Critical behaviour, equation of state, fluctuations

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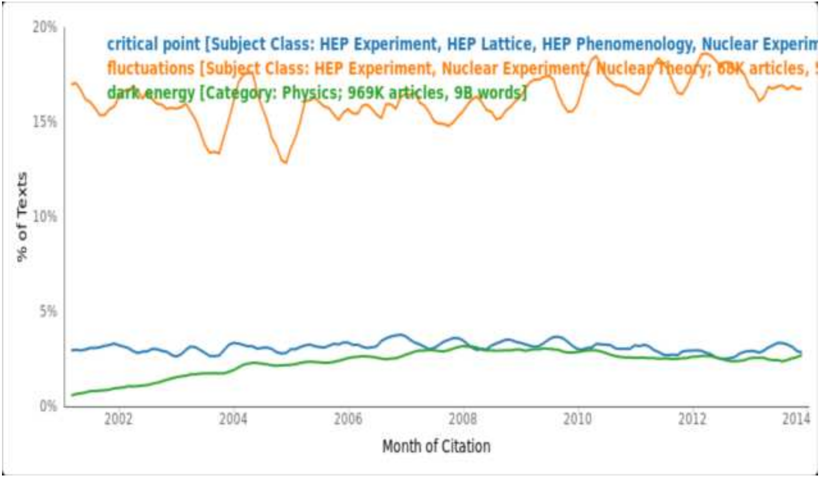
A popular problem



http://bookworm.culturomics.org/arxiv/

The mathematical problem

Perform the Maclaurin series expansion of the pressure in powers of chemical potential

$$\Delta P(\mu_u, \mu_d, T) = P(\mu_u, \mu_d, T) - P(0, 0, T) = \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m!n!}.$$

1. When does this diverge? Determine the critical point.
2. When it diverges, then how to reconstruct the function? Determine the equation of state.

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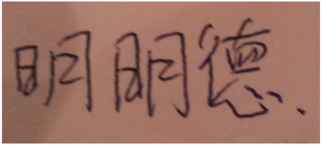

Example

Consider the series expansion of the function

$$f(x) = \frac{1}{(1-x)^3}.$$

Singularity at $x = 1$. Radius of convergence of series is 1.

number of terms	$x = 1/2$ value	$x = 1/2$ syst error	$x = 2/3$ value	$x = 2/3$ syst error
Exact	8		27	
2	4	50%	5.67	79%
3	6.19	23%	8.63	68%
4	7.28	9%	11.59	57%

The strategy

Use successive derivatives

$$n = \frac{\partial \Delta P}{\partial \mu}, \qquad \chi_B = \frac{\partial n}{\partial \mu}, \qquad m_1 = \frac{\partial \log \chi_B}{\partial \mu}.$$

Then understand the critical behaviour of the simplest of these.
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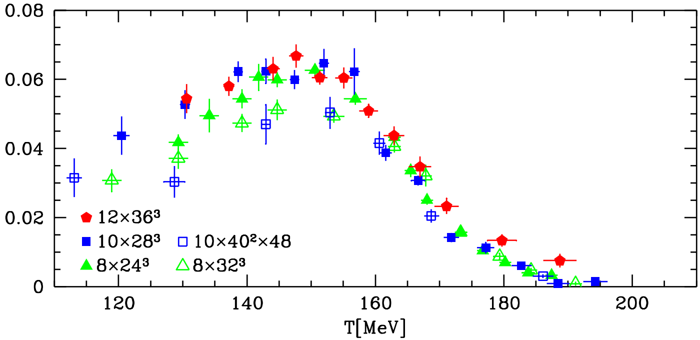
Notation: $z = \mu_B/T$. Critical divergence

$$\chi_B \simeq \frac{1}{(z_E^2 - z^2)^\psi} \qquad m_1 = \frac{2z\psi}{z_E^2 - z^2}.$$

Use Padé expansion of m_1; integrate to get χ_B, n, ΔP.

