Wilsonian renormalization

Sourendu Gupta

Mini School 2016, IACS Kolkata, India

Effective Field Theories
29 February–4 March, 2016

Outline

Outline

Spurious divergences in Quantum Field Theory

Wilsonian Effective Field Theories

Wilsonian renormalization
- The renormalization group
- The Wilsonian point of view
- RG for an Euclidean field theory in $D = 0$
- Defining QFT without perturbation theory

End matter

Outline

The old renormalization

We start with a Lagrangian, for example, the 4-Fermi theory:

$$\mathcal{L} = \frac{1}{2}\overline{\psi}\partial\!\!\!/\psi - \frac{1}{2}m\overline{\psi}\psi + \lambda(\overline{\psi}\psi)^2 + \cdots$$

Here all the parameters are finite. But anticipating the divergence of perturbative expansions, we add counter-terms

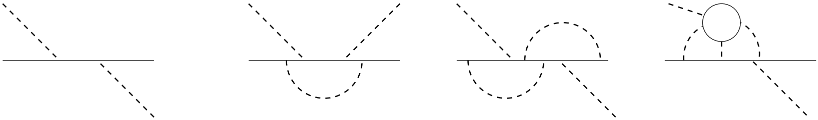
$$\mathcal{L}_c = \frac{1}{2}A\overline{\psi}\partial\!\!\!/\psi - \frac{1}{2}Bm\overline{\psi}\psi + \lambda C(\overline{\psi}\psi)^2 + \cdots$$

where A, B, C, *etc.*, are chosen to cancel all divergences in amplitudes. This gives the renormalized Lagrangian

$$\mathcal{L}_r = \frac{1}{2}\overline{\psi}_r\partial\!\!\!/\psi_r - \frac{1}{2}m_r\overline{\psi}_r\psi_r + \lambda_r(\overline{\psi}_r\psi_r)^2 + \cdots$$

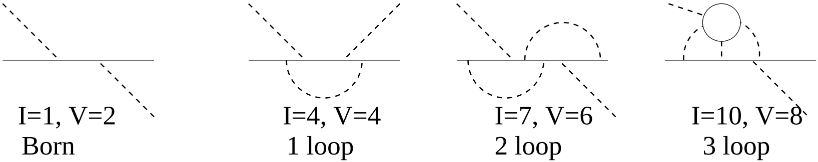
Clearly, $\psi_r = Z_\psi\psi$ where $\psi_r = \psi\sqrt{1+A}$, $m_r = m(1+B)/(1+A)$, $\lambda_r = \lambda(1+C)/(1+A)^2$, *etc.*. The 4-Fermi theory was called an unrenormalizable theory since an infinite number of counter-terms are needed to cancel all the divergences arising from $\mathcal{L}$.

Perturbation theory: expansion of amplitudes in loops



Any amplitude in a QFT can be expanded in the number of loops.

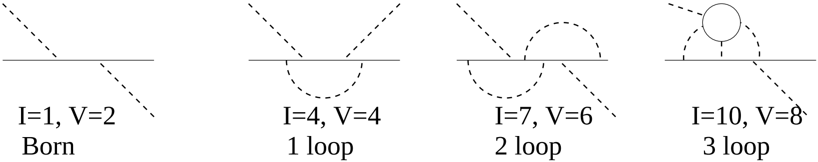
Perturbation theory: expansion of amplitudes in loops



Any amplitude in a QFT can be expanded in the number of loops.

$$L = 1 + I - V$$

Perturbation theory: expansion of amplitudes in loops



Any amplitude in a QFT can be expanded in the number of loops.

$$L = 1 + I - V$$

Problem 2.1

Prove the equation. Prove that the expansion in loops is an expansion in $\hbar$, so is a semi-classical expansion. The number of unconstrained momenta is equal to the number of loops, giving an integral over each loop momenta. (Hint: See section 6.2 of *Quantum Field Theory*, by Itzykson and Zuber.)

Ultraviolet divergences

Typical loop diagrams give rise to integrals of the form

$$I_n^m = \int \frac{d^4 k}{(2\pi)^4} \frac{k^{2m}}{(k^2 + \ell^2)^n}$$

where k is the loop momentum and ℓ may be some function of the other momenta and the masses. When $2m + 4 \geq 2n$, then the integral diverges.

