What happens when you cool mixtures

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An introduction to cooling fluids
- Preliminaries
- Pure substances
- Mixtures

Cooling of reactive ingredients
- Hydrogen atoms and radiation
- Spectral functions

The strong interactions and small bangs
- QCD
- Heavy mesons
- Light hadrons
- Light nuclei

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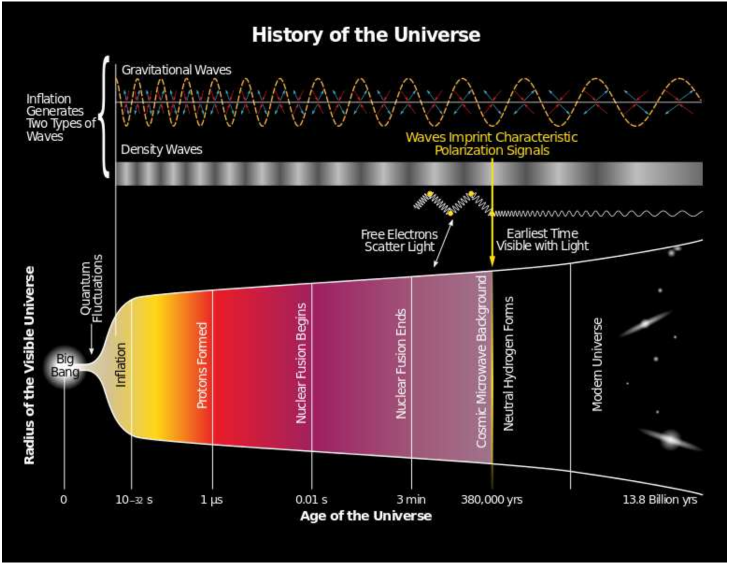
Why are we interested?



Natural units

Most of the time I use natural units:

$$\hbar = 1, \qquad c = 1, \qquad k = 1.$$

$$\begin{aligned} 1\ \mathrm{MeV} &= 11.6 \times 10^{9}\ \mathrm{Kelvin}, \\ &= 1.78 \times 10^{-30}\ \mathrm{Kg}, \\ &= 0.16 \times 10^{-12}\ \mathrm{J}. \end{aligned}$$

Charge is dimensionless; the electron's charge is given by

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.036}.$$

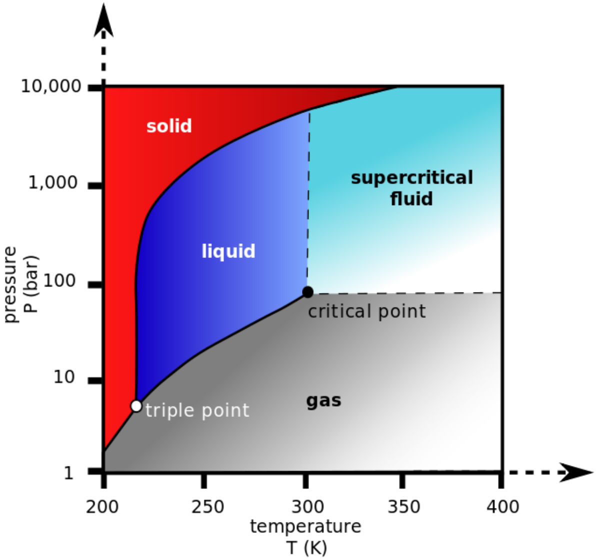
Thermodynamics: an unified theory

Consider a chemically pure substance: for example, CO_2, CH_3OH, or H_2O. Its thermodynamics in the canonical ensemble involves 3 extensive quantities: energy, U, volume, V, entropy, S. One thermodynamic state for each U and V: defines $S(U, V)$.

