Extreme QCD

Sourendu Gupta

Tata Institute of Fundamental Research

March 28, 2008

1. Quantum Chromo Dynamics
2. The phases of QCD
3. Relativistic heavy-ion collisions
4. Some phenomena
5. Summary

Scales of mass

Can you use the standard model of particle physics to compute the mass of this system?

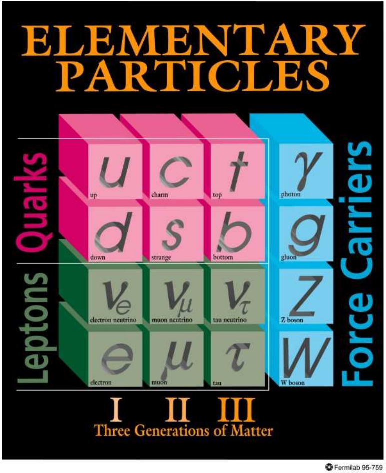
- Protein folding and other entropic terms $\sim$1–10 eV.
- Binding energy of electrons $\sim$1 KeV
- Rest mass of electrons $\sim$1 MeV
- Rest mass of protons, neutrons $\sim$1 GeV

Yes, an accuracy of about 1 part in 1000, should be possible.

The Avogadro number from particle physics

- The order of magnitude and the first three digits should come from the theory of strong interactions called **Quantum Chromodynamics** (QCD).
- The next two–three digits need the rest masses of quarks and the electron, and the **Higgs mechanism**. Unfinished agenda: the LHC experiments.
- The next two–three digits can be obtained with precision using Quantum Electrodynamics (QED). Recent progress in using QED for direct computation of bound state energies. Massive computations in atomic-molecular physics now standard.

Standard model particles



Two properties of QCD

In any quantum field theory, the vacuum is a polarizable medium. As a result, the charge of a test particle depends on the distance at which you measure it. Electrical charge is screened by the vacuum.

The QCD charge (colour charge) is anti-screened. As a result, at small distances the strength of the charge (α_s) goes to zero (**asymptotic freedom**). High energy particles have small Compton wavelengths, hence QCD is easy to handle at high energies.

At large distances the colour charge increases; as a result field strength grows. Infinite energy seems to be required to separate two colour charges to infinity (**confinement property**). The long-distance physics of QCD is complicated.

Lattice QCD

- In high-energy QCD, $\alpha_s \ll 1$ and every quantity can be expanded in a **perturbation series**

$$f(m, \alpha_s) = f_0(m) + f_1(m)\alpha_s + \frac{1}{2} f_2(m)\alpha_s^2 + \cdots$$

  QCD at long-distance is **non-perturbative**: $\alpha_s \gg 1$. Use a computer intensive method called **lattice QCD**.
- A quantum theory is a sum over all paths of a classical theory. Classical version of QCD is the analogue of Maxwell's equations (for gluons) and an equation for the motion of matter (quarks). Solve these equations repeatedly and sum over paths.
- Discretize equations in order to handle them: hence lattice. Recover proper results by taking lattice spacing to zero. This process is called renormalization. Need **supercomputers** to solve this problem.

Supercomputers

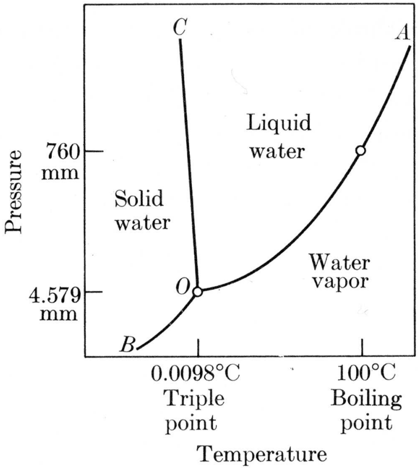
The Cray X1 at TIFR performs 2×10^{11} additions per second. Lattice QCD runs at an efficiency of over 80%.