This can be regularized by putting an UV cutoff, Λ.

$$I_n^m = \frac{\Omega_4}{(2\pi)^4} \int_0^\Lambda \frac{k^{2m+3} dk}{(k^2 + \ell^2)^n} = \frac{\Omega_4}{(2\pi)^4} \ \ell^{2(m-n)+4} F\left(\frac{\Lambda}{\ell}\right),$$

where Ω_4 is the result of doing the angular integration. The cutoff makes this a completely regular integral. As a result, the last part of the answer can be obtained entirely by dimensional analysis.

What can we say about the limit $\Lambda \to \infty$?

Dimensional regularization

The UV divergences we are worried about can be cured if $D < 4$. So, instead of the four-dimensional integral, try to perform an integral in $4+\delta$ dimensions, and then take the limit $\delta \to 0^-$. Since everything is to be defined by analytic continuation, we will not worry about the sign of δ until the end.

The integrals of interest are

$$I_n^m = \int \frac{d^4k}{(2\pi)^4}\,\frac{k^{2m}}{(k^2+\ell^2)^n} \to \int \frac{d^{4+\delta}k}{\mu^\delta(2\pi)^{4+\delta}}\,\frac{(k_\delta^2+k^2)^m}{(k_\delta^2+k^2+\ell^2)^n},$$

where we have introduced an arbitrary mass scale, μ, in the second form of the integral in order to keep the dimension of I_n unchanged. Also, the square of the $4+\delta$ dimensional momentum, k, has been decomposed into its four dimensional part, k^2, and the remainder, k_δ^2.

Doing the integral in one step

Usually one does the integral in $4+\delta$ dimensions in one step:

$$I_n^m = \mu^{4-D}\int \frac{d^Dk}{(2\pi)^D}\,\frac{k^{2m}}{(k^2+\ell^2)^n}$$
$$= \ell^{2m+4-2n}\left(\frac{\ell}{\mu}\right)^{D-4}\,\frac{\Omega_D}{(2\pi)^D}\,\frac{\Gamma(m+D/2)\Gamma(n-m-D/2)}{2\Gamma(n)},$$

where $\Omega_D = \Gamma(D/2)/(2\pi)^{D/2}$ is the volume of an unit sphere in D dimensions.

For $m=0$ and $n=1$, setting $D=4-2\epsilon$, the ϵ-dependent terms become

$$\left(\frac{\ell^2}{4\pi\mu^2}\right)^{-\epsilon}\Gamma(-1+\epsilon) = -\frac{1}{\epsilon}+\gamma-1+\log\left[\frac{\ell^2}{4\pi\mu^2}\right]+\mathcal{O}(\epsilon),$$

where γ is the Euler-Mascheroni constant.

Doing the integral in two steps

One can do this integral in two steps, as indicated by the decomposition given below

$$I_n^0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(2\pi\mu)^\delta} \int \frac{d^\delta k}{(k_\delta^2 + k^2 + \ell^2)^n},$$

Simply by power counting, one knows that the internal integral should be a k-independent multiple of $(k^2+\ell^2)^{-n+\delta/2}$. In fact, this is most easily taken care of by the transformation of variables $k_\delta^2 = (k^2+\ell^2)x^2$. This gives

$$\int \frac{d^\delta k/(2\pi\mu)^\delta}{(k_\delta^2 + k^2 + \ell^2)^n} = \frac{1}{(k^2+\ell^2)^n} \left(\frac{k^2+\ell^2}{2\pi\mu}\right)^\delta \Omega_\delta \int \frac{x^{\delta-1}dx}{(1+x^2)^n}$$

where Ω_δ is the angular integral in δ dimensions. The last two factors do not depend on k, the first factor reproduces I_n, so the regularization must be due to the second factor.

Doing the integral in two steps

One can do this integral in two steps, as indicated by the decomposition given below

$$I_n^0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(2\pi\mu)^\delta} \int \frac{d^\delta k}{(k_\delta^2 + k^2 + \ell^2)^n},$$

Simply by power counting, one knows that the internal integral should be a k-independent multiple of $(k^2+\ell^2)^{-n+\delta/2}$. In fact, this is most easily taken care of by the transformation of variables $k_\delta^2 = (k^2+\ell^2)x^2$. This gives

$$\int \frac{d^\delta k/(2\pi\mu)^\delta}{(k_\delta^2 + k^2 + \ell^2)^n} = \frac{1}{(k^2+\ell^2)^n} \left(\frac{k^2+\ell^2}{2\pi\mu}\right)^\delta \Omega_\delta \int \frac{x^{\delta-1}dx}{(1+x^2)^n}$$

where Ω_δ is the angular integral in δ dimensions. The last two factors do not depend on k, the first factor reproduces I_n, so the regularization must be due to the second factor.

