The QCD critical point

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Lattice computations

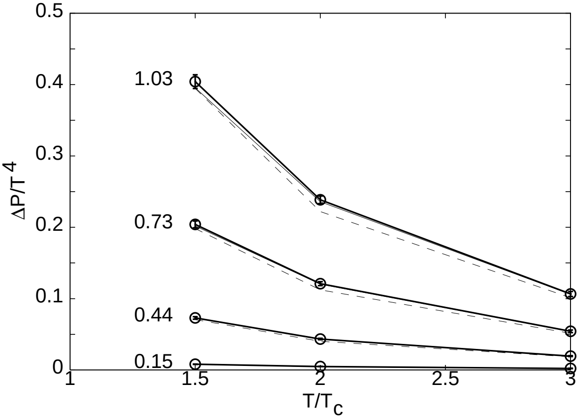
Wilson (1974): introduced a formulation of quantum field theory which made it possible to compute processes when the coupling is large.

Creutz (1975): introduced numerical computation of Wilsonian field theory.

Satz (1980): applied Wilsonian field theory to physics at finite temperature.

Method involves a Monte Carlo integration: assumes that the integrand is positive definite. At finite chemical potential the integrand is not positive. New method was needed.

EOS at $\mu \neq 0$



Gavai, SG: Phys.Rev. D68 (2003) 034506

$$\Delta P = P(\mu, T) - P(0, T).$$

The mathematical problem

Perform a series expansion of the pressure in powers of chemical potential:

$$\Delta P(\mu_B, T) = \frac{T^2}{2!}\chi_B^2(T)z^2 + \frac{T^4}{4!}\chi_B^4(T)z^4 + \frac{T^6}{6!}\chi_B^6(T)z^6 + \cdots$$

where $z = \mu_B/T$. Does this converge? Can one reconstruct the function? Well studied classical problem. Special complications: few coefficients known, with errors.

Simplest part of the problem: estimate whether the series is summable, radius of convergence and location of nearest singularity.

Next more complicated: estimating value of the function, nature of divergence.

Our computational conditions

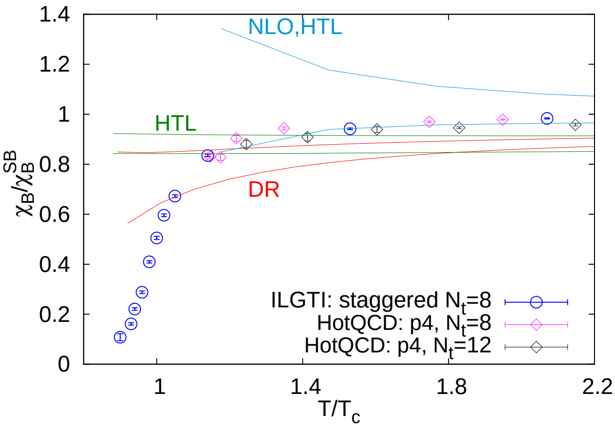
Lattice simulations with $N_f = 2$ staggered quarks and Wilson action. Used $N_t = 8$, 6 and 4; $m_\pi \simeq 0.3 m_\rho$ MeV; spatial size $L = 4/T$.

Temperature scale, T_c, found by the point at which χ_L peaks. If $T_c \simeq 170$ MeV, then $1/a = 1.36$ GeV.

Configurations: 50K+ at each coupling; large number of fermion sources used for determination of fermion traces.

Partial statistics reported in: **Datta, Gavai, SG: arXiv:1210.6784**. More statistics (0.4% of total) reported in: **Datta, Gavai, SG: Lattice (July) 2013**.

Nearing continuum physics



Continuum: $T_c = 170$ MeV; p4 with $N_t = 8$: $T_c = 180$ MeV.

HTL, DR: Andersen etal, 1307.8098; NLO: Haque etal, 1302.3228; HotQCD: Petreczky, Lattice 2013

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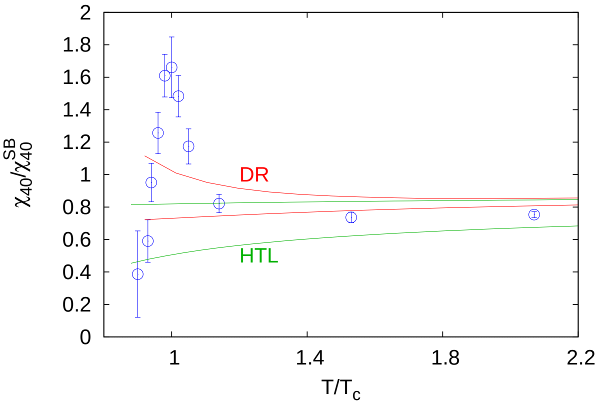