Scaling of run times with current algorithms—

- faster than volume: V^{1+z_1} ($z_1 \simeq 1/4$)
- high power of pion mass: $m_\pi^{z_2}$ ($z_2 \simeq$2–6)
- very fast with lattice spacing a^{z_3} ($z_3 \simeq 7$)

Halving the lattice spacing at constant physics implies speed of computing required changes by factor 128. Karsch's law: affordable computing speed grows by an order of magnitude every 3 years. Algorithmic improvements needed.

The phase diagram of water



1. 3 extensive thermodynamic variables: S, V, U; *i.e.*, 3-d Gibbs' space.
2. Equilibrium systems: convex relation $U(S, V)$.
3. The phase diagram is two dimensional (T, P).
4. There are lines of first-order phase transitions (two-phase coexistence), ending in critical points, and isolated points of three-phase coexistence (triple points).

Thermodynamics of QCD

5-d Gibbs' space: Matter in thermal equilibrium is characterized by the energy E, the entropy S, and the value of every other conserved quantity. For QCD the conserved quantities are the baryon number, B, the electrical charge, Q, and the strangeness, S.

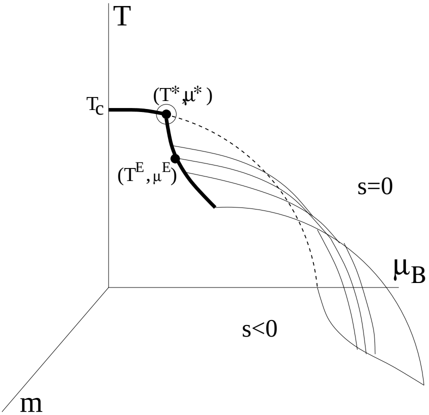
4-d phase diagram: One could describe matter in equilibrium also by the thermodynamic intensive quantities conjugate to these, *i.e.*, by the temperature, T, and three chemical potentials, μ_B, μ_Q and μ_S. One can draw the phase diagram in this 4-dimensional space.

7-d extended phase diagram: Phase diagram changes as the quarks masses are changed. There are three relevant quark masses: m_u, m_d and m_s. Hence the phase diagram can be extended to 7 dimensions. A theorist's luxury!

Phases of QCD

- At $T = 0$ and $\mu_B = 0$ the Fermi seas of q and $\bar{q}$ are level. Order parameters correspond to pairing of q and $\bar{q}$—- the chiral order parameter (s). The $T = 0$ ground state (vacuum) has a **scalar meson condensate**.
- Stressing such pairings by μ_Q, μ_S or differences in quark masses can give rise to **charged pion condensation**, or **Kaon condensation** (still $\bar{q}q$ pairings).
- These pairings persist until μ_B is large enough to break them.
- If $\mu_Q = \mu_S = 0$ then at larger μ_B pairing of quarks of different flavours likely: colour flavour locked state. Local colour symmetry is broken: Higgs mechanism, **colour Meissner effect** possible.
- Stressing by introducing μ_Q, μ_S or differences in quark masses produces interesting new superconducting phases.

The topology of the phase diagram

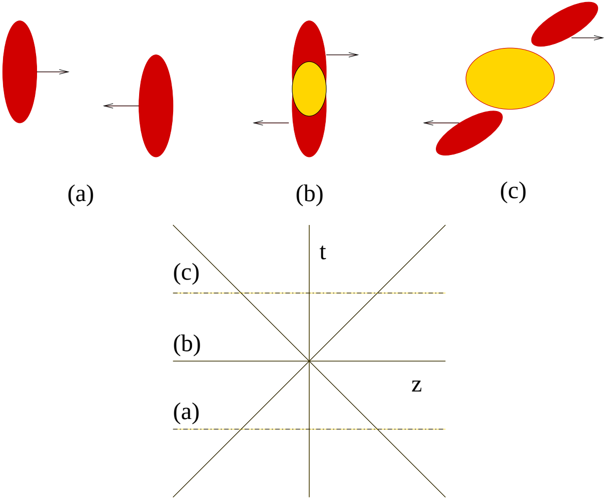


SU(2) symmetry: no distinction between u and d quarks. Common mass is m, common chemical potential is $\mu_B/3$. Topology amenable to analysis using simple models.

