Symmetries in Effective Field Theory

Sourendu Gupta

SERC Main School 2014, BITS Pilani Goa, India

Effective Field Theories
December, 2014

Outline

Outline

Symmetries of EFTs and their UV completions?

The 4-fermi theory of weak interactions has

- Has baryon and lepton number conservation, as in the SM.
- Has the V-A structure of the SM, so has broken parity.
- Does not have the gauge symmetry under SU(2)$\times$U(1) of the SM.

In general the exact symmetries of the UV completion go over to those of the effective theory. However, broken symmetries pose special problems. Their treatment in an EFT differs in the cases of softly broken symmetries, *i.e.*, symmetry breaking by a relevant operator, accidental low-energy symmetries, *i.e.*, symmetry breaking by irrelevant operators, and hard breaking of symmetry, *i.e.*, breaking by marginal operators.

Accidental symmetries: two examples

In the SM there are no flavour-changing neutral currents, in agreement with experiments. However, generic UV completions of the standard model would give rise to 4-fermi operators which could violate any of the flavour conservations, including baryon number. Therefore, flavour would be an accidental symmetry of these UV completions, protected only by the fact that Λ, the scale of new physics, is very high.

Heavy-quark effective theory exhibits a spin-flavour symmetry (named Isgur-Wise symmetry) which is reasonably well obeyed in decays of heavy-light mesons. QCD does not contain this symmetry. It is an accidental symmetry which is due to the fact that when the binding energy of the meson is much smaller than the heavy quark mass, M, then contributions due to virtual heavy-quark pairs can be neglected.

Symmetry breaking in UV by a marginal operator

Suppose that a theory is symmetric under the shift of a Higgs field $H \to H + v$, except for a small breaking by the marginal operator λH^4. At one-loop order this generates a scalar mass term

$$m^2 H^2 \simeq \lambda \Lambda^2 H^2.$$

In general, if the symmetry is broken, then all possible symmetry breaking operators will be generated by radiative corrections. Although the symmetry is broken by a marginal operator in the UV, its effect is large.

Problem 6.1: a custodial symmetry

Suppose the light quark masses were nearly vanishing. Then electromagnetic couplings of the quarks would generate a difference between the masses of the different pions. One might expect this difference to be of the order of Λ_{QCD}. Instead, one find $m^2_{\pi^+} - m^2_{\pi^0} \ll \Lambda^2_{QCD}$. Why?

Light scalar via UV breaking of conformal symmetry?

If exact conformal symmetry is spontaneously broken in the UV completion of the EFT, then the EFT contains a Goldstone boson, χ, called the dilaton. Under exact scaling symmetry

$$x^\mu \to \mathrm{e}^{-\omega} x^\mu, \quad \text{and} \quad \chi(x) \to \mathrm{e}^{\omega}\chi(x).$$

Conformal symmetry allows the term $\lambda\chi^4$ in the EFT. Now suppose there is a marginal operator in the theory with dimension $\Delta \simeq 4$, $\mathcal{O}(x) \to \exp(\omega\Delta)\mathcal{O}(x)$, and coupling constant $\kappa/\mu^{4-\Delta}$. Then the EFT would have

$$\mathcal{L}_{\rm int} = \frac{\lambda}{4!}\chi^4 - \kappa\chi^\Delta.$$

The VEV of χ can be found, and from that the dilaton mass, $m \propto (4-\Delta)$. This means that conformal symmetry breaking by a marginal operator gives a very light dilaton. 125 GeV scalar? [Chacko, Mishra; 2012. Goldberger et al; 2008].