Recognizing the regularization

The regulation becomes transparent by writing

$$\left(\frac{k^2+\ell^2}{2\pi\mu}\right)^\delta = \exp\left[\delta\log\left(\frac{k^2+\ell^2}{2\pi\mu}\right)\right].$$

For fixed μ, the logarithm goes to a constant when $k \to 0$. Also, the logarithm goes to ∞ when $k \to \infty$. As a result, the factor goes to zero provided $\delta < 0$. This is exactly the intuition we started from.

In the context of dimensional regularization, the quantity μ is called the renormalization scale. We have seen that it gives an ultraviolet cutoff. The important thing is that the scale μ is completely arbitrary, and has nothing to do with the range of applicability of the QFT.

Review problems: understanding the old renormalization

Problem 2.2: Self-study

Study the proof of renormalizability of QED to see how one identifies all the divergences which appear at fixed-loop orders, and how it is shown that taking care of a fixed number of divergences (through counter-terms) is sufficient to render the perturbation theory finite. The curing of the divergence requires fitting a small set of parameters in the theory to experimental data (a choice of which data is to be fitted is called a renormalization scheme). As a result, the content of a QFT is to use some experimental data to predict others.

Problem 2.3

Follow the above steps in a 4-Fermi theory and find a 4-loop diagram which cannot be regularized using the counter-terms shown in $\mathcal{L}_c$. Would your arguments also go through for a scalar ϕ^4 theory? Unrenormalizable theories require infinite amount of input data.

Outline

The old renormalization

In 1929, Heisenberg and Pauli wrote down a general formulation for QFT and noted the problem of infinities in using perturbation theory. After 1947 the problem was considered solved. The general outline of the method is the following:

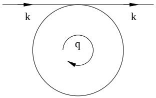
- Analyze perturbation theory for the loop integrals which have ultraviolet divergences.
- Regulate these divergences by putting an ultraviolet cutoff in some consistent way.
- Identify the independent sources of divergences, and add to the Lagrangian counter-terms which precisely cancel these divergences.
- QFTs are called renormalizable if there are a finite number of counter-terms needed to render perturbation theory useful.
- Use only renormalizable Lagrangians as models for physical phenomena.

Unrenormalizable terms

In this view, the unrenormalizable Lagrangian

$$\mathcal{L}_{\rm int} = -\lambda(\overline{\psi}\psi)^2,$$

was deemed impossible as a model for physical phenomena, since it needs an infinite number of counter-terms.



Examine its contribution to the fermion mass:

$$im\lambda \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2} \propto \lambda m \Lambda^2,$$

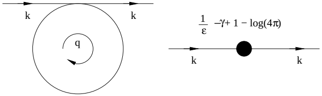
where the integral is regulated by cutting it off at the scale Λ. At higher loop orders the dependence on Λ would be even stronger. In the modern view, this analysis is mistaken because it confuses two different things.

Unrenormalizable terms

In this view, the unrenormalizable Lagrangian

$$\mathcal{L}_{\rm int} = -\lambda(\overline{\psi}\psi)^2,$$

was deemed impossible as a model for physical phenomena, since it needs an infinite number of counter-terms.



Examine its contribution to the fermion mass:

$$im\lambda\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m^2}\propto\lambda m\Lambda^2,$$

where the integral is regulated by cutting it off at the scale Λ. At higher loop orders the dependence on Λ would be even stronger. In the modern view, this analysis is mistaken because it confuses two different things.

