Thermal Photons in QGP and Non-Ideal Effects

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The Phase Diagram of QCD (Bring Your Own)

13 - 14 December, 2010, TIFR, Mumbai.

Collaborators:

- Dr. Jitesh R. Bhatt
- Dr. Hiranmaya Mishra

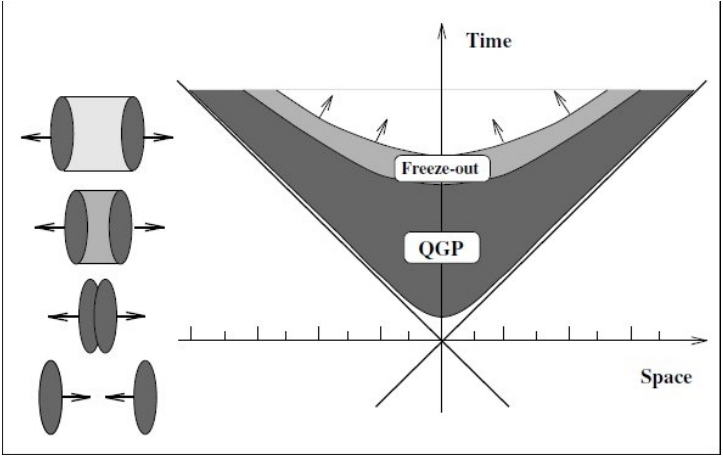
- The results are presented in:

  Jitesh R. Bhatt, H. Mishra, and Sreekanth V, JHEP 11 (2010) 106. [arXiv:1011.1969]

Plan of the Talk

- Introduction
- 2nd order causal dissipative hydrodynamics (Israel-Stewart)
- Non-ideal effects: EoS and bulk viscosity
- Hydrodyamical evolution and Cavitation
- Thermal γ from QGP
- Summary

- AIM: To study the role of *non-ideal* effects near T_c arising due to the equation of state (EoS), bulk-viscosity and cavitation on the thermal photon production from QGP.



Energy momentum tensor of the fluid element in Relativistic dissipative hydrodynamics is defined as

- $$T^{\mu\nu} = \varepsilon\, u^{\mu}\, u^{\nu}\; - P\,\Delta^{\mu\nu} + \Pi^{\mu\nu}$$
  ε, P and u^{μ} are the energy density, pressure and four velocity of the fluid element. $\Delta^{\mu\nu} \;=\; g^{\mu\nu} - u^{\mu}\, u^{\nu}$.
- Viscous contributions to $T^{\mu\nu}$ is represented by
  $$\Pi^{\mu\nu} = \pi^{\mu\nu} - \;\Delta^{\mu\nu}\,\Pi$$
- $\pi^{\mu\nu}$ (traceless) gives the contribution of shear viscosity and Π gives the bulk viscosity contribution.

Relativistic hydrodyamical equations are

$$\boxed{\begin{array}{rcl} D\varepsilon + (\varepsilon + P)\,\theta - \Pi^{\mu\nu}\nabla_{(\mu}\,u_{\nu)} & = & 0 \\ (\varepsilon + P)\,Du^{\alpha} - \nabla^{\alpha}P + \Delta_{\alpha\nu}\,\partial_{\mu}\Pi^{\mu\nu} & = & 0 \end{array}}$$

$(D \equiv u^{\mu}\partial_{\mu},\ \theta \equiv \partial_{\mu}\,u^{\mu},\ \nabla_{\alpha} = \Delta_{\mu\alpha}\partial^{\mu}$ and $A_{(\mu}\,B_{\nu)} = \frac{1}{2}[A_{\mu}\,B_{\nu} + A_{\nu}\,B_{\mu}])$

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The structure of viscous tensor can be determined with help of the definition of the entropy current s^{μ} and demanding the validity of second law of thermodynamics:

$$\partial_{\mu}s^{\mu} \geq 0 \ (s = \tfrac{\varepsilon+P}{T})$$

Causal dissipative hydrodynamics

- Second order hydrodynamics (Israel-Stewart) is obtained by using

$$s^\mu = su^\mu - \frac{\beta_0}{2T} u^\mu \Pi^2 - \frac{\beta_2}{2T} u^\mu \pi_{\alpha\beta}\pi^{\alpha\beta} + \mathcal{O}(\Pi^3)$$