Five-finger exercises

Problem 6.2

1. Why does parity appear as a good symmetry at low energies?
2. The symmetries of a lattice gauge theory are of the hypercubic Euclidean lattice and gauge invariance. When the lattice spacing goes to zero how does full rotational symmetry emerge along with gauge invariance in the continuum theory?
3. An UV completion of the SM may give rise to a term in the EFT of the form

$$\mathcal{L}_{\rm int} = \frac{\lambda_{ij}}{\Lambda}(\ell_i H^\dagger)(\ell_j H^\dagger),$$

where ℓ_i is a left-handed lepton doublet in the i-th family, H is the Higgs field, and $\Lambda > 10$ TeV. Which accidental symmetries of the SM are broken by $\mathcal{L}_{\rm int}$?

Outline

Degenerate ground states

In a QM framework, potentials with multiple wells give rise to ground states with equal energies, $|0_i\rangle$ (here i labels the well). However, in any system with a finite number of particles, there is a mixing of states which lifts this degeneracy, and creates an unique ground state.

One can check this in degenerate perturbation theory. The off-diagonal elements of the perturbation Hamiltonian are given by tunnelings between states. If the action due to this is small, then the ground state energy is lowered by a small amount.

As the number of degrees of freedom goes to infinity, the off-diagonal element goes exponentially to zero. As a result the ground states can be degenerate in a QFT.

Spontaneous symmetry breaking

Let Q be the charge of a continuous symmetry, *i.e.*, $[Q, H] = 0$, where H is the Hamiltonian of the theory. Suppose $|E\rangle$ is a state of the theory with energy E. If $|E\rangle$ is not invariant under the symmetry then $Q\,|E\rangle = |E\rangle'$, where $|E\rangle'$ is a different state, but with the same energy E. We concern ourselves with the interesting special case when E is the lowest energy state of the theory.

If the actual ground state is one of these, for example. $|E\rangle$, then we say that the symmetry is spontaneously broken.

In one of these states every local operator in the theory has a non-vanishing value, in general. This means that the field operator has a non-zero expectation value in the state $|E\rangle$; in other words the field has a non-zero vacuum expectation value (VEV).

Effective action

Consider the generating functional of the connected correlation functions of any QFT: $W[j] = \log Z[j]$. The VEV of the field

$$\Phi(x) = \frac{\partial W[j]}{\partial j(x)},$$

can be used to give the source j as a functional of the VEV. Then one can define the effective action as the Legendre transform of W—

$$\Gamma[\Phi] = W[j] - \int d^4x\, j[\Phi]\Phi(x).$$

Directly from the definition of the partition function one has a path integral representation of Γ.

Note that if the action and the path integral measure are invariant under a symmetry, then so is the effective action.

Effective potential

In a QFT which has translational symmetry, the effective action is

$$\Gamma[\Phi] = -\Omega V[\Phi],$$

where Ω is the volume of space-time, and V is called the effective potential.

Problem 6.3

Using the path-integral representation of the effective potential, check what it becomes at tree level. To all loop orders, what do the first and second derivatives tell us about the QFT? Check your intuition using the tree level expression.

Problem 6.4

In a zero dimensional QFT with a field x which can take any real value, and an action $S[x] = (x - a)^2/(2\sigma^2)$, with a and σ real, find $W[j]$, the effective action and the effective potential.

Spontaneous symmetry breaking

Assume that there is a multicomponent field ϕ invariant under a symmetry, whose generator is T. Then we can make a transformation $\phi \to \phi + i\epsilon T\phi$. Since the effective potential is invariant under this transformation, one has

$$\frac{\partial V}{\partial \phi_i} \, T_{ij}\phi_j = 0.$$

Taking another derivative with respect to the field, ϕ_k, one has

$$\frac{\partial V}{\partial \phi_i} \, T_{ik} + \frac{\partial^2 V}{\partial \phi_i \phi_k} \, T_{ij}\phi_j = 0, \quad \text{where} \quad \frac{\partial^2 V}{\partial \phi_i \phi_k} = \Delta^{-1}_{ik}(q^2 = 0).$$

The 1st term vanishes if this is taken at the minimum of V, *i.e.*, at the true VEV. If some $T_{ij}\phi_j$ do not vanish, then these must correspond to zero modes of the inverse propagator. These massless field are called Goldstone modes.

