Quantum Mechanics 1

TIFR Graduate School, 2008
Mid-semester examination
Duration: 3 hrs

1. (10 marks) Find the 3×3 matrix

$$\exp \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

2. (10 marks) A molecule is made of a closed chain of several identical atoms. Each atom has a single valence electron, and the energy of the valence shell is well separated from the energies of the core electrons, so that the binding energy of the molecule can be understood entirely from the dynamics of the valence electrons. Simplify the problem further by neglecting interactions between electrons. Now assume that the hopping to a neighbouring atom reduces an electron's energy by E_1. Then what is the ground-state binding energy of the penta-atomic molecule?

3. (15 marks) Take a particle in a one dimensional square well of depth V_0 and width $2a$. When V_0 and a are adjusted so that there is a bound state with binding energy, E which approaches zero, how do the expectation values $\langle x^2 x \rangle$, $\langle x^4 \rangle$ and $\langle p^2 \rangle$ change with E? (Perform the integrals before taking limits).

4. (15 marks) A potential, $V(x)$, is constructed from three square barriers, each of height V_0 and width $2a$. The centers of successive barriers are separated by a distance $3a$.
   (a) Find the positions of the resonances for $V(x)$.
   (b) As the incoming particle energy, E, approaches zero, the transmission coefficient vanishes as a power of E. What is this power?

5. (25 marks) Consider the matrices

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \text{and} \qquad B = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}.$$

   (a) Are these matrices simultaneously diagonalizable?
   (b) What are the eigenvalues and eigenvectors of A?
   (c) Use the eigenvectors of A to construct an unitary transformation, U. Find $U^\dagger B U$.
   (d) Construct a one-parameter (θ) set of unitary matrices $V(\theta)$ such that $V(\theta)^\dagger U^\dagger A U V(\theta)$ are diagonal for all θ. Find what happens to $V(\theta)^\dagger U^\dagger A U V(\theta)$ as a function of θ.
   (e) Is there an unique set of common eigenvectors of A and B?

6. (25 marks) For a thermal ensemble of a single spinless charged particle (of charge e and mass m) moving in a magnetic field B at temperature T, find the partition function and the expectation value of the energy.