On T_c

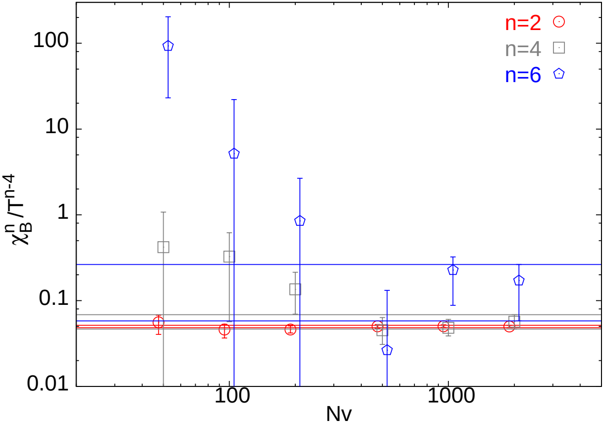


Broad crossover: even with one single measure (figure: chiral susceptibility) T_c uncertain by 20 MeV. Reflected in quoted values.

Aoki, Borsanyi, Dürr, Fodor, Katz, Krieg, Szabo: JHEP 0906 (2009) 088

Select any definition and stick with it: we use Polyakov loop susceptibility.

Numerical errors



Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.

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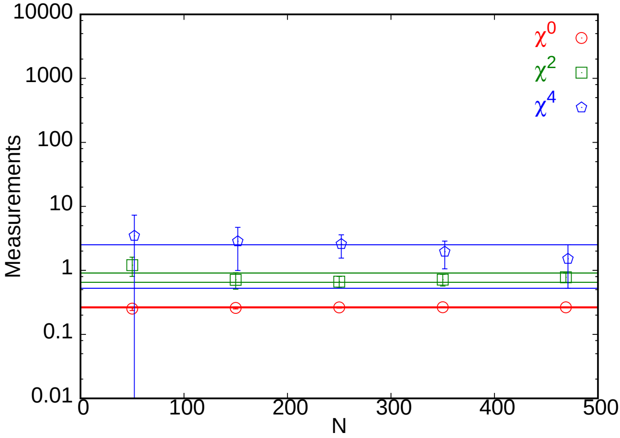

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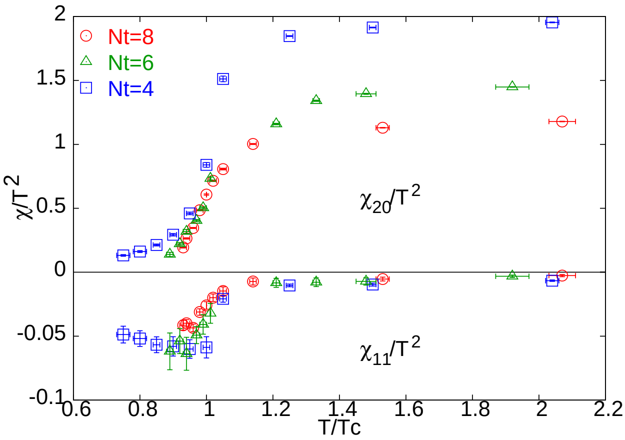
Nearing continuum physics



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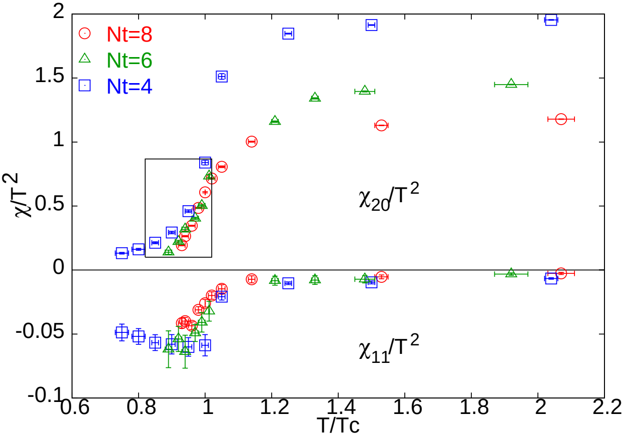