The current and the Goldstone field

If $|V\rangle$ is the vacuum with spontaneous symmetry breaking, and $J^\mu(x)$ is the current for the symmetry being broken, then the current operator acting on the vacuum produces a single Goldstone particle state. One finds

$$\langle V|\, J^\mu(x)\,|p\rangle = \frac{iF}{(2\pi)^{3/2}}\, p^\mu\, \frac{\exp(ip\cdot x)}{\sqrt{2p_0}},$$

where p is the momentum of the particle, and $[F]=1$. Clearly, $p_\mu J^\mu = 0$, so the current is conserved.
For the field operator we have

$$\langle V|\, \psi(x)\,|p\rangle = \frac{Z}{(2\pi)^{3/2}}\, \frac{\exp(-ip\cdot x)}{\sqrt{2p_0}},$$

where one has $Z=1$ by convention. These together imply

$$J^\mu(x) = F\partial^\mu\psi.$$

Some applications

1. The soft breaking of chiral symmetry is at the heart of our understanding of nuclear physics and flavour physics.
2. If supersymmetry is invoked to solve the naturalness problem of the Higgs mass, then this raises other problems. SuSy cannot be exact in nature since experiments do not find any of the super-partners. At the same time, SuSy breaking cannot be achieved in the UV, since that would spoil loop cancellations in the Higgs mass. One solution is that SuSy is softly broken, giving rise to Goldstone modes [Dimopoulos and Georgi, 1981].
3. The patterns of masses and mixings of quark and neutrino generations are quite distinct. If we assume that quarks and leptons are unified in some high scale theory, then this cannot remain true in the UV. The differences may arise due to soft breaking of the symmetry in the SM [Ma and Rajasekharan, 2001].

The Goldstone "Theorem"

If the global symmetry of a group G in a relativistic QFT is broken down to a subgroup H, then:

- There are $\dim(G) - \dim/(H)$ Goldstone boson fields. Each of these corresponds to a massless particle.
- The remaining fields take on VEVS and also give rise to massive particles.
- Due to this gap in the spectrum it is possible to write an EFT for the Goldstone modes.

Spontaneous breaking of an approximate symmetry

Suppose the effective potential is $V(\phi) = V_0(\phi) + V_1(\phi)$, where the first term satisfies the Goldstone condition, as before,

$$\frac{\partial^2 V_0}{\partial\phi_i\phi_k}\, T_{ij}\phi_j = 0.$$

We take V_1 to be a small correction which explicitly breaks the symmetry. Its effect is to shift the minimum from ϕ^0 to $\phi^0 + \phi^1$. We find the minimum of V by expanding in a Taylor series in ϕ^1. The first order term is

$$\left.\frac{\partial V_0}{\partial\phi_i}\right|_{\phi^0} = 0.$$

The next order gives

$$\left.\frac{\partial V_1}{\partial\phi_i}\right|_{\phi^0} + \left.\frac{\partial^2 V_0}{\partial\phi_i\phi_j}\right|_{\phi^0} \phi_j^1 = 0.$$

We put the Goldstone condition back into this to get a new result.

Vacuum alignment

$$\left.\frac{\partial V_1}{\partial \phi_i}\right|_{\phi^0} T_{ij}\phi_j^0 = 0.$$

This condition says that there are no tadpole corrections to pseudo-Goldstone bosons, since the PGB components of the field are $T_{ij}\phi_j^0$, and V generates all 1PI graphs. Conversely, if there were tadpoles, *i.e.*, the equation were no satisfied, then large corrections would be generated.

This condition also forces the symmetry broken vacuum to align with the direction in field space chosen by V_1. Asa result, this generates a small mass for the PBG, proportional to the perturbation giving rise to V_1.

The effective potential is a great aid to visualization, and therefore to intuition.

