_{qq}(y^2) = \frac{2}{p^+} \int dy^- \langle p|\mathcal{O}_\mu(0)\mathcal{O}_\nu(y)|p\rangle,$$

where y^- is a light-cone variable, $\mathcal{O}_\mu(y) = \bar{q}(y)\gamma_\mu q(y)$, and the states are proton states of momentum p. The matrix element becomes

$$\langle p|\mathcal{O}_\mu(0)\mathcal{O}_\nu(y)|p\rangle = p_\mu p_\nu F(y^2, p\cdot y).$$

The integral then reduces to one over $p\cdot y$. The lattice computation can be performed for spacelike p and y, and therefore cannot completely determine the Mellin moment.

It is possible to check models using lattice computations. Experimental data and lattice computations together can restrict possible models of parton correlations in hadrons.

Double parton scattering

1. There are clear kinematic, flavour and colour signals for double parton scattering. A smoking gun signal is of same-sign W pair production.
2. Double parton scattering is not a higher twist effect. So should expect it to be visible at $x \simeq 0.1$, *i.e.*, at $Q \simeq 1$ TeV. However should be critical at $x \simeq 10^{-3}$, *i.e.*, at $Q \simeq 10$ GeV.
3. There may be no disagreement between CDF/D0 and LHCb extractions of $\sigma_{\rm eff}$. The apparent disagreement may be due to subtleties of the analysis.
4. A theoretical framework is fully in place for analysis of any data which may be thrown up in the LHC experiments.