Irrelevant terms

Today the same Lagrangian is written as

$$\mathcal{L}_{int} = -\frac{c_6}{\Lambda^2}(\overline{\psi}\psi)^2,$$

where Λ is interpreted as a scale below which one should apply the theory.
The contribution to the mass is

$$\frac{imc_6}{\Lambda^2}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m^2}=\frac{c_6 m^3}{16\pi^2\Lambda^2}\left(-\frac{1}{\epsilon}+\gamma-1+\log\left[\frac{m^2}{4\pi\mu^2}\right]\right),$$

where the integral is regulated by doing it in $4-2\epsilon$ dimensions. In the $\overline{\rm MS}$ renormalization scheme the counter-term subtracts the pole and the finite parts $\gamma-1-\log 4\pi$, leaving

$$\frac{\delta m}{m}=\frac{c_6}{16\pi^2}\left(\frac{m}{\Lambda}\right)^2\log\left[\frac{m^2}{\mu^2}\right].$$

Separation of scales

The cutoff scale in the problem, Λ, is dissociated from the renormalization scale, μ, in dimensional regularization. This is not true in cutoff regularization. This separation of scales allows us to recognize two things:

- There is no divergence in the limit $\Lambda \to \infty$; instead the coupling becomes irrelevant. The theory remains predictive, because the effect of these terms is bounded.
- There are no large logarithms such as $\log(m/\Lambda)$. The amplitudes, computed to all orders are independent of μ, although fixed loop orders are not. In practical fixed loop-order computations, it is possible to choose $\mu \simeq m$, and reduce the dependence on this spurious scale.

Regularization schemes which do this are called mass-independent regularization. They are a crucial technical step in the new Wilsonian way of thinking about renormalization.

Is cutoff regularization wrong?

All regularizations must give the same results when the perturbation theory is done to all orders. Cutoff regularization is just more cumbersome.

Cutoff regularization retains all the problems of the old view: since the cutoff and renormalization scales are not separated, higher dimensional counter-terms are needed to cancel the worsening divergences at higher loop orders. When all is computed and cancelled, the m^2/Λ^2 and $\log(m/\Lambda)$ emerge.

In mass-independent regularization schemes, higher dimensional terms give smaller corrections because of larger powers of m/Λ.

In a renormalizable theory, since the number of counter-terms is finite and small, the equivalence of different regularizations is easier to see.

Outline

Outline

Spurious divergences in Quantum Field Theory

Wilsonian Effective Field Theories

Wilsonian renormalization

- The renormalization group
- The Wilsonian point of view
- RG for an Euclidean field theory in $D = 0$
- Defining QFT without perturbation theory

End matter

The physical content of renormalization

Wilson fixed his attention on the quantum field theory which emerges as the cutoff Λ is pushed to infinity while the low-energy physics is held fixed. According to him, one should define a renormalization group (RG) transformation as the following—

1. Integrate the momenta over $[\zeta, \zeta\Lambda]$, and perform a wave-function renormalization by scaling the field to the same range as the original fields. This changes $\Lambda \to \zeta\Lambda$.
2. Find the Hamiltonian of the coarse grained field which reproduces the dynamics of the original system. The couplings in the Hamiltonians "flow" $g(\Lambda) \to g(\zeta\Lambda)$. This flow defines the Callan-Symanzik beta-function

$$\beta(g) = \frac{\partial g}{\partial \zeta}.$$

A fixed point of the RG has $\beta(g) = 0$.

Linearized Renormalization Group transformation

Assume that there are multiple couplings G_i with beta-functions B_i. At the fixed point the values are G_i^*. Define $g_i = G_i - G_i^*$. Then,

$$\beta_i(G_1, G_2, \cdots) = \sum_j B_{ij} g_j + \mathcal{O}(g^2).$$

Diagonalize the matrix B whose elements are B_{ij}. In cases of interest the eigenvalues, y, turn out to be real. Under an RG transformation by a scaling factor ζ an eigenvector of B scales as $v \to \zeta^y v$

Eigenvectors corresponding to negative eigenvalues scale away to zero under RG, and so correspond to super-renormalizable couplings. We have already set up the correspondence of these with relevant couplings. For positive eigenvalues, we find un-renormalizable couplings, *i.e.*, irrelevant couplings. Those with zero eigenvalues are the marginal operators.

Understanding the beta-function

We examine the β-function in a model field theory with a single coupling g. If the β-function is computed in perturbation theory then we know its behaviour only near $g = 0$. But imagine that we know it at all g.