Chiral symmetry

The fermion kinetic term, $\mathcal{L}_0 = i\overline{\psi}\partial\!\!\!/\psi/2$, has chiral symmetry. If we transform

$$\psi \to \mathrm{e}^{i\zeta\gamma_5}\psi \quad \textit{and} \quad \overline{\psi} \to \overline{\psi}\mathrm{e}^{i\zeta\gamma_5},$$

then $\mathcal{L}_0$ remains unchanged. However, the mass term $\mathcal{L}_m = m\overline{\psi}\psi$, is not invariant; so mass terms cause explicit breaking of chiral symmetry.

Another way to exhibit this symmetry is to write the left and right helicity projections of a Dirac field: $\psi_L = (1-\gamma_5)\psi/2$ and $\psi_R = (1+\gamma_5)\psi/2$. Then, $\mathcal{L}_0 = i\overline{\psi}_L\partial\!\!\!/\psi_L + i\overline{\psi}_R\partial\!\!\!/\psi_R$ and $\mathcal{L}_m = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$. Using $\exp(i\zeta\gamma_5) = \cos\zeta + i\sin\zeta\gamma_5$, we write

$$\mathrm{e}^{i\zeta\gamma_5}\psi_L = \cos\zeta\,\psi_L + i\sin\zeta\,\psi_R, \quad \mathrm{e}^{i\zeta\gamma_5}\psi_R = \cos\zeta\,\psi_R + i\sin\zeta\,\psi_L.$$

As a result, we see the same invariance properties again.

Flavour symmetries

Since QCD has multiple flavours, the kinetic term is

$$\mathcal{L}_0 = \frac{i}{2}\sum_{i=1}^{N_f}\left[\overline{\psi}^i_L \partial\!\!\!/ \psi^i_L + \overline{\psi}^i_R \partial\!\!\!/ \psi^i_R\right],$$

and one can independently perform unitary transformations $\mathrm{U}(N_f)$ on each chirality. So, in the massless limit, QCD has chiral $\mathrm{U}_L(N_f)\times\mathrm{U}_R(N_f)$ symmetry.

The $\mathrm{U}_A(1)$ symmetry has an anomaly; the measure in the path integral does not respect this symmetry. So QCD has a symmetry $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)\times\mathrm{U}_V(1)$. This is spontaneously broken to $\mathrm{SU}_V(N_f)$ symmetry. The conserved vector U(1) is called baryon number, and the remaining $\mathrm{SU}(N_f)$ are the symmetries of the quark model. Since the axial symmetries are broken, the Goldstone bosons are pseudo-scalars.

Explicit chiral symmetry breaking

The mass terms in QCD are

$$\mathcal{L}_m = \overline{\psi} M \psi = \sum_{i=1}^{N_f} m_i \left[\overline{\psi}^i_L \psi^i_R + \overline{\psi}^i_R \psi^i_L \right],$$

where M is a diagonal mass matrix. The masses break chiral symmetry explicitly, so that the pseudoscalar mesons become (at best) pseudo-Goldstone bosons. By taking the m_i to be unequal one explicitly breaks $\mathrm{SU}_V(N_F)$.

Note A: If M were a dynamical field (a spurion) transforming inversely as ψ, then the chiral symmetry could be recovered.

Note B: A vector symmetry is one in which the same transformation is made on the L and R components, and an axial symmetry has Hermitean conjugate transformations on L and R.

A contrafactual theory

Problem 6.5

Assume that QCD with massless quarks is a conformal field theory (CFT). This would mean, of course, that the β-function of QCD vanishes and therefore the theory is characterized by a constant gauge coupling g. Since this violates our observations, such a theory is contrafactual. However, we may still want to examine other features of such theories.

What observed features of QCD would be violated by such a CFT? Could your answer depend on the value of g?

Would it be possible to have spontaneous breaking of chiral symmetry in such a theory? If yes, then would it be possible to have an EFT for the Goldstone bosons, and how would it differ from chiral perturbation theory? If not, what would you expect for the spectrum of the theory?