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- Now $\partial_\mu s^\mu \geq 0$ gives *dynamical evolution equations* for $\pi_{\mu\nu}$ and Π

$$\pi_{\alpha\beta} = \eta\left(\nabla_{<\alpha}u_{\beta>} - \pi_{\alpha\beta}TD\left(\frac{\beta_2}{T}\right) - 2\beta_2 D\pi_{\alpha\beta} - \beta_2\pi_{\alpha\beta}\partial_\mu u^\mu\right) ,$$
$$\Pi = \zeta\left(\nabla_\alpha u^\alpha - \frac{1}{2}\Pi\ TD\left(\frac{\beta_0}{T}\right) - \beta_0 D\Pi - \frac{1}{2}\beta_0\Pi\partial_\mu u^\mu\right) ,$$

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\end{aligned}$$

The coefficients β_0 and β_2 are related with the relaxation time by
$$\tau_\Pi = \zeta\,\beta_0\,, \tau_\pi = 2\eta\,\beta_2.$$

- Unlike first order (Navier-Stokes) this description is *causal* and no *instabilities* [Hiscock and Lindblom (1985), Baier et. al (2006)]

1D boost invariant flow

Bjorken's prescription to describe the dimensional boost invariant expanding flow:-

- convenient parametrization of the coordinates using the proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $y = \frac{1}{2}\, ln[\frac{t+z}{t-z}]$;

$$t = \tau \cosh y \text{ and } z = \tau \sinh y$$

- in the local rest frame of the fireball $u^\mu = (\cosh y, 0, 0, \sinh y)$, form of $T^{\mu\nu} = \mathrm{diag.}(\varepsilon, P_\perp, P_\perp, P_z)$
- Viscosities can contribute in the effective pressure in the transverse and longitudinal directions

$$\begin{aligned} P_\perp &= P + \Pi + \frac{1}{2}\Phi \\ P_z &= P + \Pi - \Phi \end{aligned}$$

- Φ is the shear ($\pi^{ij} = \mathrm{diag}(\Phi/2, \Phi/2, -\Phi)$) and Π is the bulk viscosity contributions to the equilibrium pressure P.

Equations governing longitudinal expansion

In this 1D formalism;

$$
\begin{aligned}
\frac{\partial\varepsilon}{\partial\tau} &= -\frac{1}{\tau}(\varepsilon + P + \Pi - \Phi)\,, \\
\frac{\partial\Phi}{\partial\tau} &= -\frac{\Phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\beta_2\tau} - \frac{\Phi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_2}\partial_\tau\left(\frac{\beta_2}{T}\right)\right], \\
\frac{\partial\Pi}{\partial\tau} &= -\frac{\Pi}{\tau_\Pi} - \frac{1}{\beta_0\tau} - \frac{\Pi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_0}\partial_\tau\left(\frac{\beta_0}{T}\right)\right].
\end{aligned}
$$

- $\tau_\pi(T) = \tau_\Pi(T)$ as we don't have any reliable prediction for τ_Π and $\tau_\pi = \frac{2-\ln 2}{2\pi T}$
- EoS is needed to close the system.

Non-ideal effect: Equation of State

- We use lattice QCD result for *non-ideal* ($\varepsilon - 3P \neq 0$) EoS [A. Bazavov *et al*. (2009)]
- Important near T_c
- Parametrised form of their result for trace anomaly is given by

$$\frac{\varepsilon - 3P}{T^4} = \left(1 - \frac{1}{\left[1 + \exp\left(\frac{T - c_1}{c_2}\right)\right]^2}\right)\left(\frac{d_2}{T^2} + \frac{d_4}{T^4}\right) ,$$

with $d_2 = 0.24$ GeV^2, $d_4 = 0.0054$ GeV^4, $c_1 = 0.2073$ GeV, and $c_2 = 0.0172$ GeV