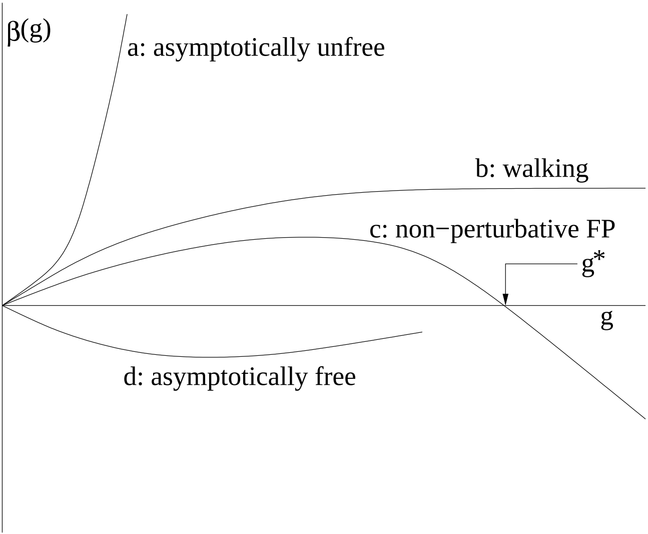
The solution of the Callan-Symanzik equation gives us a running coupling, obtained by inverting the equation

$$\zeta = \int_0^{g(\zeta)} \frac{dg}{\beta(g)}.$$

This happens since the coupling which gives a fixed physics can change as we change the cutoff scale.

Since larger ζ means that we can examine larger momenta, the behaviour of $g(\zeta)$ at large ζ tells us about high-energy scattering.

The behaviour of model field theories



Enumerating the cases

1. Asymptotically unfree: if $\beta(g)$ grows sufficiently fast, then the integral converges. This means that the upper limit of the integral can be pushed to infinity with ζ finite. This happens with the one-loop expression for QED and scalar theory.
2. Walking theories: if $\beta(g)$ grows slowly enough, then the integral does not converge. As a result, $g(\zeta)$ grows very slowly as $\zeta \to \infty$.
3. Non-perturbative fixed point: there is a new fixed point at g^*. The scaling dimensions of the fields may be very different here.
4. Asymptotic freedom: if $\beta(g) < 0$ near $g = 0$, then, the coupling comes closer to $g = 0$ as $\zeta \to \infty$. There is no special significance to $\beta(g)$ changing sign at some g^*, except that it means that for all couplings below g^*, the renormalized theory is asymptotically free.

Wilson's change of perspective

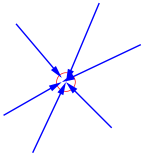
In a QFT we want to compute amplitudes with bounded errors, and to systematically improve the error bounds, if required. With just a small change in the point of view, Wilsonian renormalization gives a new non-perturbative computing technique.

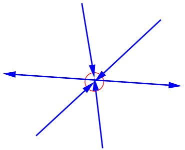
If we need amplitudes at a low momentum scale, then we can use the RG to systematically lower the cutoff scale, by integrating over the range $[\Lambda/\zeta, \Lambda]$. This corresponds to coarse graining the fields and examining the long-distance behaviour of the theory. Now the couplings follow the changed equation

$$\frac{\partial g}{\partial \zeta} = -\beta(g).$$

Asymptotically unfree theories may be perturbative at long distances; while asymptotically free theories may become highly non-perturbative if the corresponding beta-function crosses zero at some $g^* \equiv 0$.