Non-linear fields

We write the pseudoscalar fields as π_α. The soft-breaking "theorem" assures us that there is an intrinsic scale defined by the axial current: $J^\mu = F\partial^\mu\psi$. Since F is also the normalization of the matrix element of the axial vector current between a one-pion state and the QCD vacuum (*i.e.*, a state without any hadrons), we can identify $F = 92$ MeV with the pion decay constant.

Since F has dimensions of mass, we can define the exponential

$$\Sigma = \exp\left(2i\,\frac{\pi_\alpha T_\alpha}{F}\right).$$

Under the chiral symmetry of QCD this transforms as $L^\dagger \Sigma R$. Invariance under chiral symmetry requires that the only non-derivative terms which can arise are $\mathrm{Tr}\,(\Sigma\Sigma^\dagger)^n$, which are trivial since $\Sigma\Sigma^\dagger = 1$. As a result only derivatives appear, and all interactions vanish at low momenta.

The Goldstone EFT

The leading term in the Goldstone EFT is

$$\mathcal{L}_0 = \frac{F^2}{4} \mathrm{Tr}\, \partial_\mu \Sigma \partial_\mu \Sigma = \frac{1}{2} \partial_\mu \pi^\alpha \partial_\mu \pi^\alpha + \frac{1}{2F^2} (\pi^\alpha \partial_\mu \pi^\alpha)^2 + \cdots$$

Note that the coefficients of all the terms are fixed by the normalization of the kinetic term. Higher order terms are organized by powers of the derivatives, since these are the only dimensional terms. QED can be included as usual.

So the EFT is organized as a derivative expansion

$$\mathcal{L} = F^2 \left[\mathcal{L}_2 + \frac{1}{\Lambda^2} \mathcal{L}_4 + \frac{1}{\Lambda^4} \mathcal{L}_6 + \cdots \right],$$

where $\mathcal{L}_n$ contains all terms with n derivatives.

The scale of QCD and an effective theory

The scale anomaly ensures that there are massive particles in QCD even in the chiral symmetric limit, *i.e.*, when $M = 0$. Since there is a finite temperature phase transition in QCD in this limit, one can use the transition temperature, T_0, as an intrinsic scale. For the mass of a hadron H, one can generally write $M_H = G_H T_0 + \mathcal{O}(M)$. However, as we saw before, the mass of the PGB vanishes in the chiral limit, so that $M^2_{PGB} \propto M$.

In order to write an EFT of hadrons, one has to find whether there is a significant gap in the spectrum. If there is, then one might be able to include only the light hadrons. The clearest such gap occurs between the pions and other hadrons, so one might try to write an EFT for pions.

Since $M_\pi \simeq 140$ MeV, and the next lowest non-strange particle is $M_\rho \simeq 770$ MeV, one will have the EFT cutoff in this range. Since kaons are tricky, with $M_K \simeq 550$ MeV, we might use $N_f = 2$ or 3.

Weinberg's power counting

Suppose one has a diagram with ℓ internal lines, V_i vertices coming from the $\mathcal{L}_i$ term, and L loops. Then the amplitude will be schematically

$$\int (d^4k)^L \, \frac{1}{(k^2)^\ell} \, \Pi_i (k^i)^{V_k}.$$

The UV divergence is of order $D = 4L - 2\ell + \sum_i iV_i$. Since $V - \ell + L = 1$, where V is the total number of vertices, one has

$$D = 2(L+1) + \sum_i (i-2) V_i.$$

Since $k \geq 2$, all terms are positive. All Born contributions from $\mathcal{L}_2$ have $D = 2$. All other contributions are suppressed by Λ^{2-D}.

The leading irrelevant contributions are due to $D = 4$. These come from either $L = 0$ or 1. The Born terms must come from a single vertex generated by $\mathcal{L}_4$ ($V_4 = 1$) and any number of vertices from $\mathcal{L}_2$. The one loop term can have contributions only from $\mathcal{L}_2$.

