Informal problems of inference

Sourendu Gupta

TIFR, Mumbai, India

Methods in Science 2013
26 February, 2013

Outline

Problems of inference

Physics of coin tossing

Keywords and References

Keywords

cards, cheating, prior knowledge, probability, inference, Monty Hall, Rolls Royce, Martin Gardner, AIDS testing, search engine terms, tossing a coin, angular momentum, phase space, symmetry

Books

Martin Gardner

Copyright statement

Games with cards

- How many colours, suits, cards per suit.
- What is the probability that the first card I draw from a complete suit is red? Second card? Third card?
- What is the probability that the pack is not shuffled?
- What is the probability that the pack is not normal?

My prior knowledge: the pack is constructed by merging the red cards from two cards.
Your prior knowledge: the pack is normal.

The meaning of probability. Did you infer something about the problem by observing the successive trials? Is there inference?

The Monty Hall problem

In the Monty Hall game show *Let's Make a Deal*, the host shows you three doors. Behind one is a Rolls Royce, the others hold a goat. You guess one door. The host opens one of the other doors and shows you that there is a goat behind it.

Will you change your guess?

Interpreting a medical test

Somebody develops a test for AIDS which costs only Rs. 200 and gives correct results 98% of the time. The alternative costs Rs. 15000 and gives correct results 99.7% of the time. You go to a lab which offers you a choice. Which one would you take?

You have to balance the frequency of AIDS in the population against the efficiency of the test.

When using a search engine with common terms, what are the chances that you will hit the right page quickly?

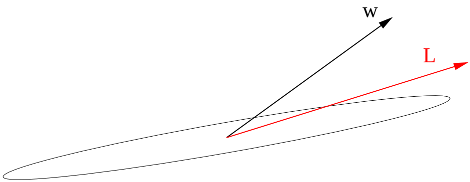
Tossing a coin

When you toss a coin, what is the probability of heads? Why?

The flip of a coin can be analyzed in terms of the conservation of angular momentum. The initial force on it imparts an angular momentum, **L**, about the CM. The CM describes a parabolic path. Gravity also gives a small torque which causes **L** to change. If we neglect the torque then **L** is conserved and the angular velocity, ω, precesses about **L**.

The moment of inertia tensor of a coin has cylindrical symmetry. One principal axis is normal to the coin; this corresponds to the smallest moment of inertia, I_3. The other two axes are in the plane of the coin, with component $I_1 \gg I_3$. For generic ω, $L_3 = I_3\omega_3$ is much smaller than L_1 and L_2, so **L** lies nearly on the surface of the coin, even if ω_3 is not small.

The coin is a top



The component of ω normal to the coin can be rather large, although **L** is almost in the plane of the coin. So precession can cause the coin to rotate around its own plane. Since ω_3 is not easily controlled, we have no knowledge of whether the number of rotations is integer or half integer when the CM touches the ground.

Viewed looking from the arrowhead of **L** towards the coin, one always sees the same face of the coin. Does this help to control the toss?

Hamiltonian analysis of a tossed coin

A coin is a rigid body. So the physics of a tossed coin can be placed in a 12 dimensional phase space. Any point in phase space is a possible initial condition for the toss. Given a particular method of tossing a coin (such as: starting from Tanay's hand, going at least 4 m high, and landing on the ground), some parts of phase space are ruled out as initial conditions. Each point in the remaining part can be called H or T, according to whether the initial condition starting from there corresponds to a result of Heads or Tails. The same point can have different labels if we specify a different procedure for determining the outcome. Suppose the initial vertical velocity, $\dot{z}$, is such that the coin revolves exactly 100 times before stopping and showing H. Then a 0.5% change causes the coin to show T, and a 1% change causes it to show H again. So phase space is sliced into thin ribbons labelled H and T, along $\dot{z}$. It is sliced into similar ribbons in direction ω_3.

Estimates of accuracy

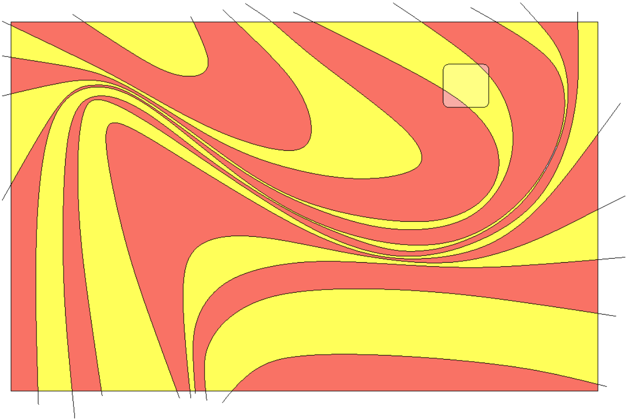
The Lagrangian of the problem is symmetric under relabelling of H and T (a Z_2 symmetry), so exactly half of the phase space must be labelled either way.

The coin must initially have $\dot{z} \geq \sqrt{2gh} \simeq 9$ m/s in order to travel at least 4 m up. The time of flight is about $2\dot{z}/g \simeq 1$ s. Control to better than a few percent is hard to achieve. The initial torque can be high, so a spin rate of $\omega = 0.05$ Hz is possible. Control of ω_3 requires controlling the initial orientation of the coin, and high manual accuracy in this is also not feasible. If R is the region within which we know the initial conditions, then

$$p(H) = \Gamma(H)/\Gamma(R),$$

where Γ denotes the phase space volume. Since the size of R is large compared to the width of the H ribbons, then due to the Z_2 symmetry of the problem, we can write $p(H) = 1/2$.

Foliation of phase space



However, if we are allowed to toss gently, the phase space ribbons are wide, and we may have $\Gamma(H) \simeq \Gamma(R)$, so that it is possible to control the toss.