- $$\frac{P(T)}{T^4} - \frac{P(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \, \frac{\varepsilon - 3P}{T'^5} ,$$

with $T_0 = 50$ MeV and $P(T_0) = 0$

Non-ideal effect: Bulk Viscosity

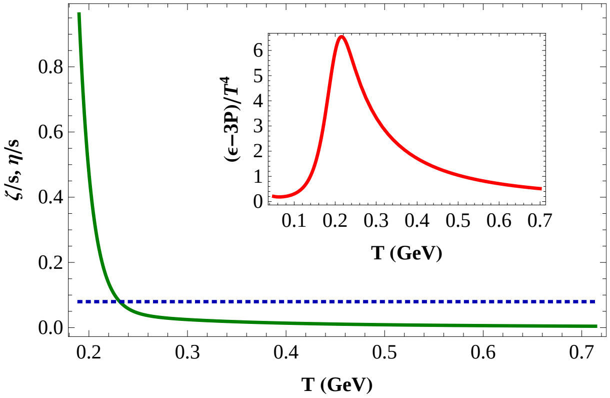
- Unlike *ideal* EoS ($\varepsilon - 3P = 0$), bulk viscosity is not negligible with non-ideal case.
- Recent studies show that near the critical temperature T_c effect of bulk viscosity becomes important
- We use the result of Meyer (2008) based upon Lattice QCD calculations, for ζ/s [K. Rajagopal (2010)]

$$\frac{\zeta}{s} = a \exp\left(\frac{T_c - T}{\Delta T}\right) + b\left(\frac{T_c}{T}\right)^2 \quad \text{for } T > T_c,$$

which indicate the existence a peak of ζ/s near T_c, however the height and width of this curve are not well understood.

- The parameter $a = 0.901$ controls the height and $\Delta T = \frac{T_c}{14.5}$ controls the width of the ζ/s curve
- We use the lower bound of the shear viscosity to entropy density ratio $\eta/s = 1/4\pi$ [KSS (2005)]

Non-ideal Equation of State



- $(\varepsilon - 3P)/T^4$, ζ/s (and $\eta/s = 1/4\pi$) as functions of temperature T. Around critical temperature ($T_c = .190$ GeV) $\zeta \gg \eta$ and departure of equation of state from ideal case ($\varepsilon = 3P$) is large.

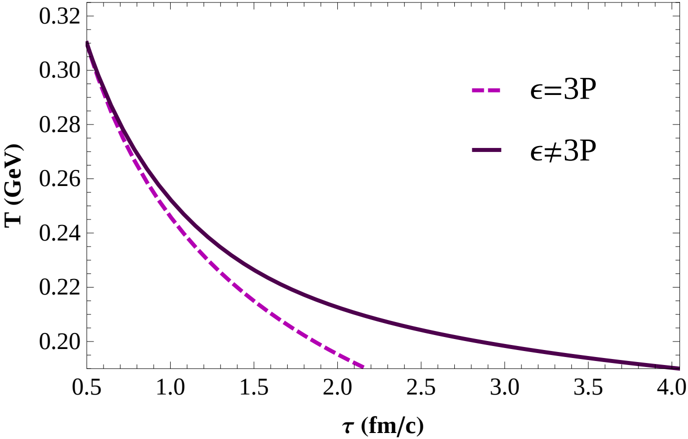
Hydrodyamical Evolution

- In order to understand the temporal evolution of temperature $T(\tau)$, pressure $P(\tau)$ and viscous stresses - $\Phi(\tau)$ and $\Pi(\tau)$, we numerically solve the hydrodynamical equations describing the longitudinal expansion of the plasma
- Initial conditions: we use relevant initial condition for RHIC [D.K. Srivastava (1999)]