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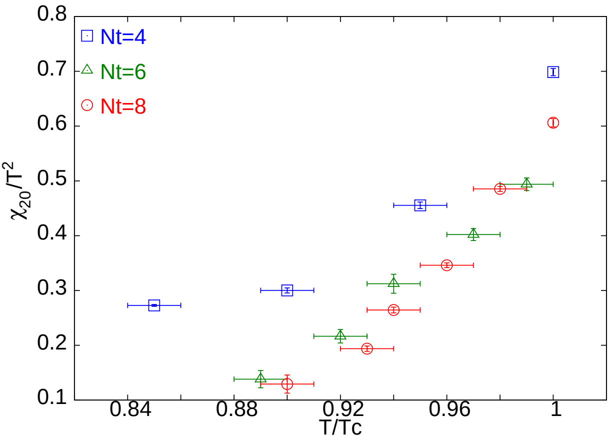
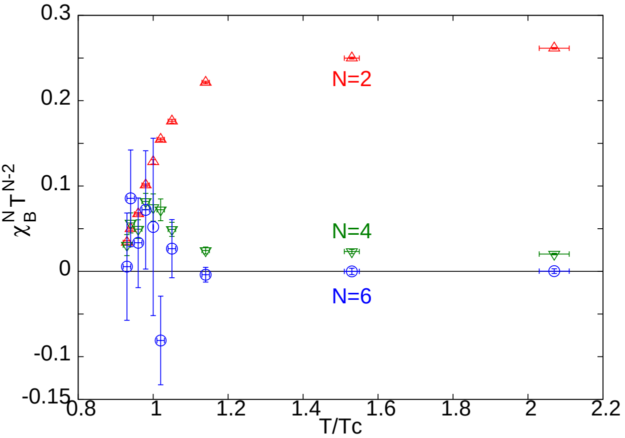


Susceptibilities at $\mu = 0$

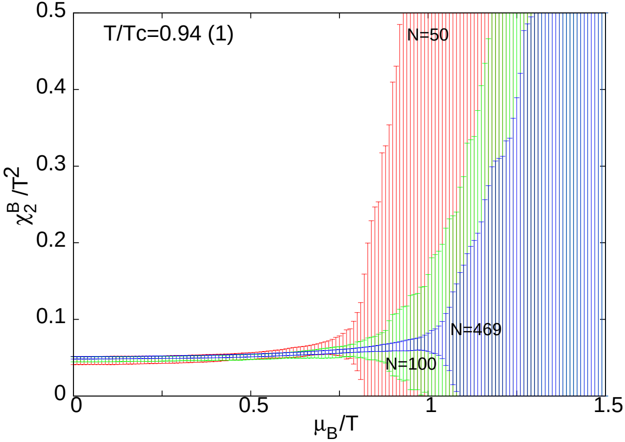


$N_t = 8$

The radius of convergence

m_π/m_ρ		$N_t = 4$	$N_t = 6$	$N_t = 8$
0.6	μ_B^E/T^E	$1.5^{+0.2}_{-0.1}$		
	T^E/T_c	0.94 ± 0.02		
0.4	μ_B^E/T^E			1.8 ± 0.2
	T^E/T_c			0.94 ± 0.01
0.35	μ_B^E/T^E	$1.5^{+0.5}_{-0.2}$	1.8 ± 0.1	
	T^E/T_c	0.95 ± 0.01	0.94 ± 0.01	
0.25	μ_B^E/T^E	$1.4^{+0.4}_{-0.2}$		
	T^E/T_c	0.96 ± 0.01		

Critical slowing down



9-fold increase in statistics not help control errors near the critical point. Exponential demand for statistics: critical slowing down.

Widom scaling

Widom scaling for the order parameter gives

$$|\Delta\mu| = |\Delta n|^{\delta} J\left(\frac{|\Delta T|}{|\Delta n|^{1/\beta}}\right),$$

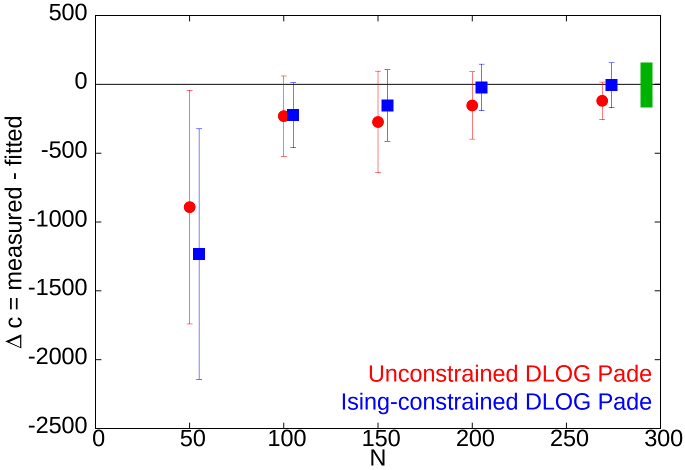
where $\Delta T = T - T_E$ and $\Delta\mu = \mu - \mu_E$. For $\Delta T = 0$ one finds $\Delta n \propto |\Delta\mu|^{1/\delta}$ in the high density phase. Then clearly one has

