Effective Field Theory Methods in Atomic and Nuclear Physics

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Effective Field Theories
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Outline

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Shell models

In atomic and nuclear spectroscopy, phenomenologically reasonable results are often obtained assuming that many of the excitations are single particle excitations of electrons or nucleons.

The Coulomb potential between electrons in an atom, V_{ee}, and those between electron and nucleus, V_{ne}, are of the order

$$V_{ee} \simeq \frac{\alpha}{a_0}, \quad \text{and} \quad V_{ne} \simeq \frac{\alpha Z}{a_0},$$

where Z is the atomic number, $a_0 = 1/(m\alpha Z)$ is the Bohr radius of the atom, and m is the mass of the electron. Since there are Z electrons, the potential on an electron due to all the others is $(Z-1)V_{ee} \simeq V_{ne}$. As a result, one finds it hard to understand why the shell model works.

In the nuclear shell model it is even harder to understand why a central potential should work.

Quasi particles

Both the atomic and nuclear spectroscopy problems are non-relativistic, and we would like to write a Schrödinger theory for them, in terms of single particle excitations over the ground state energy, E_0. The fermionic nature of the particles is important in order to fill shells, so we will have to use non-relativistic fermions. The single particle terms in the Lagrangian can be written as

$$L_1 = \int d^3p \sum_{m=\pm 1/2} \left[i\psi_m^\dagger(p)\partial_t\psi_m(p) - \{E(p) - E_0\}\,\psi_m^\dagger(p)\psi_m(p) \right].$$

Here $E(p)$ is the exact energy of the single particle state.

We need to define dimensions in a way appropriate to the experiments we would like to describe. In relativistic problems we chose to use an energy scale so that typical energy scales of experiment, Q, and of the system, Λ could be compared.

Scaling dimensions

Let us choose to use the momentum p to give a scale. Introduce the notation $K = \nabla_p E(p)$ for the momentum which correspond to energy E_0. Since we are only interested in excitations above E_0, we define $p = K + q$ and investigate RG flows for $q \to \xi q$. If a quantity scales as ξ^ℓ, then we say that it has scaling dimension ℓ. As a result,

$$[q] = 1, \quad [K] = 0, \quad [\int d^3p] = [\int d^2K dq] = 1, \quad [E - E_0] = 1.$$

The last scaling dimension comes from the fact that $E(p) - E_0 = q \cdot v_F(K) + \mathcal{O}(q^2)$.

Finally, since all terms in the action must have the same dimension, and the action cannot scale, one find

$$[\partial_t] = 1 \qquad \text{and so} \qquad [\psi] = -\frac{1}{2}.$$

Two-particle interactions

The most general two-particle interaction in this QFT is given by

$$L_2 = \int \prod_{i=1}^{4} d^3 p_i \delta^3(p_1 + p_2 - p_3 - p_4) \sum_{mm'} \lambda(p_1, p_2, p_3, p_4) \times \\ \times \psi_m^\dagger(p_1)\psi_m(p_2)\psi_{m'}^\dagger(p_3)\psi_{m'}(p_4).$$

For generic p_i the delta function constrains essentially the K_i, so $[\delta^3] = 0$, and hence $[\lambda] = -1$. The four-fermi coupling is therefore irrelevant. This is called the Landau-Fermi liquid

The fact that the only relevant terms in these QFTs are the single particle terms tells us why the spectroscopic problem is conceptually easy. Of course, the hard (multiparticle) problem is hidden in $E(p)$.

Pairing

For the specific case when $p_1 + p_2$ is nearly zero, then $[\delta^3] = -1$. As a result, $[\lambda] = 0$, and hence marginal. A detailed computation shows that when the interaction is attractive the coupling becomes relevant, and leads to pairing of quasiparticles near the Fermi surface. This is the case of superconductivity.

Pairing is absent in atomic spectra, as expected. But it has been known for a long time that it is necessary to use paired nucleons to explain some systematics of nuclear spectra. A pairing term was also included in Wigner's semi-empirical mass formula.

In metals, electrons form bands. Typical transport properties are due to particles with the quantum numbers of electrons: the electron fermi liquid. In special cases phonons mediate attractive interactions between these electrons, giving rise to superconductivity.

Evidence for pairing in finite nuclei

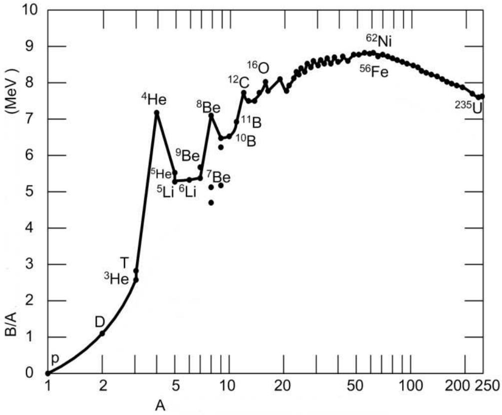
Pairing of nucleons in a finite nucleus due to attractive interactions between them results in the pair having zero angular momentum. This manifests itself in three ways:

1. The ground states of nuclei with one more or less nucleon are contain a term due to this pairing. The change in binding energy is

$$\delta B = \begin{cases} \Delta & (\mathrm{N\ and\ Z\ even}) \\ 0 & (\mathrm{N+Z\ odd}) \\ -\Delta & (\mathrm{N\ and\ Z\ odd}) \end{cases}$$

2. Ground states of even-even nuclei have zero angular momentum, since the nucleons are paired.
3. The first excitation energy of even-even nuclei are significantly higher than those of nearby nuclei.

Binding energies of nuclei



Note the even-odd structure: even-even nuclei most bound.

Some problems

Problem 5.1

Pairing of fermions in fluids (for example, in ^{3}He) gives rise to superfluidity. In nuclei, why does pairing not give rise to superfluidity? What limits must be taken to observe superfluidity in systems of nucleons? What would be the observable properties of such superfluids?

Problem 5.2

It is possible to replace electrons in atoms by other negatively charged particles: for example pions. If one replaces two electrons by pions, what changes does one expect in the electronic shell structure of atoms? Apart from these relatively simpler effects, should one expect any effects of pairing between the pions due to strong interaction effects?

Outline

Keywords and References

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Shell model, Landau-Fermi liquid; superconductivity; Wigner's semi-empirical mass formula.

References

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