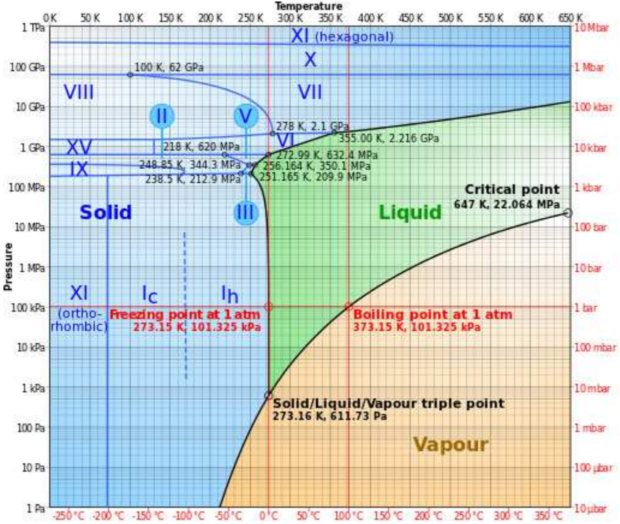
There are two intensive conjugate variables: T and P. The free energy is a function of these two: $F(T, P)$. Usually one thermodynamic state at each T and P.

Different states of a substance have different functional forms for F. The phase diagram traces the values of T and P at which two of these phases coexist, *i.e.*, the $F_1(T, P) = F_2(T, P)$.

Cooling a pure fluid: CO_2

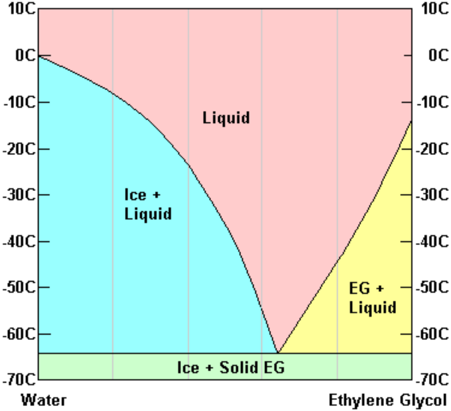


Cooling a pure fluid: H_2O



Two-component fluids: freeze out

Examples: ethylene glycol in water, salt in water, steel. Number of molecules of substance A or B separately conserved. New intensive variable, composition: % of substance A or B in the mixture.



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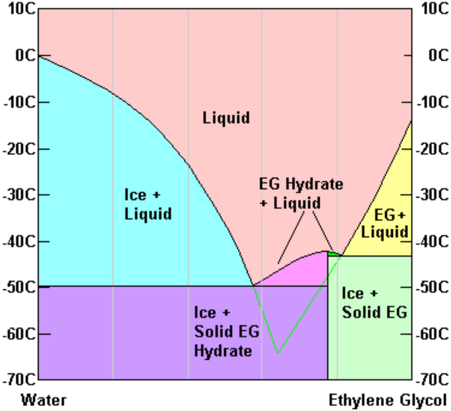
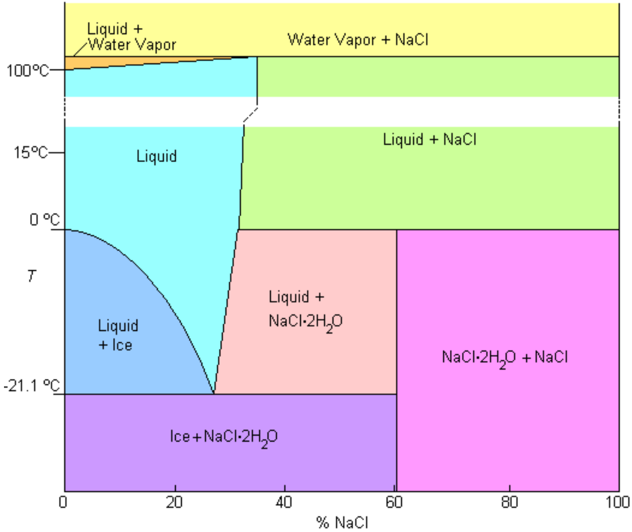


Just for fun: freezeout of salt in water



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The hydrogen atom

The hydrogen atom has a dimensionless parameter α, a mass scale m of the electron and a second mass scale M from the proton.

Since $M \gg m$, a first approximation is to take $M \to \infty$. The velocity of the electron $v \simeq \alpha$. The natural momentum scale of the atom gives the Bohr radius, $r_0^{-1} \simeq mv = m\alpha$. The natural energy scale is the Rydberg, $R = mv^2 \simeq m\alpha^2$. Corrections: spin-orbit coupling at order $\alpha^2 R$, Lamb shift at order $\alpha^3 R$, hyperfine splitting at order $(m/M)\alpha^2 R$.

Put atomic hydrogen in a bath of radiation. Make the gas so dilute that the mean free time between atomic collisions is much larger than the duration of the experiment. Chemical reactions occur: $H + \gamma \leftrightarrow H^*$.

