Effective Field Theory of Dark Matter Detection

Sourendu Gupta

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Effective Field Theories
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Outline

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Units

Since we believe that classical general relativity is the proper theory for cosmology, its natural units should use $[c] = [G] = 1$, which means $[M] = [L] = [T]$. So the natural length scale associated with any mass is of the order of its Schwartzschild radius. Since a parsec (pc) is the normal length unit used in astrophysics and cosmology, we have

$$\hbar = 0.2743 \times 10^{-102}\ \mathrm{pc}^2, \qquad 1\ \mathrm{yr} = 0.3066\ \mathrm{pc},$$
$$M_s = 47.85 \times 10^{-15}\ \mathrm{pc}, \qquad H_0 = 0.224(4)\ \mathrm{Gpc}^{-1}.$$

Problem 4.1

There is a strong mismatch of the actual length scales of galaxies and galaxy clusters with their natural length scales. Is this a fine-tuning problem?

Typical densities

The universe is observed to be nearly flat. The density required for this is called the critical density

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G} = 8.855 \times 10^{-3}\ {\rm Gpc}^{-2}.$$

Recent observations yield

$$\Omega_B = \frac{\rho_B}{\rho_{\rm crit}} = 0.0499(22), \quad \text{and} \quad \Omega_{\rm CDM} = \frac{\rho_{\rm CDM}}{\rho_{\rm crit}} = 0.265(11).$$

The number density of photons in the current epoch is

$$n_\gamma = 12.066 \times 10^{57}\ {\rm pc}^{-3}.$$

Using the baryon mass and ρ_B, we find

$$n_B = 7.30(8) \times 10^{48}\ {\rm pc}^{-3}, \quad \text{so} \quad \eta = \frac{n_B}{n_\gamma} = 6.05(7) \times 10^{-10}.$$

Some problems

Problem 4.2

Assume that $\rho_{\rm CDM}$ is dominated by one kind of particle, just like ρ_B. Is it natural for the ratio of the masses of these two kinds of particles to be approximately the same as the ratio of their densities? Your arguments should be able to give further properties of the CDM particle, χ, using dimensional analysis and minor assumptions. Try to extract as much physics as possible from your arguments.

Problem 4.3

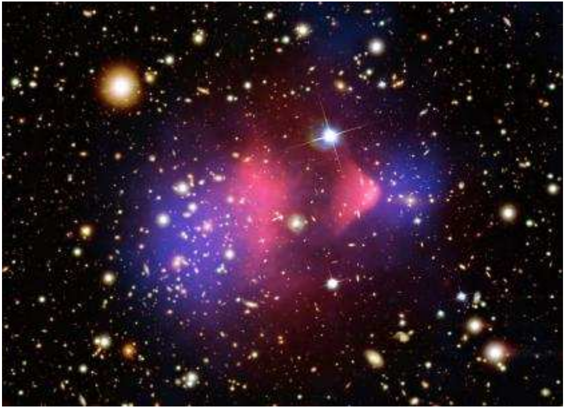
The dimensional analysis presented till now gives a good account of the largest scale structures in the universe in the sense that once the Hubble scale, H_0, is known, everything else becomes natural in these units. Test whether the same dimensional analysis also works at the scale of galactic clusters and galaxies. If not, why not?

Evidence for dark matter

The existence of dark matter is inferred from its gravitational influence on visible matter. The main observations are:

1. The angular velocity of stars in a galaxy around its center is often independent of the distance from the center (flat rotation curves), even out to the very large distances. This can happen only if there is more mass than visible.
2. Velocity distributions of galaxies in clusters also imply more mass than is visible.
3. In the merger of two clusters of galaxies, the distribution of observable stars and gas is mapped. Then by observing lensing due to this system, the gravitational potential can also be mapped. The two are not the same. [arxiv:0608407]
4. CMB anisotropies imply $\Omega_B h^2 = 0.02207(27)$ and $\Omega_{CDM} h^2 = 0.1198(26)$, where scale factor for Hubble expansion is $h = 0.673(12)$.

NASA image for the Bullet Cluster



Hot gas detected by Chandra in X-rays is seen as two pink clumps in the image and contains most of the "normal," or baryonic, matter in the two clusters. The blue areas in this image show where astronomers find most of the mass in the clusters; determined using the effect of gravitational lensing. Most of the matter in the clusters (blue) is clearly separate from the normal matter (pink).