$$\psi = 1 - \frac{1}{\delta}.$$

For the 3d Ising model, $\delta = 1.49$, so $\psi = 0.79$.
In mean field theory one has $\delta = 3$, so $\psi = 0.66$.

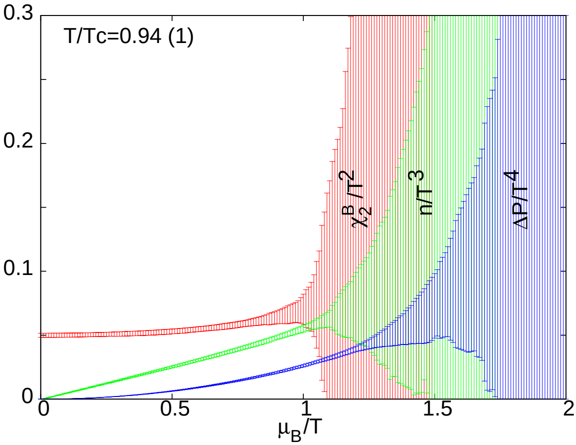
Our computations consistent with both: cannot distinguish between them yet.

Testing the DLOG Pade

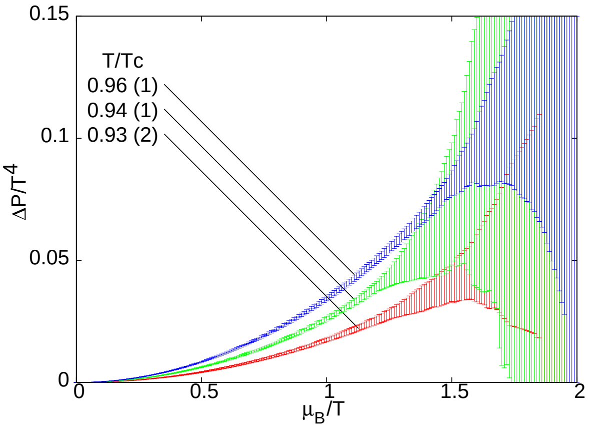


Padé uses 2 terms of the series for m_1. Predicts the 3rd term! Test of pole ansatz = test of criticality.

Successive integrations



The equation of state with $N_t = 8$



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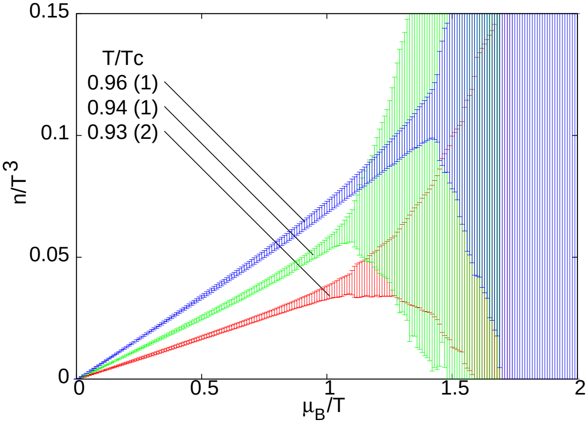