The hydrogen atom with blackbody radiation

Is the probability of finding a photon with energy ω is

$$P_\gamma(\omega) \simeq \frac{\omega^2}{\exp(\omega/T) - 1}\,?$$

Is the probability of the electron being in a state of energy ω

$$P_e(\omega) \simeq \frac{\omega^2}{\exp(\omega/T) + 1}\,?$$

No. Interactions between the electron and radiation have to be taken into account. Very low frequency modes cause Stark and Zeeman effects. Others cause transitions between atomic states, leading to finite lifetimes, τ. So, atomic states no longer have well-defined energies.

Reason: a new Hamiltonian: atom + radiation.

Transition rates

The dipole transition rate between two states (i and f), with energy splitting ω_{if}, is given by

$$\Gamma_{if} \simeq \alpha r_0^2 \omega_{if} P_\gamma(\omega_{if})$$

Taking $\omega_{if} = y_{if} R$, we obtain

$$\Gamma_{if} \simeq \frac{y_{if}^3 \alpha^3 R}{\exp(y_{if} R/T) - 1}.$$

For $R \ll T \ll m$, we find $\Gamma_{if} \simeq y_{if}^2 \alpha^3 T$. The lifetime of the state i is given by

$$\Gamma_i = \tau_i^{-1} = \sum_f \Gamma_{if} \simeq N\overline{y^2}\alpha^3 T,$$

where N is the number of states joined to i by dipole transitions, and $\overline{y^2}$ is the average of y_{if}^2.

How important is the heat bath?

Consider the dimensionless number

$$\varphi = \frac{\Gamma_i}{\Delta\omega_i},$$

where $\Delta\omega_i$ is the mean level spacing near the state i. $\Delta\omega_i$ is large for ground state, but decreases as we approach the unbound continuum.

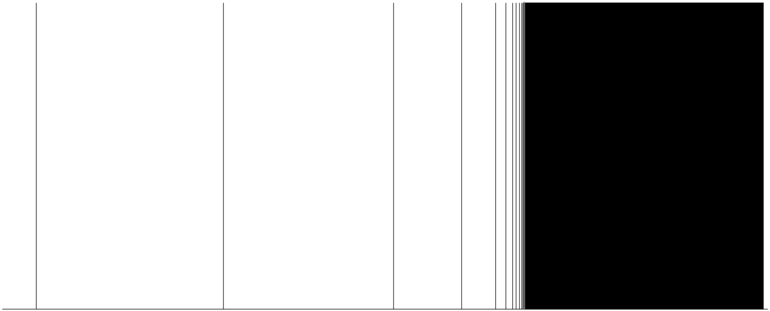
If φ is small, then the state is hardly modified, and the Fermi distribution for the electron may be an useful approximation.

On the other hand, if $\varphi \simeq 1$, then there is strong mixing between the various nearby states, and the initial Fermi distribution is no longer an useful approximation. Instead one has to take into account states of $|pe\gamma\gamma\cdots\rangle$, *i.e.*, try to solve the quantum field theory.

The density matrix

1. The density matrix of a hydrogen atom at finite temperature has matrix elements between states $\langle pe\gamma|\exp(-H/T)|pe\gamma\gamma\rangle$, *etc.*.
2. It cannot be approximated by a density matrix of Hydrogen without accounting for radiation, because this will not give correct predictions for conductivity, refractive index, *etc.*
3. Such a drastic approximation will also give wrong predictions for the population of excited states.
4. Since the density matrix gives U, S, P, *etc.*, all these quantities will be predicted wrongly unless the full density matrix is taken into account. Especially, P will be wrongly predicted, since the mass of a free electron is much smaller than the mass of H, and hence gives very large contribution to P. As a result, $S = (U + P)/T$ will also be wrong.

Cooling a dilute hydrogen gas



Atomic hydrogen gas, when heated will be a plasma, *i.e.*, a mixture of ions and atoms in various states of excitation. There is no phase transition of hydrogen into a plasma. As a result, when cooled, the density of plasma decreases continuously to zero.