Physical properties of dark matter

There are very few constraints on the properties of dark matter. These arise from indirect inferences:

1. Since Ω_B inferred from CMB anisotropies is consistent with that from nucleosynthesis models, this implies that $\Omega_{\rm CDM}$ is not baryonic.
2. If dark matter interacted strongly enough to clump together, then there would have been some clumps closer to the center of the galaxy than the sun. The absence of lensing events due to such clumps then implies that dark matter interacts weakly. This inference is subject to many caveats.
3. Some inferences on the strength of interactions of dark matter come from gravitational N-body simulations of the large scale structure of the universe. However, since these are subject to many errors, the inferences are subject to large uncertainties.

Direct experiment seems to be the only way of determining the properties of dark matter.

Dark matter around the sun

Since the sun lies within the disk of the Milky Way, it is hard to measure properties of our own galaxy. But this is the only known way to estimate the local density and velocity of dark matter. As a result, these properties are known with great imprecision. The standard values adopted are

$$\begin{aligned} \rho_\chi &= 0.3\mathrm{GeV/cm}^3 = 2.3\ \mathrm{eV}^4 = 42.7 \times 10^3 \rho_{\mathrm{crit}} \\ \overline{v}_\chi &= 270\mathrm{Km/s} = 0.9 \times 10^{-3}. \end{aligned}$$

The uncertainty in ρ_χ ranges from 0.05–1 GeV/cm^{-3}, with a corresponding uncertainty in $\overline{v}$.

In typical collisions of dark matter with nuclei, $S \simeq 100$ GeV^2, if $M_\chi \simeq 10$ GeV. Weak interactions would then give $\sigma \simeq G_W^2 S \simeq 10^{-8}/\mathrm{GeV}^2$, *i.e.*, about 10^{-12} barns or 10^{-40} cm^2.

Why not $\sigma_{N\chi} = 0$?

If the only interaction of normal matter and dark matter was gravitational, then dark matter produced in the early universe would not annihilate. The relic density, ρ_χ then would be just the primordial density, ρ_χ^0, diluted by Hubble expansion

$$\rho_\chi = \rho_\chi^0 \exp\left[-\frac{1}{3}\int \frac{dt}{H(t)}\right],$$

where $H(t)$ is the Hubble expansion rate at epoch t.

The primordial density would be governed by the rate of production of χ by gravitational processes: for example, graviton fusion to give χs. This process would naturally generate $\rho_\chi^0 \simeq M_{\rm Pl}^{-2} \simeq 1/\hbar$ around the inflationary epoch. Even assuming 60 e-fold dilution since then, the relic density of χ would overclose the universe by a factor of nearly 10^{100}. So dark matter must have been in chemical equilibrium with normal matter at early times, implying $\sigma_{N\chi} > 0$.

A research problem

Problem 4.4

Is it possible to refine the argument of the previous transparency and give a lower bound to $\sigma_{\chi N}$ such that $\Omega_{\rm CDM} \simeq \Omega_{\rm crit}$? The argument should utilize the fact that $\sigma_{\chi N}$ depends on the temperature. Why does this bound go by the name of WIMP miracle?

Is it possible to tweak the same argument to give an upper bound to the cross section using the argument that $\Omega_B < \Omega_{\rm CDM}$?

If one were to try to frame these arguments within the an effective field theory, what would be the cutoff Λ for such a theory?

Search through elastic collisions

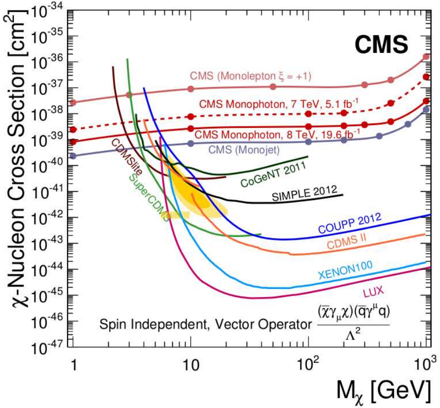
Laboratory searches for CDM assume that there is mainly one type of dark matter particle, and it scatters elastically, *i.e.*, the dark matter particle does not change. Most often experiments also look for elastic scattering of a nucleus against χ, *i.e.*,

$$\chi + N \to \chi + N.$$

The signal is a nucleus recoiling against nothing.

