The physics of plasmas

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Outline

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Keywords and References

Keywords

plasma, electrical resistance, Ohm's law, resistivity, conductivity, ballistic transport, collisions, Rutherford scattering, plasma parameter, space charge, screening, Debye screening length, plasma oscillations, Langmuir waves, plasmon, Landau damping, quantum plasmas, diffractive scattering, relativistic plasmas, plasmas in quantum field theory.

Books

Statistical Plasma Physics, by S. Ichimaru, Ch 1.
Qualitative methods in QM, by A. B. Migdal, Ch 1.

A neutral conductor

Definition

A plasma is any material which is overall charge neutral but contains mobile charge.

Any charge neutral medium which conducts electricity is likely to be plasma. Tap water is the most common plasma: it conducts well enough that a wet hand is dangerous near an electrical mains. The cellular material of animals, plants and bacteria are plasmas. Most gas in galaxies are plasmas. All plasmas need not be fluid; metals are plasmas.

Sometimes the notion of a charged plasma is also introduced. These are of technological importance: for example beams of electrons which are used to produce x-rays. In this lecture we will confine ourselves to plasmas in the older sense: those which are electrically neutral.

Electrical conductivity

The electrical resistance of a material is defined to be the ratio of the current carried by the material to the applied potential difference: $R = V/J$. Since $[J] = M^{1/2}L^{3/2}T^{-2}$, and $[V] = M^{1/2}L^{1/2}T^{-1}$, we have $[R] = L^{-1}T$ in Gaussian units.
The resistance may depend on the geometry of the material, so an intensive quantity is required to characterize the material. This is the resistivity, $\rho = RA/L$, where A is the cross sectional area of the current carrying medium and L the length through which the current flows. So $[\rho] = T$. The conductivity of a material is the inverse of the resistivity: $\sigma = 1/\rho$, so $[\sigma] = T^{-1}$.
In SI $\{J\} = \mathrm{A}$, $\{V\} = \mathrm{kg\,m^2/(A\,s^3) = V}$
$\{R\} = \mathrm{kg\,m^2/(A^2 s^3) = V/A} = \Omega$, where the ohm ($\Omega$) is the derived SI unit of resistance. So $\{\rho\} = \Omega m$, $\{\sigma\} = S/m$, where Siemens (S=1/Ω) is the SI derived unit of conductivity.

A small exercise

Suppose we make a small modification of Millikan's experiment. Initially balance some charges between the plates of a capacitor, separated by a distance ℓ. Then suddenly change the voltage to a different value, V. The charges accelerate towards the plates. Taking the number of charges projected per unit area of a plate as n, one can neglect interactions between the charges if n is small enough. Then can we compute the conductivity of the charges?

Dimensional analysis involves $[n] = L^{-2}$, $[\ell] = L$, $[m] = M$, $[e] = M^{1/2}L^{3/2}T^{-1}$, $[V] = M^{1/2}L^{1/2}T^{-1}$, we find $[neV/m] = T^{-2}$. Then dimensionally one might expect σ^2 to be this expression. The appearance of V in the formula is unsatisfactory. From the remaining quantities, we find $[ne\ell^{1/2}/m^{1/2}] = T^{-1}$. This solution is unsatisfactory because the shape variable ℓ appears in the formula for σ.

Ohm's law is not due to ballistic transport

Why does dimensional analysis fail? Solve the problem to find why. The acceleration is constant: $a = eE/m = eV/(m\ell)$. The charge travels a distance d in time $\sqrt{2dm/(eE)}$, and then its velocity is $\sqrt{2eEd/m}$. So there is a pulse of particles passing any surface, and not a continuous current of particles. So ballistic motion of charges is not compatible with our observations of materials. This is the reason why dimensional analysis failed.

Is it enough to introduce collisions between particles? If the number of collisions is large enough, then perhaps the velocities can be fully randomized by collisions. If the mean drift velocity is v, and the mean free path is λ, then $\rho \propto \lambda/v$. A question which would arise then is whether this is independent of V. The theory of transport is open to interesting dimensional analysis.

The mythical one-component plasma

We assume that the mobile charges are in thermal equilibrium when no external field is present. The temperature, Θ, then sets the scale of kinetic energy of the charges. Assume that the charge of each particle is e, and their number density is n. The average distance between charges is $r_0 = 1/\sqrt[3]{n}$.
The scattering cross section between charges must involve e^2. Since $[e^2] = ML^3T^{-2}$, dimensional considerations dictate that the Rutherford scattering cross section is

$$\sigma \simeq \frac{e^4}{\Theta^2}.$$

The Rutherford scattering formula, σ_R has another factor for the angular dependence, which is divergent for small angle scattering: $1/\psi^4$. Small angle scattering happens when the charges approach each other with large impact parameter. Then should other charges in the plasma be taken into account?