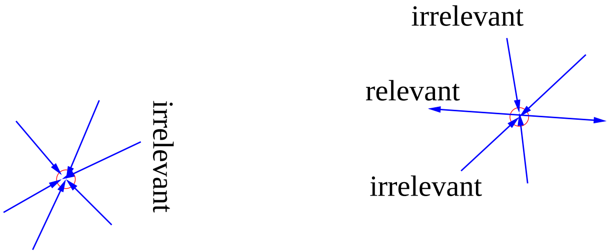
Renormalization Group trajectories





$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories

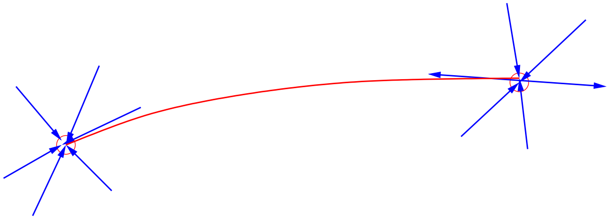


$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories

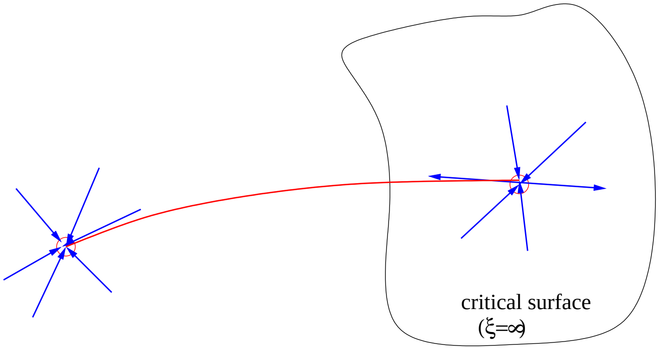
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



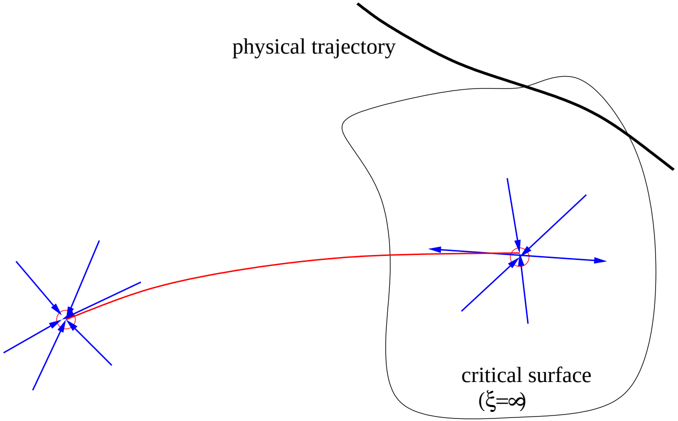
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



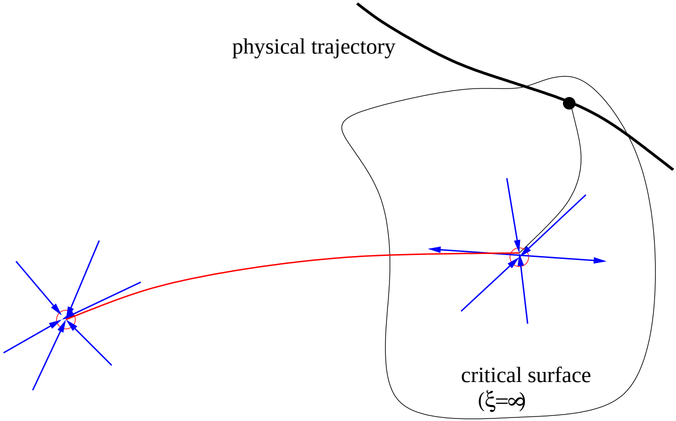
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



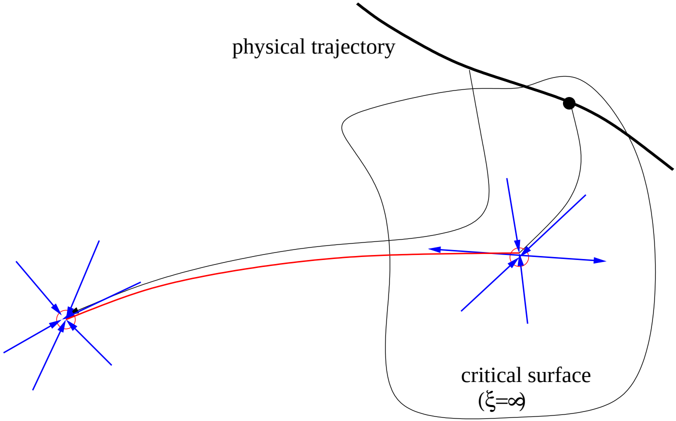
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