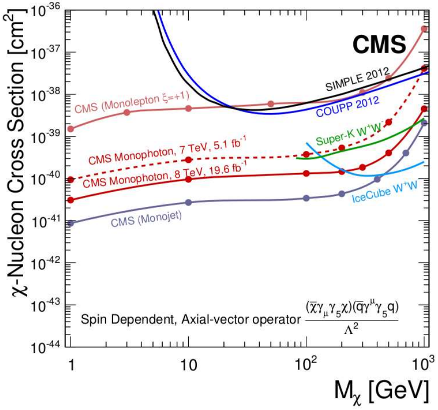
It would also be possible to look in channels where the scattered nucleus is excited, so that it emits a photon and relaxes back to the ground state. However, then there is a natural background due to radioactivity which must be subtracted. Similarly, one can look for pair produced electrons with a recoil nucleus; again there are backgrounds to be subtracted.

Cross section bounds from laboratory searches



Search for elastic collisions between dark matter and nuclei.
LUX arxiv:1310.8214; CMS arxiv:1410.8812

Cross section bounds from laboratory searches



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Outline

Why an effective field theory?

Since dark matter must interact with normal matter, there must be an unified theory at sufficiently high scale. Guesses about such theories may be constrained by collider data, but not heavily constrained.

The details of this high-scale theory should not matter in low-energy collisions. As a result, an EFT is the best possible tool to parametrize the interactions. This parametrization then constrains the UV completion, which is the high-scale theory.

If sufficiently general sets of terms are allowed to appear in the EFT then the symmetry structure of the UV completion can be constrained from low-energy data. However, this is a complicated issue, which will be taken up in more detail later.

Kinematics

Since the dark matter particle has very small velocity, the kinematics is non-relativistic (NR). Assume the mass of the nucleus is $M \simeq 100$ GeV, its initial 3-momentum is $p = MV$, and the dark matter particle has mass M_χ, and initial 3-momentum $p' = M_\chi V'$. Galilean invariance implies that physics depends only on invariants built from

$$q = p - p', \quad \text{and} \quad v = V - V'.$$

Additionally, there may be terms involving the spin S of the nucleus and the the spin S_χ of χ (if it has spin).

In the CM frame expressions involve the reduced mass $\mu = MM_\chi/(M + M_\chi)$. The initial and final energies are $E_{in} = \mu v^2/2$ and $E_{fin} = \mu(v + q/\mu)^2/2$. Equating them, we have

$$v \cdot q = -\frac{q^2}{2\mu}.$$

Listing basic operators

In NRQM, Hermitean conjugation exchanges incoming and outgoing particles, so q is anti-Hermitean. It will be more convenient to work with the Hermitean quantity iq. Under Hermitean conjugation, $v \to v + q/\mu$. So,

$$u = v + \frac{q}{2\mu}, \qquad \text{with} \qquad u \cdot q = 0,$$

is Hermitean, as are S and S_χ.

Since this non-relativistic theory comes from a Lorentz invariant action, CP symmetry must be equal to invariance under T. The action under P is also important. It is easy to see the transformation properties

$$\begin{aligned} T: &\qquad S \to -S, \quad S_\chi \to -S_\chi, \quad u \to -u, \quad iq \to iq \\ P: &\qquad S \to S, \quad S_\chi \to S_\chi, \quad u \to -u, \quad iq \to -iq. \end{aligned}$$

Field operators and the kinetic term

The NR fields involve only creation operators

$$\chi(x) = \int \frac{d^3k}{(2\pi)^3} \, \mathrm{e}^{-ik\cdot x} a^\dagger(k),$$

with an UV momentum cutoff Λ. If the maximum observed recoil energy E_m, then $\Lambda > \sqrt{2\mu E_m} \simeq 200$ MeV, since $\beta \simeq 10^{-3}$.

With this normalization, the NR kinetic term is

$$\mathcal{L}_1 = \chi^\dagger(x) \left[i\partial_t - \frac{1}{2M_\chi} \nabla^2 \right] \chi(x),$$

and a similar kinetic term can be written for nuclei, whose field operator we denote by $N(x)$.

