Quantum Mechanics 1

TIFR Graduate School, 2008
Drop test

1. (5 marks) Does the following matrix have unique eigenvectors?

$$\begin{pmatrix} 1 & 7 & 7 & 7 \\ 7 & 1 & 7 & 7 \\ 7 & 7 & 1 & 7 \\ 7 & 7 & 7 & 1 \end{pmatrix}.$$

2. (5 marks) Compute the operator A such that $\exp A = \exp(ix\hat{\mathbf{r}})\exp(iq\hat{\mathbf{p}})$ where $\hat{\mathbf{r}}$ is the position operator and $\hat{\mathbf{p}}$ is the momentum operator.

3. (5 marks) A set of polynomials is generated by the recurrence relation $P_n(x) = xP_{n-1}(x) - P'_{n-1}(x)$, where $P_0(x) = 1$ and prime denotes differentiation. Are these polynomials orthogonal under the inner product rule given below?

$$\langle f|g\rangle = \int_{-\infty}^{\infty} dx e^{-x^2} f(x)g(x).$$

4. (5 marks) A stream of Ag^+ ions with fixed momentum are polarized in a uniform magnetic field transverse to the direction of the momentum. They are then allowed to pass through a tube of fixed length containing a constant magnetic field pointing along the axis of the tube. What fraction of the atoms emerging from the tube retain their original polarization?

5. (5 marks) Find the commutator of the Hamiltonian for a one-dimensional harmonic oscillator with a generic translation operator in one dimension.

6. (10 marks) How many real numbers are required to specify the most general 3×3 Hermitean matrix? Write down a set of traceless Hermitean matrices Σ_j, analogous to the Pauli matrices, such that any 3×3 Hermitean matrix, $H = a_0 + \sum_j a_j \Sigma_j$ for real a_0. How many of the Σ_j commute among themselves?

7. (10 marks) Write down the most general density matrix for a system consisting of two spin 1/2 particles. Construct the density matrix (in the same basis) in the special case when the two-particle wavefunctions are projected into total spin 1.

8. (10 marks) Find the 2×2 scattering matrix for waves of a fixed magnitude of the momentum, q, for the one-dimensional single step potential: $V(x) = V_0 > 0$ when $|x| \leq a$ and 0 otherwise. From this find the resonant values of q.

9. (20 marks) A molecule consists of six atoms arranged in a hexagonal ring. Assume that the atom contains a single valence electron and the energy splitting between the filled and valence shells is large enough that the coupling between them can be neglected, and that one can also neglect the energy of interaction of the valence electrons with each other. The energy of the valence electron in each of the isolated atoms is E_0. In the molecule each valence electron lowers its energy by E_1 by hopping to either of its neighbours. What are the energies in the eigenstates of the Hamiltonian of the molecule for each valence electron?

10. (25 marks) Consider the energy level $E = 5\hbar\omega/2$ of an isotropic three dimensional harmonic operator. What is the degeneracy of this level? A basis for the eigenstates of the three dimensional oscillator is that of three independent harmonic oscillators along each of the three orthogonal directions, x, y and z. Another basis is that in which the angular momentum operator is diagonalized simultaneously with the Hamiltonian. For the energy level given above, find the unitary matrix which performs the transformation between the two bases.