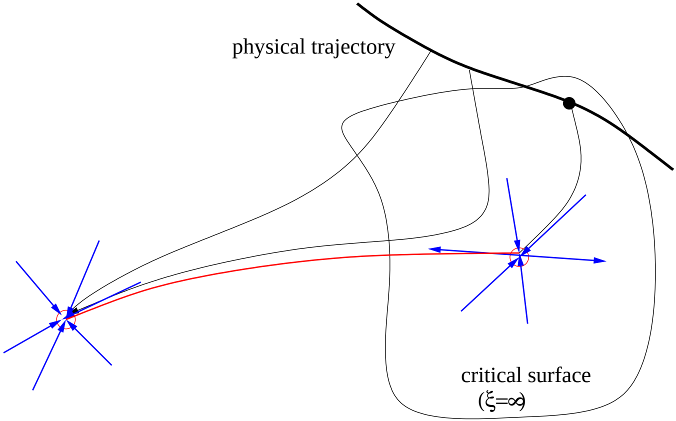
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



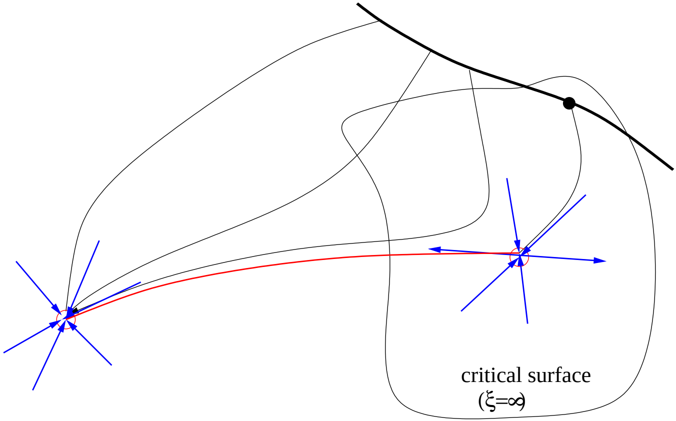
$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Renormalization Group trajectories



$\xi = \Lambda/m$. Fixed points: $\xi = 0$ (stable) or $\xi = \infty$ (unstable).

Probability theory as a trivial field theory

Consider a random variate x with a probability density $P(x)$. In the commonest applications x is real. One needs to compute

$$\langle f \rangle = \int_{-\infty}^{\infty} dx\, f(x) P(x), \quad \text{where} \quad \langle 1 \rangle = 1.$$

Since $P(x) \geq 0$, one finds $S(x) = -\ln P(x)$ is real.
Define the characteristic function, $Z[j]$ and cumulants

$$Z(j) = \int_{-\infty}^{\infty} dx \mathrm{e}^{-S(x)-jx}, \quad \text{and} \quad [x^n] = \left. \frac{\partial^n F(j)}{\partial j^n} \right|_{j=0},$$

where the generating function: $F[j] = -\log Z[j]$. Note the analogy of $S(x)$ with the action of a zero dimensional field theory, of $Z(j)$ with the path integral and $F(j)$ with the generating function for the correlators. The cumulants, $[x^n]$, and are just connected parts of n-point functions of the field x. The connection between the cumulants, $[x^n]$ and the moments, $\langle x^n \rangle$, is left as an exercise in `Mathematica`.

Setting up the RG

Now suppose we take m of the random variates and average them, then what are the cumulants of the distribution of

$$y_m = \frac{1}{m}\sum_{i=1}^{m} x_i?$$

This is an RG. The sum over many random variates corresponds to taking low-frequency modes of quantum fields, and m corresponds to Λ.

Clearly,

$$Z_m(j) = \int \left\{\prod_{i=1}^{m} dx_i P(x_i)\right\} \mathrm{e}^{-jy}\delta\left(y - \frac{1}{m}\sum_{i=1}^{m} x_i\right) = \left[Z\left(\frac{j}{m}\right)\right]^m.$$

So the RG gives us $F_m(j) = mF(j/m)$.

The central limit theorem

Since the cumulants are Taylor coefficients of the generating function, one has

$$F(j) = \sum_{n=1}^{\infty} [x^n] \frac{j^n}{n!},$$

and similarly for $F_m(j)$. Then comparing the coefficients of j^n gives the RG flow

$$[y^n] = \frac{1}{m^{n-1}} [x^n].$$

This procedure corresponds to matching the "low-momentum" correlation function.

The mean is unchanged by the RG, and the variance scales as $1/m$. All the higher cumulants scale by successively higher powers of m, and can be neglected if m is large enough. The RG flow proves the central limit theorem: the fixed point of probability distributions under RG is the Gaussian distribution.