It is also possible to have a normalization for χ which has an extra factor of $1/\sqrt{2M_\chi}$. In this normalization, the kinetic term has to be multiplied by $2M_\chi$.

Power counting

When we did power counting of fields in the relativistic case, we used the fact that momentum and energy have the same dimensions. Since this is no longer true, we must establish new power counting rules in the NR field theory.

In the NR theory we would like to perform an RG transformation which takes $q \to \zeta q$. Since $M_\chi \gg E_m$, this mass will not scale at all. As a result, we will have

$$[q] = 1, \quad [M_\chi] = 0, \quad [v] = 1, \quad [S_\chi] = 0, \quad [i\partial_t] = [E] = 2.$$

The last power tells us that $[t] = -2$. Since the action is dimensionless, $[\mathcal{L}] = 5$ in $3 + 1$ dimensions. This implies

$$[\chi] = \frac{3}{2}.$$

This scaling of the field variable is independent of whether it is a fermion or boson.

Interaction terms for elastic scattering

All the interaction terms will be of the kind

$$\mathcal{L}_{\rm int} = \chi^\dagger(x)\mathcal{O}\chi(x)\, N^\dagger(x)\mathcal{O}' N(x),$$

where the operators are constrained by symmetries. Also, since q^2 is completely invariant under symmetries, $\mathcal{O}$ and $q^2\mathcal{O}$ have the same symmetries, so we can collect together all terms of the kind

$$f_0\mathcal{O} + f_1 q^2\mathcal{O} + f_2 q^4\mathcal{O} + \cdots = F\left(\frac{q^2}{\Lambda^2}\right)\mathcal{O},$$

where F is a form factor. We will count the dimensions of F later.

The energy scale associated with the scattering is given by

$$\omega_q = \frac{q^2}{2\mu} \simeq 0.2\ \ {\rm MeV},$$

which is usually smaller than the excitation energy of nuclei.

Listing terms by symmetry

T-even, P-even

S_χ independent	$\mathcal{O}_1 = 1, \quad \mathcal{O}_2 = u^2, \quad \mathcal{O}_3 = S \cdot (iq \times u)$
S_χ dependent	$\mathcal{O}_4 = S \cdot S_\chi, \quad \mathcal{O}_5 = S_\chi \cdot (iq \times u),$
	$\mathcal{O}_6 = S_\chi \cdot q\, S \cdot q$

T-even, P-odd

S_χ independent	$\mathcal{O}_7 = S \cdot u$
S_χ dependent	$\mathcal{O}_8 = S_\chi \cdot u, \quad \mathcal{O}_9 = S_\chi \cdot (S \times iq)$

T-odd, P-odd

S_χ independent	$\mathcal{O}_{10} = S \cdot iq$
S_χ dependent	$\mathcal{O}_{11} = S_\chi \cdot iq$

All these terms arise out of relativistic spin 0 or spin 1 exchange. In addition, the following also arise: $\mathcal{O}_{10}\mathcal{O}_5$, $\mathcal{O}_{10}\mathcal{O}_8$, $\mathcal{O}_{11}\mathcal{O}_3$, $\mathcal{O}_{11}\mathcal{O}_7$. Powers of S or S_χ can be reduced using the algebra of angular momenta. Scalar dark matter has no S_χ dependent terms.

Cross sections

Problem 4.5

T-odd, P-even operators are not listed in the table in the previous slide. What are these operators? Is the list of operators otherwise complete?

Problem 4.6

Count the powers of the operators shown in the previous slide, and therefore the powers of Λ in their couplings. Construct Born level spin-independent elastic cross sections. Which are the most important operators (in the sense of power counting) to contribute to the spin-independent cross sections? Which contribute to spin-dependent elastic cross sections. What simplifications occur if CDM consists of scalar particles?

Outline

Keywords and References

Keywords

Classical general relativity; natural length scale; Schwartzschild radius; critical density; Hubble scale; scale of galactic clusters; flat rotation curves; Velocity distributions of galaxies in clusters; CMB anisotropies; local density and velocity of dark matter; relic density; primordial density; chemical equilibrium; WIMP miracle; Galilean invariance; power counting rules; spin-independent elastic cross sections; spin-dependent elastic cross sections.

References

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A. L. Fitzpatrick *et al.*, *The EFT of Dark Matter Direct Detection*, arxiv:1203.3542.

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