The plasma parameter

In fact, the microscopic variables in the plasma can be used to create a dimensionless variable, called the plasma parameter,

$$\Lambda = \frac{e^2}{\sqrt[3]{n}\Theta} = \frac{e^2}{r_0\Theta}.$$

This is a comparison between the average electrostatic potential energy between two charges in the plasma, e^2/r_0, and the kinetic energy, Θ.

If $\Lambda \ll 1$, then one says that the plasma is weakly coupled. On the other hand, if $\Lambda \simeq 1$ or greater, then the plasma is said to be strongly coupled. $1/\sqrt{\Lambda}$ is the smallest angle that can be taken into account through two-body scattering in a plasma.

The length scale, $r_D = r_0/\Lambda$ is called the Debye screening length. At distances of approach below this, the Coulomb force between two particles in a plasma is not strongly modified.

Space charge formation and screening

The number of charge carriers in a volume of radius r_D is $nr_D^3 \simeq 1/\Lambda^3$. For a weakly coupled plasma this is large. The modification of the Coulomb potential is due to a many-body effect.

This many body effect is the formation of a space charge. If an external charge is introduced into a plasma, then it attracts opposite charges towards itself, and repels similar charges. For $\Lambda \ll 1$, the thermal energy is much larger than the Coulomb energy, so the charges can knock each other around. So a larger volume is needed to shield (screen) the charge, and appreciably decrease the Coulomb force between it and a distant charge.

Another way to state this is to say that the charge e depends on the distance at which it is measured, $e(r)$. This can happen because of the generation of a new length scale: $e(r) = ef(r/r_D)$.

Excitation by external fields

If an electromagnetic wave impinges on the plasma, the response of the medium depends on the frequency. Write $k_D = 1/r_D = e^2/(\Theta r_0^2)$. For waves with $k \gg k_D$, the wave will drive charged particles individually. The energy of the wave will be lost in accelerating each particle separately.

In the opposite limit $k \ll k_D$, there will be collective effects. In the limit $k \to 0$, *i.e.*, for static fields, there will be a space charge separation. If the field begins to oscillate slowly, so will the space charge. So, for very slow and long waves, the different charges in the plasma begin to separate out, and then oscillate against each other. These are called plasma oscillations.

Plasma oscillations are also called Langmuir waves. Quantized plasma oscillations are important in metals; they are called plasmons.

The plasma frequency

The quantities of interest are e, n and the mass of the charges, m. Since $[e] = M^{1/2}L^{3/2}T^{-1}$, and $[n] = L^{-3}$, it is clear that the only frequency one can construct is the plasma frequency

$$\omega_p \simeq \sqrt{\frac{e^2 n}{m}} = \sqrt{\frac{e^2}{mr_0^3}}.$$

An incident wave with $\omega = \omega_p$ can destabilize the plasma by pumping in energy and separating out the charges.

This formula is true for strongly interacting plasmas. When $\Lambda \ll 1$ thermal motion can randomize the coherent effect of the wave. As a result, Θ will control the magnitude of the dissipative terms.

Since, the charges have speed $v \simeq \sqrt{\Theta/m}$, one might expect the plasma wave dispersion relation to be modified to

$$\omega^2 = \omega_p^2 + k^2 v^2.$$

The plasma parameter again

It is also interesting to construct the submicroscopic length scale

$$r_c = \Lambda r_0 = \frac{e^2}{\Theta}.$$

Clearly, this is the distance at which the Coulomb and thermal energies become equal, *i.e.*, the distance of closest approach of two charges. For a weakly coupled plasma we have $r_c \ll r_0 \ll r_D$, and the plasma parameter, Λ, controls the separation of these scales.

A typical microscopic time scale in the plasma is the frequency of low angle scattering of charge carriers

$$\tau = \frac{r_D}{v} \simeq \frac{r_0^2 \Theta}{e^2} \sqrt{\frac{m}{\Theta}}. \quad \text{So} \quad (\tau\omega_p)^2 = \frac{r_0\Theta}{e^2} = \frac{1}{\Lambda}.$$

Another time scale in the plasma is the frequency of large angle scattering, $\tau_\ell = r_0/v = \tau/\Lambda$. This means that there is a similar hierarchy of time scales, $\tau_\ell \ll \tau \ll 1/\omega_p$.

Landau damping

Take a generic electromagnetic wave travelling in a plasma with wavenumber $\mathbf{k}$ and frequency ω. If there is a charged particle with velocity $\mathbf{v}$ which satisfies $\mathbf{k} \cdot \mathbf{v} = \omega$, then it is resonantly coupled to the wave because it sees a static field: the particle position is $\mathbf{v}t$, and the wave has phase $(\mathbf{k} \cdot \mathbf{v} - \omega)t = 0$ at the position of the particle.

Particles travelling slightly faster than resonance will be decelerated and lose energy to the wave. Particles travelling slower will be accelerated, and will gain energy from the wave.