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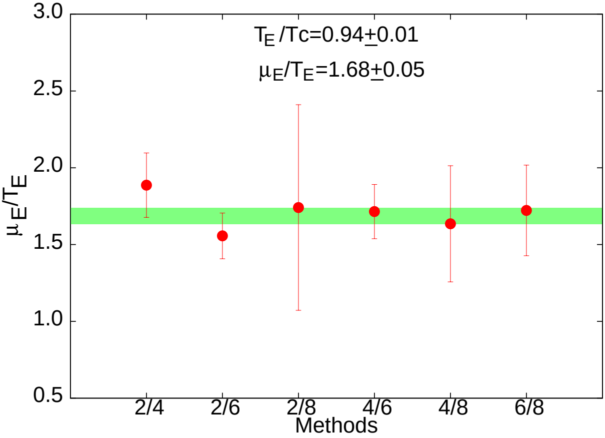
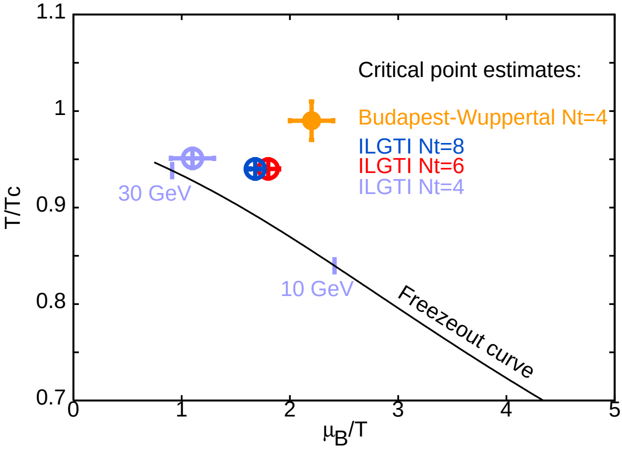
The radius of convergence



For $N_t = 6$, $\mu_E/T_E = 1.7 \pm 0.1$ **Gavai, SG: 2008**

Critical point and critical region



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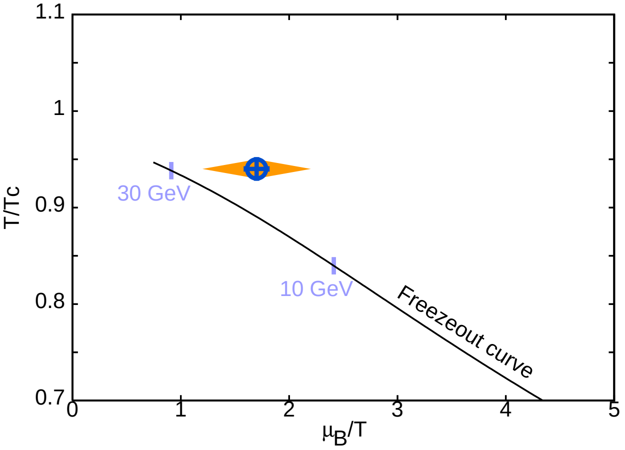

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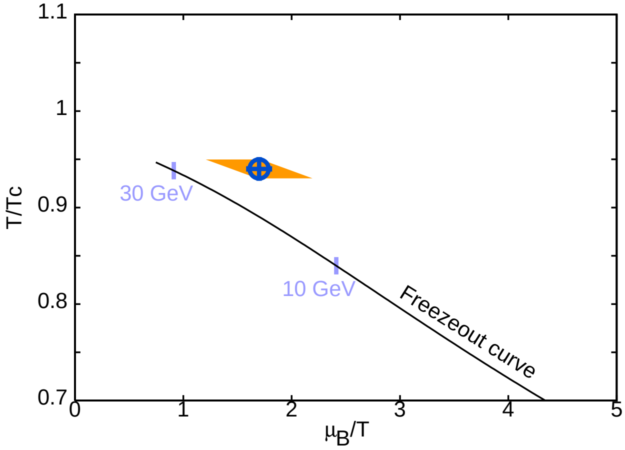