The cutoff scale

A 1-loop graphs which diverge with $D = 4$ is shown. The vertex comes from the $(\pi\partial\pi)^2$ term, which is suppressed with respect to the kinetic term by F^2. The loop integral is

$$F^2 \int \frac{d^4k}{(2\pi)^4} \, \frac{1}{k^4} \, \left(\frac{k}{F}\right)^4 = \frac{p^4 \log(p/\mu)}{16\pi^2 F^2},$$

where μ is the renormalization scale and p is a generic external momentum.

A Born term with $D = 4$ is given by cp^4/F^2, without dependence on μ. Since we are allowed to shift μ, we can change the log by a number of order 1. This should be absorbed by a change in c, which is therefore of order $1/(16\pi^2)$. This means that the theory works fine if we allow

$$\Lambda \simeq 4\pi F \simeq 1 \ \mathrm{GeV}.$$

Quark masses

The way M enters into the action is fixed by treating it as a spurion and then requiring terms to be invariant under chiral symmetry. Then one can add the mass term

$$\mathcal{L}_m = \Lambda F^2 \frac{a}{2} \mathrm{Tr}\left(M\Sigma^\dagger + \mathrm{hc}\right) = a\Lambda m_\alpha^2 \pi^\alpha \pi^\alpha + \cdots = \frac{1}{2} m_\alpha^2 \pi^\alpha \pi^\alpha + \cdots$$

This shows that the pion mass vanishes as the square root of the quark masses. Every power of M then counts as dimension 2. A mass term is a perturbation about the chiral limit, so the analysis of such terms are called chiral perturbation theory (χPT).

Problem 6.6

Using a tree level analysis of $\mathcal{L}_m$, and the known values of the pion, kaon and η masses, show that

$$\frac{m_u}{m_d} = 0.55, \qquad \frac{m_s}{m_d} = 20.1.$$

The quark condensate

Q^α are the generators of the broken symmetry $SU_A(3)$. We will evaluate the VEV of its commutator with the pseudoscalar density $\overline{\psi}\gamma_5\lambda^\alpha\psi$, in the chiral limit, where λ^α are the Gell-Mann matrices. In the symmetry broken vacuum

$$\langle 0| [Q^\alpha, \overline{\psi}\gamma_5\lambda^\beta\psi] |0\rangle = -\frac{2}{3}\delta^{\alpha\beta} \langle 0| \overline{q}q |0\rangle .$$

Since the symmetry is spontaneously broken, the quark condensate $\langle 0| \overline{q}q |0\rangle$ does not vanish. This is an order parameter for χSB.

Problem 6.7

How would you compute the numerical value of the quark condensate from low-energy data on hadron physics? Implement your computation. What value do you get? Why is there a mismatch?

Outline

Keywords and References

Keywords

Accidental low-energy symmetries; hard breaking of symmetry; softly broken symmetry; flavour-changing neutral currents; Isgur-Wise symmetry; custodial symmetry; conformal symmetry; Goldstone boson; vacuum expectation value; generating functional; effective action; effective potential; Goldstone modes; no tadpole corrections; pseudo-Goldstone bosons; chiral symmetry; explicit breaking of symmetry; helicity projection; anomaly; baryon number; spurion; pion decay constant; chiral perturbation theory; quark condensate.

References

D. B. Kaplan, *Five Lectures on Effective Field Theory*, nucl-th/0510023;
S. Weinberg, *The Quantum Theory of Fields*, Vol II.

Copyright statement

Copyright for this work remains with Sourendu Gupta. However, teachers are free to use them in this form in classrooms without changing the author's name or this copyright statement. They are free to paraphrase or extract material for legitimate classroom or academic use with the usual academic fair use conventions.

If you are a teacher and use this material in your classes, I would be very happy to hear of your experience. I will also be very happy if you write to me to point out errors.

This material may not be sold or exchanged for service, or incorporated into other media which is sold or exchanged for service. This material may not be distributed on any other website except by my written permission.