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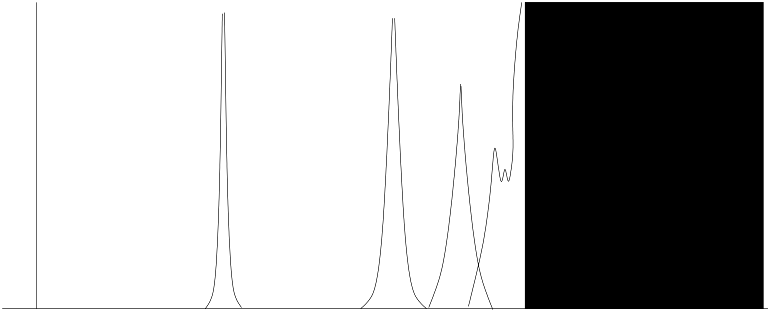

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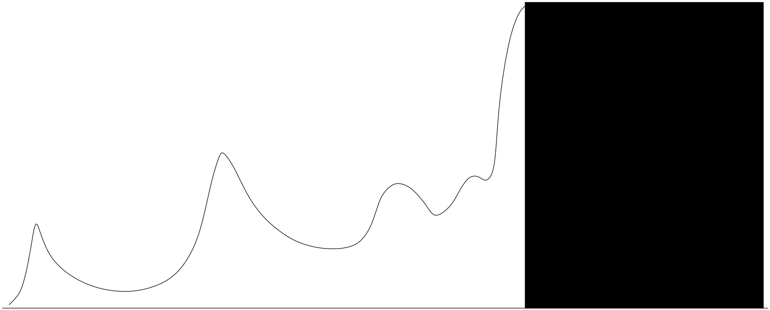

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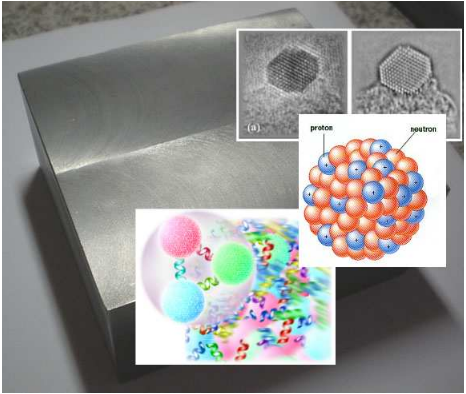
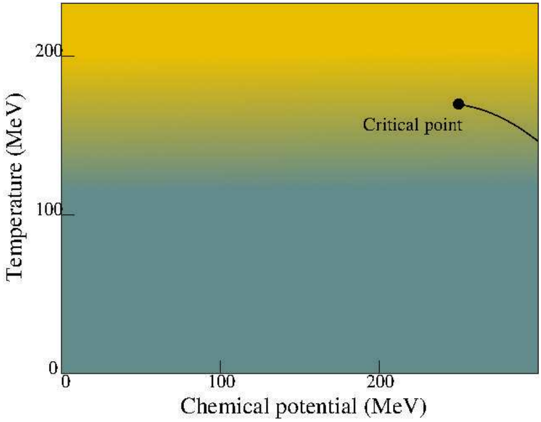
Strong interactions: $\alpha_s \geq 1$



Protons and neutrons contain 3 quarks. Mesons contain a quark and an antiquark. At $T = 0$ there are no unbound states of quarks: confinement, a property of strong interactions. Typical scale, $\Lambda \simeq m_\rho/2 \simeq m_p/3$. Many excited states of hadrons known.

Hot strong interactions



The phase diagram of strongly interacting matter has a phase coexistence line and a critical point. No phase transition at $\mu = 0$.

Small bangs

Bang two gold nuclei together at relativistic energies. Result: a hot fireball with T greater than the μs universe. Produce a fireball of quarks and gluons. The fireball expands and cools.

As the fireball expands, the matter becomes more dilute. Mean free path increases, until the constituents can no longer remain in equilibrium. So there is a point at which the system falls out of equilibrium.

Out of equilibrium— tough problem!

First take a sub-problem where the dynamics takes place while the system remains in equilibrium.