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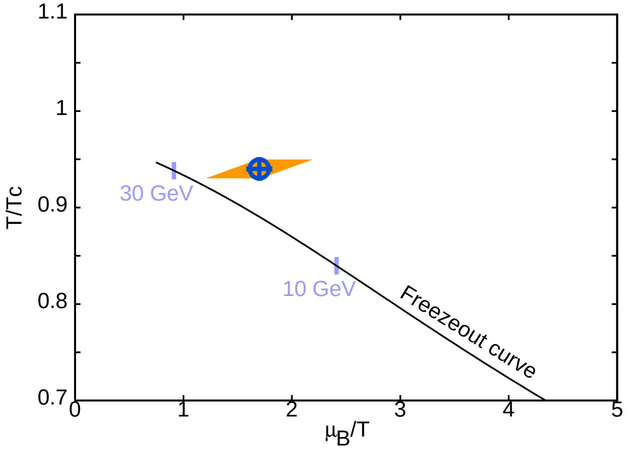


Experimentalists know how to make fireballs

" We didn't have flint when when I was a kid, we had to rub two sticks together. "

What to compare with QCD

The cumulants of E/E distribution are related to the Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi_B^2}{T^2}\right), \quad [B^3] = T^3 V \left(\frac{\chi_B^3}{T}\right), \quad [B^4] = T^3 V \chi_B^4.$$

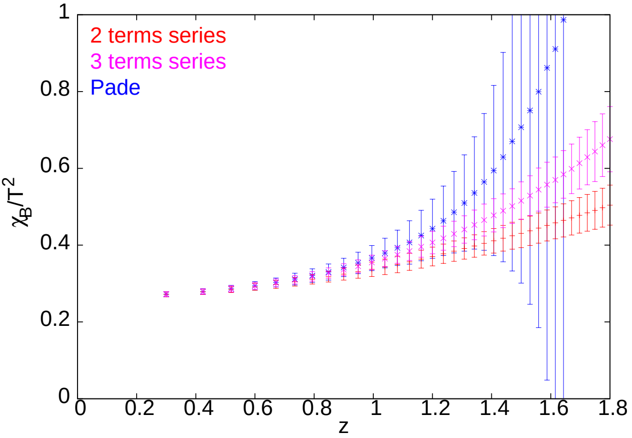
T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD provided all other fluctuations removed, and lattice results extrapolated to freezeout conditions.

SG, 0909.4630 (2009)

Must resum a series expansion



Truncated series sum is regular even at the radius of convergence, so is missing something important.

Critical behaviour of m_1

If $\chi_B(z) \simeq (z_* - z)^{-\psi}$, then $m_1 = d\log\chi_B/dz$ has a pole. Series expansion of χ_B gives series for m_1. Resum series into a Padé approximant:

$$[0,1]: \qquad m_1(z) = \frac{c}{z_* - z}$$

Width of the critical region? If we define it by

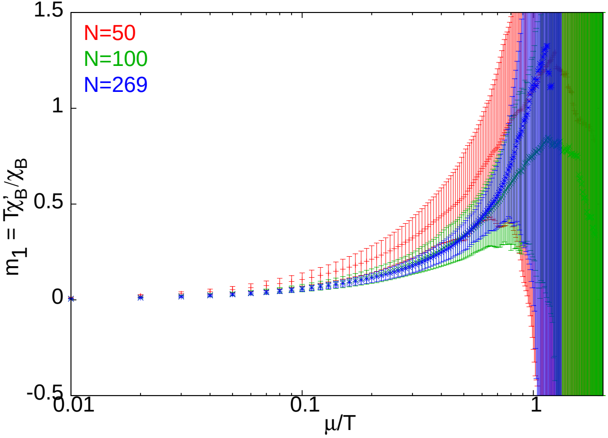
$$\left|\frac{m_1(z)}{m_1(0)}\right| > \Lambda,$$

then $|z - z_*| \le z_*/\Lambda$.
Errors in extrapolation? We have

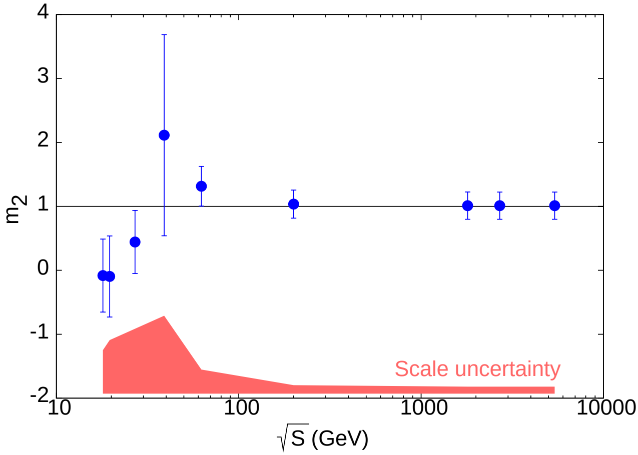
$$\left|\frac{\Delta m_1}{m_1}\right| > \frac{1}{1-\Lambda\delta},$$

where δ is fractional error in z_*.