$$\tau_0 = 0.5 fm/c,\, T_0 = .310 GeV$$

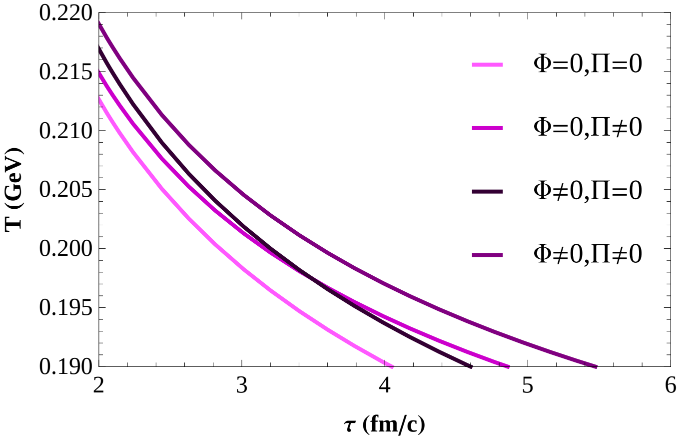
- We will take initial values of viscous contributions as $\Phi(\tau_0) = 0$ and $\Pi(\tau_0) = 0$.

Temperature profile (No viscosity)

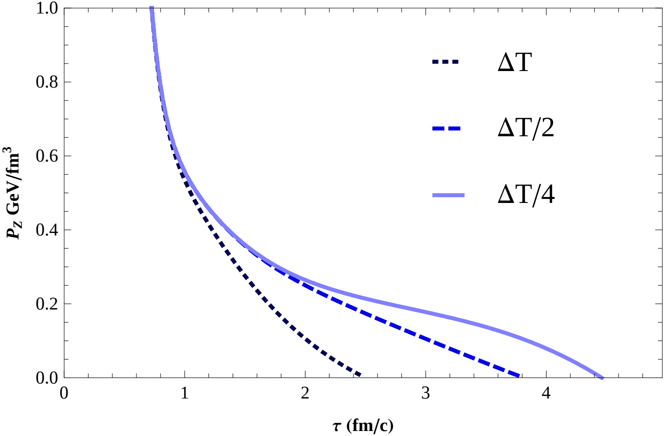


- Temperature profile using massless (*ideal*) and *non-ideal* EoS in RHIC scenario. Viscous effects are neglected in both cases. System evolving with *non-ideal* EoS takes a significantly larger time to reach T_c as compared to *ideal* EoS scenario.

Temperature profile



- Time evolution of temperature with *non-ideal* EoS for different combinations of bulk (Π) and shear (Φ) viscosities.



- Longitudinal pressure P_z for various bulk viscosity cases.

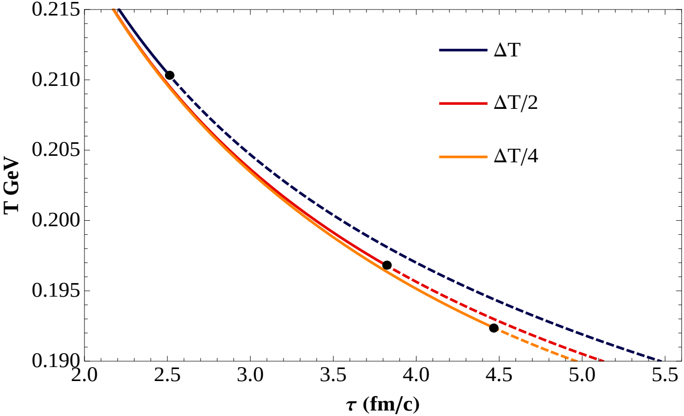
Cavitation

- Since $\Pi < 0$, from the definition of longitudinal pressure $P_z = P + \Pi - \Phi$ it is clear that if either Π or Φ is large enough it can drive P_z to negative values.

- $Pz = 0$ defines the condition for the onset of *cavitation*

- At this instant when of P_z becoming zero the expanding fluid will break apart in to fragments and *hydrodynamic treatment looses its validity* [K. Rajagopal et. al. (2010)]

Cavitation



Temperature is plotted as a function of time. With peak value (a) of ζ/s remains same while width (ΔT) varies. Solid line in the curve ends at the time of cavitation, while the dashed lines shows that how system would continue till T_c if cavitation is ignored. Figure shows that larger the ΔT shorter the cavitation time.