Perturbation theory is insufficient

- The β-function of QED, obtained at 1-loop order, is positive and grows so fast that the running coupling becomes infinite at finite energy: this is called the Landau pole. As a result, QED does not work at high energy.
- The 1-loop effective action for non-Abelian gauge fields is minimized at a finite constant field strength [Savvidy: 1977]. In such a background, the gauge fields have an instability [Nielsen, Olesen: 1978]. So a perturbative expansion around this does not work.
- There are arguments which lead us to believe that the Euclidean path integral of a non-Abelian gauge theory is not dominated by a minimum of the classical action [Pagels, Tomboulis: 1978]. As a result a perturbative expansion around the quantum ground state cannot work.

What is quantum field theory?

The quantum theory of fixed number of particles can be solved in many different ways. Perturbation theory is only one of these.

The older view of renormalization tied the definition of a quantum field theory completely to the perturbation expansion. But since perturbation theory is insufficient, it became necessary to develop a definition, *i.e.*, a computational method, for quantum field theory independent of the perturbation expansion.

The Wilsonian view of renormalization yields a new way of defining computational techniques for quantum field theory: the method of effective field theory. These can be treated in perturbation theory (as in this course). Or one can treat it exactly by creating a Wilson flow in the space of Lagrangians, as in lattice field theory.

A space-time lattice

If a Green's function has an UV divergence, then that means that the product of field operators separated by short distances diverges. An UV cutoff means that the shortest distances are not allowed.

A simple way to implement this is to put fields on a space-time lattice. If the lattice spacing is a, then this corresponds to an UV cutoff, $\Lambda \simeq 1/a$. Derivative operators are simple:

$$\partial_\mu \phi(x) = \frac{1}{a} \left[\phi(x + a\hat{\mu}) - \phi(x) \right].$$

The discretization of the derivative operator is not unique; there are others which differ by higher powers of a. This means that the difference between different definitions of the derivative are irrelevant operators.

The reciprocal lattice: momenta

Making a lattice in space-time means putting an upper bound to the momenta. It is also possible to make an infrared (IR) cutoff by putting the field theory in a finite box. If the box size is $L = Na$, and one puts periodic boundary conditions, then only the momenta $2\pi n/(Na)$ are allowed. The spacing between allowed momenta is $2\pi/(Na)$, the lowest momentum possible is 0, and the highest possible momentum is $2\pi(N-1)/(Na)$. This range is called the Brillouin zone.

Fourier transforms of fields become discrete Fourier series, and momentum integrals become computable sums.

Problem 2.4

Explicitly construct the Fourier transforms of scalar and Dirac fields with periodic and anti-periodic boundary conditions on a hypercubic lattice in 4-dimensions of size N^4.

Pure Higgs theory

Take the scalar field theory in Euclidean space-time:

$$S = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \lambda\phi^4\right], \qquad (m, \lambda > 0),$$

and put it on a space-time lattice. The discretisation of the derivative in the kinetic term gives products of fields at neighbouring lattice sites. Everything else becomes an on-site interaction of the fields. If we take $\lambda \to \infty$, then the fields are pinned to the minimum of the potential. We can render the fields dimensionless using the lattice spacing a, and scale the field value at the minimum of the potential to ± 1. Then the scalar field theory reduces to

$$S = \sum_{x,\mu} s_x s_{x+\hat{\mu}}, \qquad (s_j = \pm 1),$$

which is the Ising model. (Problem 2.5: Complete this construction.)

Outline

Keywords and References

Keywords

Loop integrals; ultraviolet cutoff scale; cutoff regularization; large logarithms; dimensional regularization; mass-independent regularization; counter-terms; renormalization scheme; renormalization scale; *msbar* renormalization scheme; un-renormalizable theory; renormalizable Lagrangians; super-renormalizable couplings; Chiral Ward identities; wave-function renormalization; Callan-Symanzik beta-function; fixed point; running coupling; coarse-graining; central limit theorem; Landau pole; lattice field theory; Ising model.

References

David B. Kaplan, *Effective Field Theories*, arxiv: nucl-th/9506035;
Aneesh Manohar, *Effective Field Theories*, arxiv: hep-ph/9606222;
S. Weinberg, *The Quantum Theory of Fields* Vol II.

Copyright statement