Critical slowing down



Fourth cumulant



The DLOG Pade

At a critical point

$$\chi_B = \frac{\partial^2 (P/T^4)}{\partial z^2} \simeq (z_*^2 - z^2)^{-\psi}.$$

Continuity and finiteness of P at the CEP forces $\psi \leq 1$.

Since

$$m_1(z) = \frac{d \log \chi_B}{dz} \simeq \frac{2\psi z}{z_*^2 - z^2},$$

use the series to estimate the critical exponent. Series for m_1 has one term less than series for χ_B.

Accurate results require fine statistical control of at least 3 series coefficients of χ_B: 2 of m_1.

Widom scaling

Widom scaling for the order parameter gives

$$|\Delta\mu| = |\Delta n|^{\delta} J\left(\frac{|\Delta T|}{|\Delta n|^{1/\beta}}\right),$$

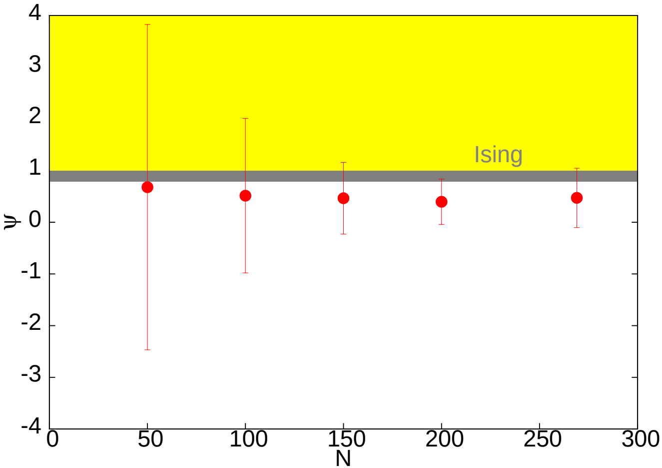
where $\Delta T = T - T_E$ and $\Delta\mu = \mu - \mu_E$. For $\Delta T = 0$ one finds $\Delta n \propto |\Delta\mu|^{1/\delta}$ in the high density phase. Then clearly one has

$$\psi = 1 - \frac{1}{\delta}.$$

For the 3d Ising model, $\delta = 1.49$, so $\psi = 0.79$. Since the identification of the two scaling directions is arbitrary, one can vary these. This gives $0.79 \leq \psi \leq 1$.

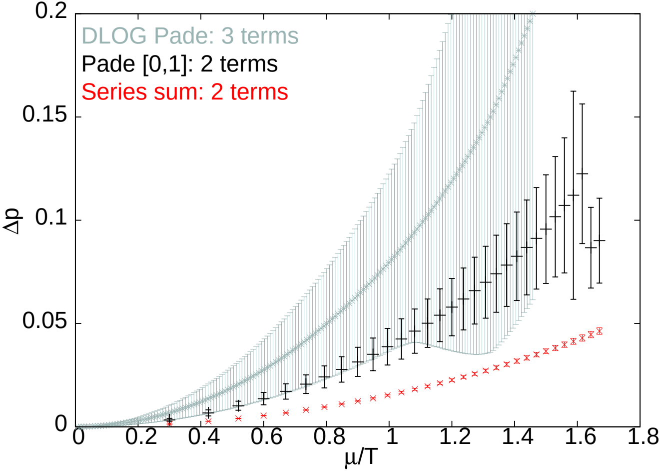
In mean field theory one has $\delta = 3$, so $0.66 \leq \psi \leq 1$. The data cannot yet distinguish between these cases.

Critical exponent



Large errors in ψ, but $\psi < 1$ as expected from continuity of pressure. Ising prediction: $\psi \geq 0.79$.

The pressure



Critical point and critical region

1. The method of Taylor expansion of the pressure now the method of choice for all major lattice computations.
2. Estimate of the critical end point does not move significantly if lattice cutoff is changed from 1 GeV to 1.4 GeV. End point seems to be at

$$\mu_E/T_E = 1.68 \pm 0.05, \quad \text{and} \quad T_E/T_c = 0.94 \pm 0.01.$$

3. Increasing control over extrapolation of measurable quantities to finite μ: however control over at least 3 terms of the series required. Has been hard to get; technical innovations were necessary.
4. First attempt to obtain critical indices from lattice computations. Consistent with all known constraints, but extremely statistics hungry.