Thermal Photons in QGP

- Thermal photons emitted from the hot fireball created in relativistic heavy-ion collisions is a promising tool for providing a signature of quark-gluon plasma
- Spectra of thermal photons depend upon the fireball temperature and they can be calculated from the scattering cross-section of the processes like *Compton scattering* $q(\bar{q})g \rightarrow q(\bar{q})\gamma$ and annihilation processes $q\bar{q} \rightarrow g\gamma$ and higher order processes *bremsstrahlung* etc.
- Thermal photons can be used as a tool to measure the viscosity of the strongly interacting matter produced in the collisions [J. Bhatt et. al (2010), K Dusling (2010)]

Viscous corrections to the distribution functions

- Viscous corrections to the particle ditribution functions are determined by [Teaney (2008)]

$$f(p) = f_0 + \delta f = f_0 + \delta f_\eta + \delta f_\zeta$$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu f = T_o^{\mu\nu} + \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu}\,\Theta$$

- restricting corrections to f upto quadratic order in momentum,

$$f(p) = f_0 \left(1 + \frac{\eta/s}{2T^3} p^\alpha p^\beta \nabla_{\langle\alpha} u_{\beta\rangle} + \frac{\zeta/s}{2T^3} p^\alpha p^\beta \Delta_{\alpha\beta} \Theta \right)$$

Total Photon Production

Once the evolution of temperature is known from the hydrodynamical model, the *total photon spectrum* is obtained by integrating the total rate over the space time history of the collision,

$$\left(\frac{dN}{d^2p_\perp dy}\right)_{y,p_\perp} = \int d^4x \left(E\frac{dN}{d^3pd^4x}\right)$$
$$= Q\int_{\tau_0}^{\tau_f} d\tau\ \tau \int_{-y_{nuc}}^{y_{nuc}} dy^{'} \left(E\frac{dN}{d^3pd^4x}\right)$$

- τ_0 and τ_f (with $T(\tau_f) = T_c$) are the initial and final values of time we are interested.

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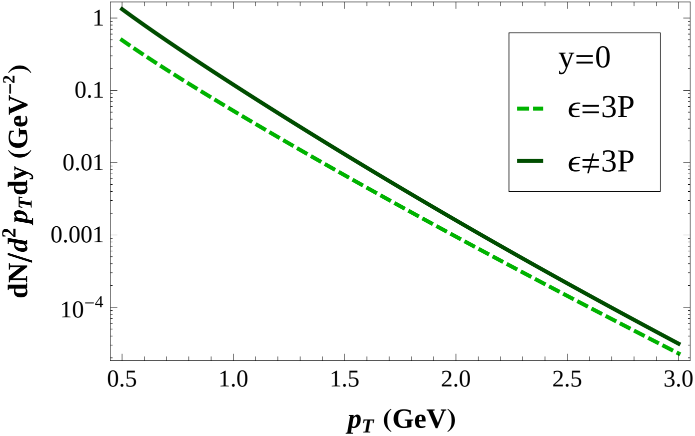
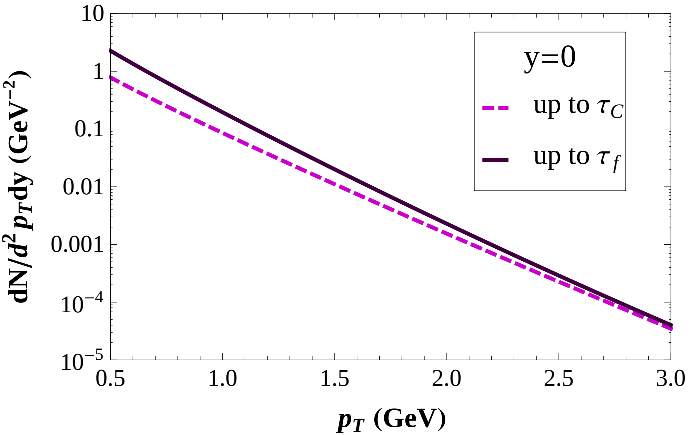
- τ_0 and τ_f (with $T(\tau_f) = T_c$) are the initial and final values of time we are interested.
- If cavitation occurs at τ_c, we have to replace τ_f by τ_c in photon yield expression, since hydrodynamics looses its validity after cavitation.