Heavy-quark mesons: the hydrogen atom of QCD

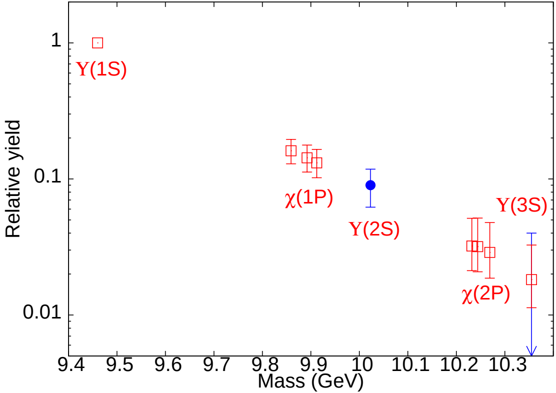
Consider heavy quarks Q and antiquarks $\overline{Q}$, with mass M. Two more scales in the problem: T and Λ. In small bangs $T \simeq \Lambda$. Quark is heavy: $M \gg T$, $M \gg \Lambda$.

In the meson, $\overline{Q}Q$, the binding energy, $B \simeq Mv^2 \simeq \Lambda$, as a result, $v^2 \simeq \Lambda/M \ll 1$. So, $B \simeq T$, and thermal effects can completely modify the meson.

Many arguments and computations lead to $\Gamma \simeq T$. As a result bound mesons and free Q, $\overline{Q}$ remain in thermal equilibrium with each other until the temperature falls below the expansion rate of the fireball.

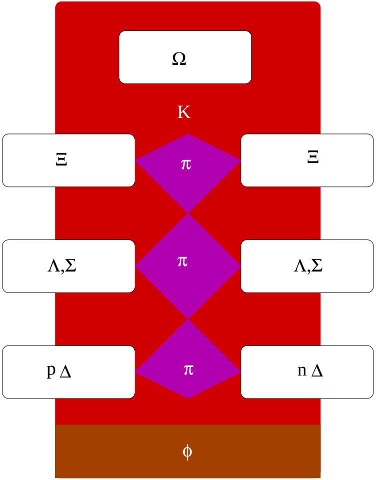
At this point Q and $\overline{Q}$ freeze out into mesons: the population of different excited states given by the Bose distribution.

Relative yields



Freezeout temperature is 222 ± 29 MeV: central rapidity, all centrality, low transverse momentum.

Light hadron chemistry



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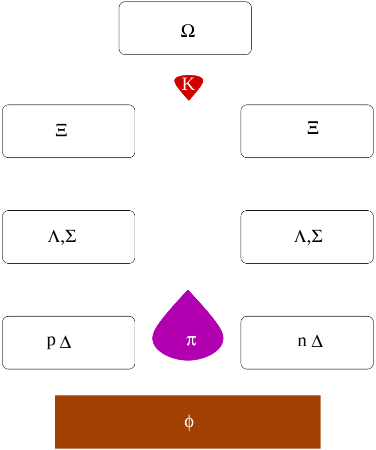

1CFO: T, μ, V, γ_S

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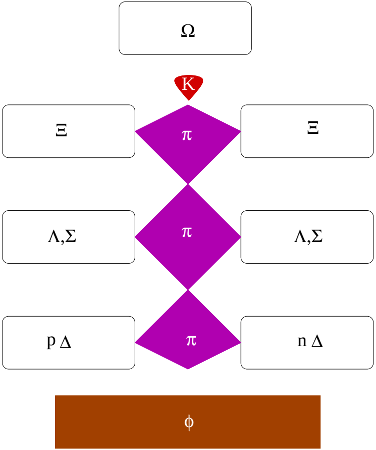
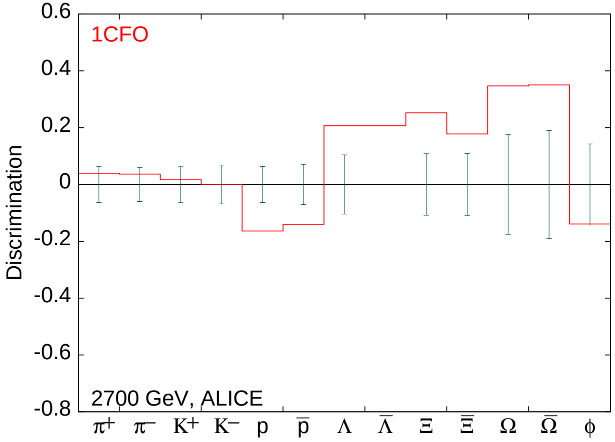