If there are more particles slightly slower than the wave, then they will damp out the wave. This process is called Landau damping. This is common because momentum distributions of particles typically drop with increasing momentum.

Some plasmas

System	n (m^{-3})	T (K)	Λ	r_D (m)
Interstellar gas	10^6	10^4	2321	2.3×10^1
Gaseous nebulae	10^8	10^4	1077	2.3×10^0
Ionosphere	10^{12}	10^3	73	7.3×10^{-3}
Solar				$\times$
corona	10^{12}	10^6	2321	2.3×10^{-1}
atmosphere	10^{20}	10^4	11	2.3×10^{-6}
interior	10^{33}	10^7	2	2.3×10^{-11}
Lab plasma				$\times$
tenuous	10^{17}	10^4	34	7.3×10^{-5}
dense	10^{22}	10^5	16	7.3×10^{-7}
thermonuclear	10^{22}	10^8	500	2.3×10^{-5}
Metal	10^{29}	10^2	0.03	7.3×10^{-12}

Quantum plasmas

In any quantum many-body system at finite temperature there is a new length scale, $r_T = 1/mv = 1/\sqrt{m\Theta}$, called the thermal wavelength. So there is a new dimensionless variable

$$\eta = \frac{r_0}{r_T} = \frac{r_0}{mv} = \frac{r_0}{\sqrt{m\Theta}}.$$

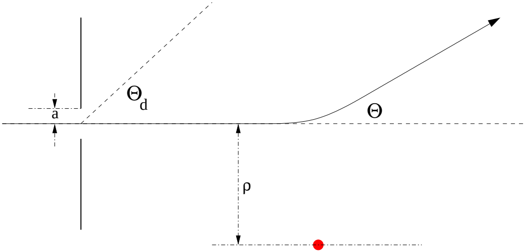
This parameter is also important for an ideal quantum gas. Note also that $[e^2] = LT^{-1}$, so $\Phi = e^2/v = \Lambda\eta$.

The Debye screening length is now a function of both variables,

$$r_D = r_0 Q(\Lambda, \Phi).$$

We are interested in $\Lambda \ll 1$, as before, but whether or not the classical computation is valid depends on Φ. In order to understand this, we need to figure out when the classical scattering formulae stop being valid.

Diffractive scattering in quantum mechanics



For potentials, V, which fall faster than Coulomb, at large impact parameter classical mechanics can be wrong due to diffraction. The classical scattering angle, $\psi \simeq mV(\rho)/p^2$, for a particle with momentum p. The diffraction angle due to a slit of radius a is $\psi_d \simeq 1/(pa)$. For classical scattering to be accurate, we must have $\psi \gg \psi_d = 1/(p\rho)$.

Diffraction and quantum plasmas

For a classical plasma the Coulomb potential is essentially unmodified for $\rho \ll r_D$. For classical scattering to be useful in this region one must have $e^2/v = \Phi \gg 1$. However, the many body physics of the plasma arises at longer length scales, and one must have other criteria to decide whether classical physics works there. At distances comparable with r_D, classical scattering works only for

$$\frac{mV(\rho)}{p^2} \gg \frac{1}{p\rho}.$$

For a Debye screened potential this condition implies

$$\rho \ll r_D \log \Phi.$$

When $\Phi \gg 1$, *i.e.*, for large Θ, the domain of classical physics could include r_D. At smaller temperatures, quantum effects on plasmas must be taken into account.

Relativity

In relativistic plasmas, there is a new energy scale, m. So there is a new dimensionless number,

$$\gamma = \frac{\Theta}{m}.$$

This is the usual relativistic Lorentz factor. When $\gamma \ll 1$ the previous classical theory is applicable.

When $\gamma \gg 1$, the system is ultra-relativistic, and the mass of the particle is much less than the kinetic energy. Under these circumstances, it is easy to create and destroy particles. It is therefore unproductive to examine relativistic classical plasmas. It is more useful and realistic to examine plasmas within quantum field theory.

Quantum field theory

In the quantum field theory of electron-positron plasmas, Θ, r_0, m_e and $\alpha = e^2$ are the parameters. Now m_e/Θ, $r_0\Theta$ and α are three dimensionless parameters in the problem. One should be able to write $n = \Theta^3 f(\alpha, m_e/\Theta)$. It turns out that n does not have a good limit as $(m_e/\Theta) \to 0$, since low-energy electron-positron pairs can be produced in arbitrarily high numbers. However, the energy density, $\epsilon = \Theta^4 g(\alpha, m_e/\Theta)$ has a good limit $\epsilon = \Theta^4 g(\alpha)$.

Debye screening occurs in such a plasma, and one should be able to write $r_D^{-1} = \Theta\rho(\alpha, m_e/\Theta)$. This has interesting, curable, infra-red problems in the limit $m_e \ll \Theta$.

There is a plasmon in the problem, with dispersion relations shown before. The plasma frequency, ω_p, acts like a mass in a relativistic dispersion relation.