Thermal Photon Production

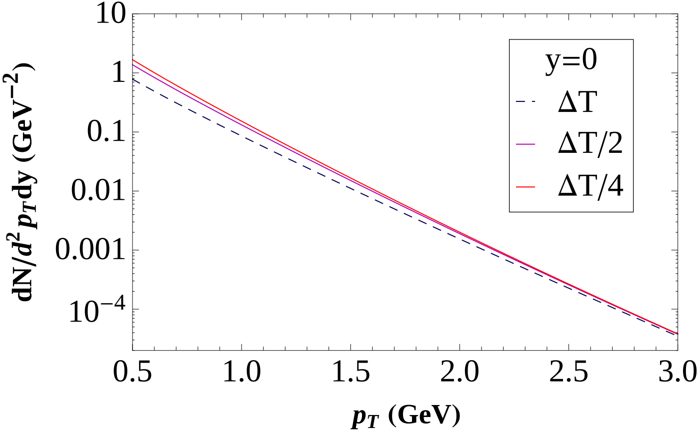


- Photon flux as function of transverse momentum p_T of the photon for different equation of states. No effect of viscosity included in the hydrodynamical equations. At energy $E = 1$ GeV, photon flux for the *non-ideal* EoS is 60% larger than that of *ideal* EoS case.

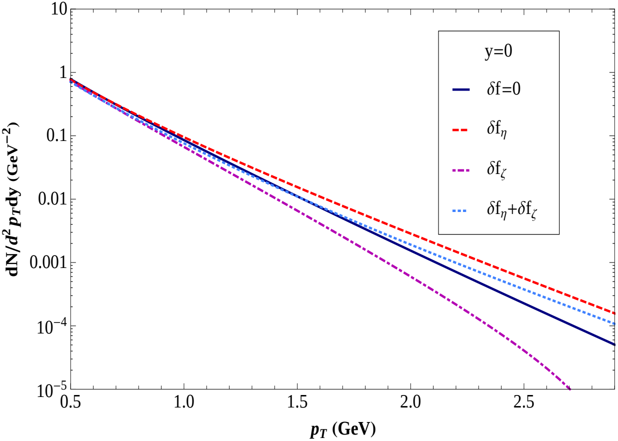
Rate over-estimation



- Photon spectrum obtained by considering the effect of cavitation (dashed line). For a comparison we plot the spectrum without incorporating the effect of cavitation (solid line). (At p=0.5 GeV overestimation of rate is 200% and at p=2 GeV it is 50%).



- Photon production rates showing the effect of different cavitation time.



- Viscous corrections to the distribution function and photon production rate.

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- We have shown using non-ideal EoS using the recent lattice results that the hydrodynamical expansion gets significantly slow down as compared to the case with the massless EoS. This in turn enhances the flux of hard thermal photons.
- Bulk viscosity plays a dual role in heavy-ion collisions: On one hand it enhances the time by which the system attains the critical temperature, while on the other hand it can make the hydrodynamical treatment invalid much before it reaches T_c.

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- Using second order causal relativistic hydrodynamics we have analyzed the role of non-ideal effects near T_c arising due to the equation of state, bulk-viscosity and cavitation on the thermal photon production.
- We have shown using non-ideal EoS using the recent lattice results that the hydrodynamical expansion gets significantly slow down as compared to the case with the massless EoS. This in turn enhances the flux of hard thermal photons.
- Bulk viscosity plays a dual role in heavy-ion collisions: On one hand it enhances the time by which the system attains the critical temperature, while on the other hand it can make the hydrodynamical treatment invalid much before it reaches T_c.
- We have shown that if the phenomenon of cavitation is ignored one can have erroneous estimates of the photon production.

THANK YOU

Ideal Equation of State

In order to understand the effect of *non-ideal* EoS in hydrodynamical evolution and subsequent photon spectra we compare these results with that of an *ideal* EoS ($\varepsilon = 3P$).