2CFO: T_S, μ_S, V_S, T_{NS}, μ_{NS}, V_{NS}

Large Hadron Collider data



Large non-thermal fluctuations? **Bleicher et al, Becattini et al**

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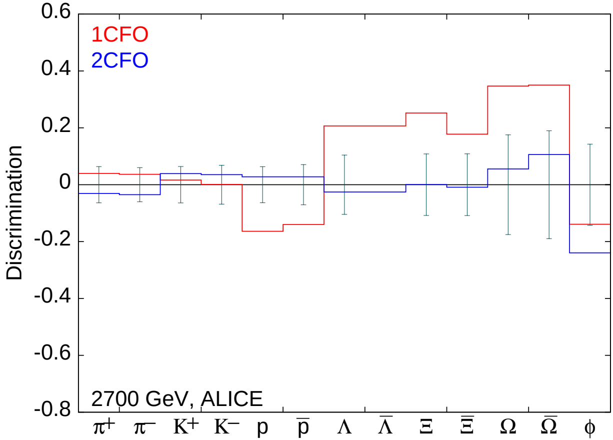
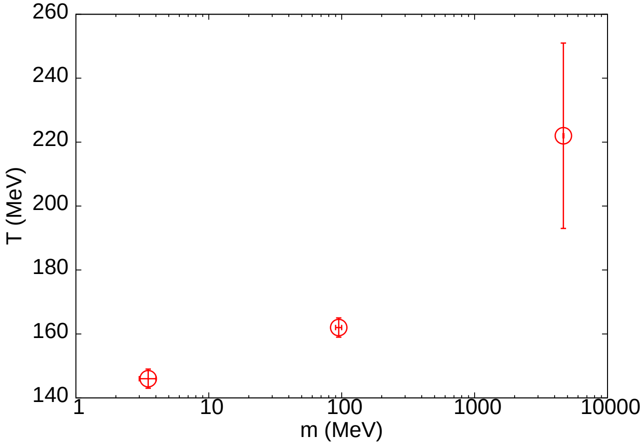


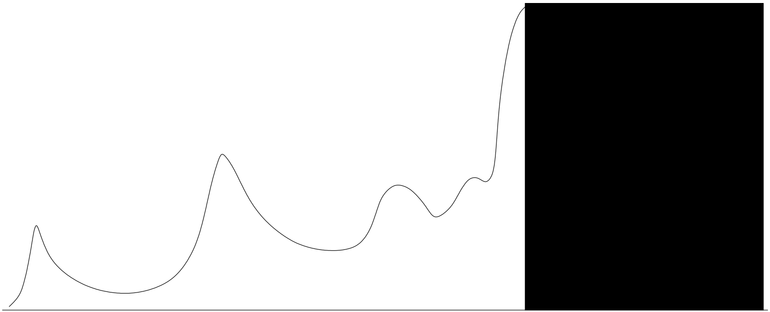
Large non-thermal fluctuations? **Bleicher et al, Becattini et al**

Or normal late-stage kinetics? **Chatterjee et al, Bugaev et al**

Systematics of quark/hadron freezeout



No nuclei in equilibrium!



Nuclear binding energies $B \simeq 10$ MeV, so $B/\Lambda \ll 1$. This is like Hydrogen atoms at high temperature: $B \ll T \ll M_p$. Nuclei cannot form in equilibrium fireball.

Post-thermal kinetics

What happens if nuclei form long after freeze out?

Consider $p + n \leftrightarrow d$, with $B \simeq 2$ MeV. The kinetic energies of p and n are (on average) equal to their freezeout temperature: 100–150 MeV $\gg B$. So, d forms largely at zero relative velocity. So, the velocity of d is characteristic of the freezeout temperature of p and n.

Similarly, hypernuclei also form late, but their freezeout velocities and kinetic energies are characteristic of the freezeout temperature of their constituents.

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1. Cooling of multi-component reactive fluids typically leads to a phase separation: freeze out.
2. The example of Hydrogen atoms in reactive equilibrium with black body radiation illustrates that it is not possible to describe hot reactive substances in terms of a mixture of ideal gases. Freeze out must be understood as a dynamic process.
3. Freeze out of particles after the big bang can be studied in small bangs (relativistic heavy-ion collisions) produced in the lab. These allow us to study the history of the universe from sub-μs ages to the age of 3 minutes. Many open questions: much scope for theory and experiment to evolve together.