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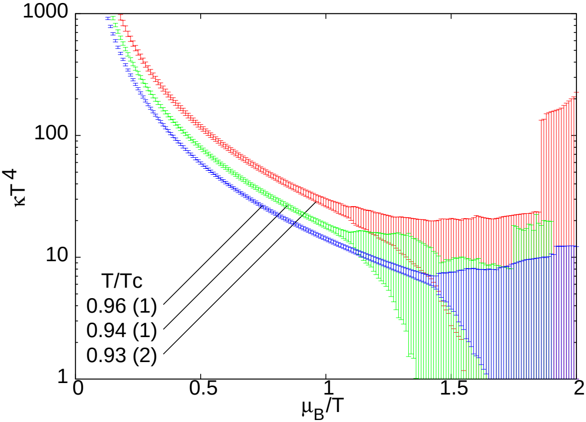
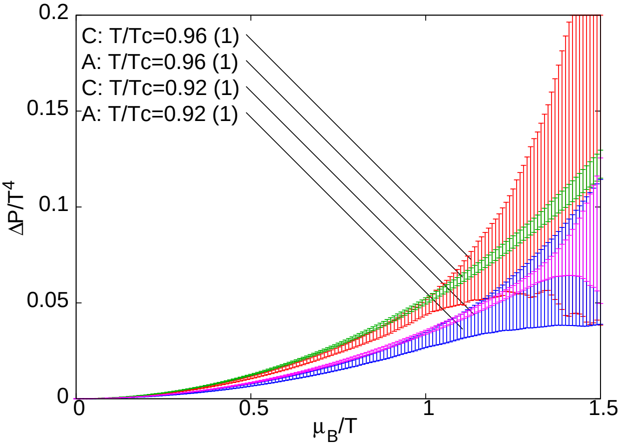


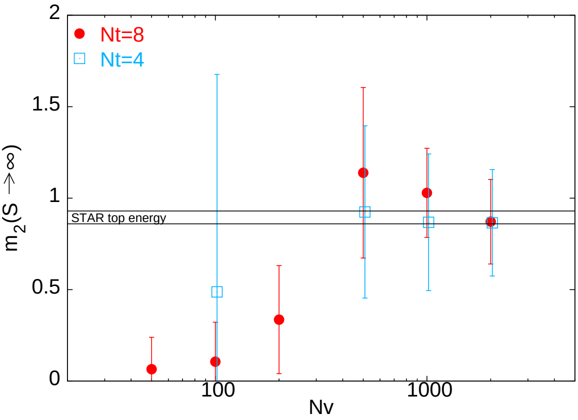
$$\frac{1}{\kappa T^4} = V \left. \frac{\partial P}{\partial V} \right|_T .$$

Little m_π dependence of ΔP



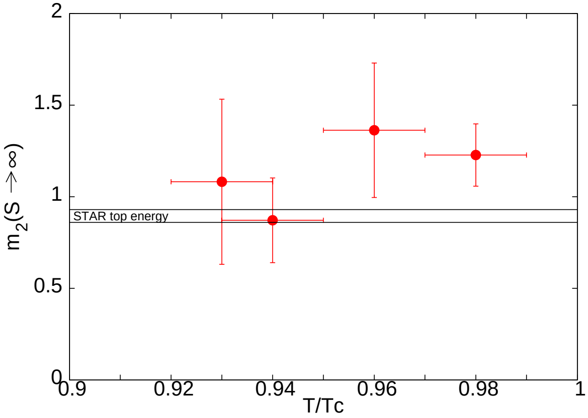
Set A: $m_\pi/m_\rho \simeq 0.25$; set C: $m_\pi/m_\rho \simeq 0.6$; both with $N_t = 4$.

Little N_t dependence of m_2



Defined $m_2 = \chi_B^4/(\chi_B^2/T^2)$. No N_t dependence seen; GLMRX scale setting remains valid.

m_2 and the freezeout parameters



Highest energy data (LHC, maybe also RHIC 200 GeV) at $z \simeq 0$. Direct comparison with lattice; no extrapolation required. Require more precision from the lattice: factor 10 in statistics.

Critical point and the pressure

- QNS require huge CPU expenses; we have up to the 8th order. Momentum cutoff of 0.7 GeV, 1 GeV and 1.4 GeV. Able to see the approach to the renormalized values: $T^E \simeq 0.94\,T_c$, $\mu_B^E/T^E \simeq 1.7$.
- When the series diverges then ΔP at finite μ_B cannot be obtained from a partial resummation of the series.
- Since $\chi_B \simeq |\mu_B - \mu_B^E|^{-\psi}$, the ratio $m_1 = \chi'_B/\chi_B$ has a simple pole. Resum the series expansion into a simple pole. Integrate this to find χ_B and ΔP. First results for pressure at finite μ_B are reported.
- Lattice uses m_1 along a path of constant T and varying μ_B. Event-to-event fluctuations of baryon number can measure m_1 along the freezeout curve.