- We consider the EoS of a relativistic gas of massless quarks and gluons. The pressure of such a system is given by

$$P = a\ T^4\,;\ a = \left(16 + \frac{21}{2}N_f\right)\ \frac{\pi^2}{90}$$

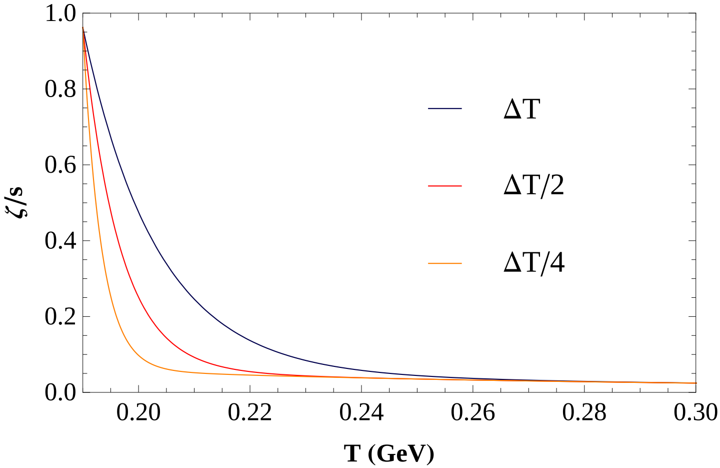
  where $N_f = 2$ in our calculations.

- Hydrodynamical evolution equations of such an EoS within ideal (without viscous effects) Bjorken flow can be solved analytically and the temperature dependence is given by

$$T = T_0\ \left(\frac{\tau_0}{\tau}\right)^{1/3},$$

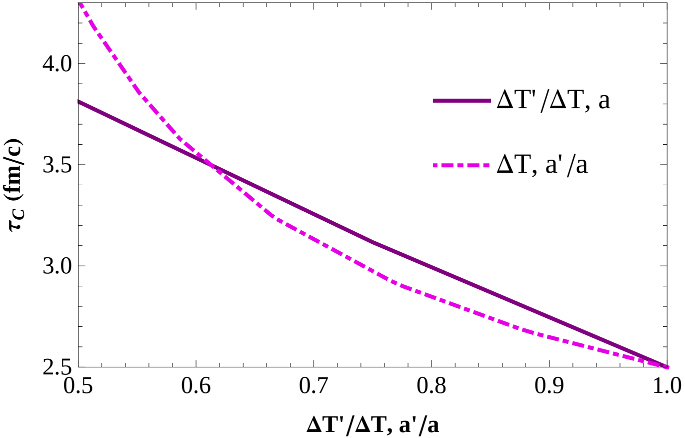
  where τ_0 and T_0 are the initial time and temperature.

- effect of bulk viscosity can be neglected in the relativistic limit when the equation of state $3P = \varepsilon$ is obeyed

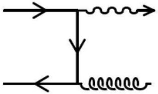


- Various bulk viscosity scenarios by changing the width of the curve through the parameter ΔT.

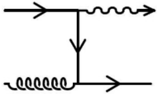
Cavitation time



- Cavitation time τ_c as a function of different values of height (a') and width $(\Delta T')$ of ζ/s curve.



$q\,\bar{q} \rightarrow \gamma\, g$



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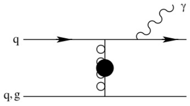

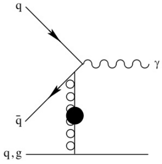

$$E\frac{dN}{d^4xd^3p}\Big|_{cs+ann.} = 0.0281\alpha\alpha_s\ T^2\ e^{-E/T}\ \ln\left(\frac{0.23\mathrm{E}}{\alpha_s \mathrm{T}}\right)$$

$$E\frac{dN}{d^4xd^3p}\Big|_{brems.} = 0.0219\ \alpha\alpha_s\ T^2\ e^{-E/T}$$

$$E\frac{dN}{d^4xd^3p}\Big|_{aws.} = 0.0105\ \alpha\alpha_s\ ET\ e^{-E/T}$$

Viscous corrections to the distribution functions

- In 1D Bjorken flow viscous corrections to the distribution function takes the form

$$
\begin{aligned}
f = f_0\Bigg(1 \quad &+ \quad \frac{\eta/s}{2T^3}\left[\frac{2}{3\tau}p_T^2 - \frac{4}{3\tau}m_T^2 sinh^2(y-y')\right] \\
&- \quad \frac{2}{5}\frac{\zeta/s}{2T^3}\left[\frac{p_T^2}{\tau} + \frac{m_T^2}{\tau} sinh^2(y-y')\right]\Bigg)
\end{aligned}
$$