Some questions

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Quantum Mechanics 1
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What is a density matrix

- When a single quantum system is in a state $|\phi\rangle$, the expectation value of an operator, O, is defined to be $\langle O\rangle = \langle\phi|O|\phi\rangle$. This means that when we make measurements on **many identical systems** (*i.e.*, an ensemble), then the **average** of the measurement over these repetitions is $\langle O\rangle$.
- Now we make an entirely new formulation of quantum mechanics. We postulate that when we have an **ensemble** of quantum systems, then they are described by a density matrix ρ. Every measurement is defined by a Hermitean operator O (exactly as in the usual formulation). The expectation value of a measurement over an ensemble is $\langle O\rangle = \mathrm{Tr}\,\rho O$.
- This formulation must be the same as the standard formulation when we discuss the same system. For a **pure state ensemble**, *i.e.*, an emsemble in which every quantum state is identical, one has to write $\rho = |\phi\rangle\langle\phi|$. Then $\langle O\rangle = \mathrm{Tr}\,\rho O = \langle\phi|O|\phi\rangle$. However, in the most general case we can only say that $\rho^\dagger = \rho$ and $\mathrm{Tr}\,\rho = 1$.

A two-particle density matrix

Take a the density matrix of a two-particle system where each particle has spin $j = 1/2$. The two-particle Hilbert space is 4-dimensional and spanned by the orthonormal states $|0, 0\rangle$ and $|1, m\rangle$ where $m = \pm 1$, 0. The most general density matrix is a 4×4 unitary matrix with unit trace, and hence needs 15 real components for its specification. Hence one has

$$\rho = \begin{pmatrix} \rho_{00,00} & \rho_{00,11} & \rho_{00,10} & \rho_{00,1-1} \\ \rho^*_{00,11} & \rho_{11,11} & \rho_{11,10} & \rho_{11,1-1} \\ \rho^*_{00,10} & \rho^*_{11,10} & \rho_{10,10} & \rho_{10,1-1} \\ \rho^*_{00,1-1} & \rho^*_{11,1-1} & \rho^*_{10,1-1} & \rho_{1-1,1-1} \end{pmatrix}.$$

The trace of this matrix is unity.
Using this we can find expectation values of operators symmetric in the exchange of the particles (a special case is of a scalar operator) or an operator which is odd under such an exchange (for example, a vector operator). Not all the matrix elements enter into each expectation value.