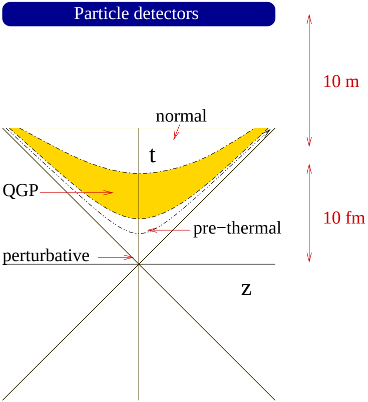
Computing the phase diagram

1. Lattice: only tool for computing location of phase transition.
2. Till 2000, all investigations for $T > 0$ and $\mu = 0$. Computation straightforward: Monte Carlo technique with importance sampling. No phase transition, **narrow cross over: $T_c = 190 \pm 5$ MeV**. Properties of matter on both sides of the cross over studied in detail.
3. At finite μ_B lattice computations difficult: importance sampling fails because weight is complex. Similar problems arise also in some problems in condensed matter physics. **Sign problem**
4. Several methods developed since 2000. The best is to use a series expansion in μ_B and locate the critical end point by examining the divergence of the series. **Our result: $T^E \simeq 0.95 T_c$ and $\mu_B^E \simeq T_c$**.
5. Other results in the same ballpark: μ_B^E / T^E =1–2. Push to smaller lattice spacing (a) important.

Heavy-ion collisions: kinematics



Heavy-ion collisions: evolution



- AGS (Brookhaven): Au, 10 GeV, ??– 1990
- SPS (CERN): S, Au, 20 GeV, 1987 – 1995
- RHIC (Brookhaven), d, Au, 60, 120, 200 GeV, 2000 – 2015
- LHC (CERN), Pb, 5.4 TeV, 2010 – ??
- FAIR (GSI Darmstadt), U, 10–40 GeV, 2015 –

Observables

- **Calorimetry**: measures energy densities reached. High enough to pass through the crossover.
- **Particle species abundances**: chemistry; signals possible chemical equilibration. Results from SPS (CERN) and RHIC (Brookhaven Lab) show low chemical potential.
- **Particle momentum distribution**: signals of hydrodynamic flow, hence of the equation of state. Very nice results from RHIC signal **almost** ideal fluid flow.
- **Hard particles**: may measure temperature, and give information on detailed properties of matter. Detailed results on "jet quenching" from RHIC test aspects of models of hot QCD matter.
- **Heavy quark mesons**: may signal deconfinement. Results are still to settle. LHC is the ideal machine for this.

Finding the critical point in experiment

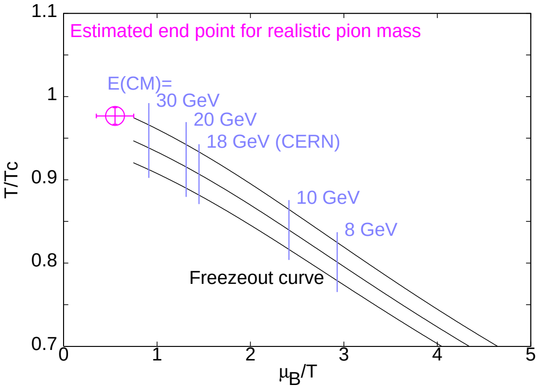


Figure: Statistics, quark mass, finite volumes, method, are now under control. Lattice spacing is the final frontier. In quenched computations this ratio increases with N_t (Gavai and Gupta, Phys. Rev. D 68, 034506, 2003). Continuum limit could be 25% larger, if quenched is an accurate guide.

Kinetic theory and hydrodynamics

- Navier-Stokes hydrodynamics does not obey relativistic causality.
- Example of diffusion equation: continuity equation obeys causality, Fick's law breaks it. Expand the kinetic theory in powers of $\tau\omega$ where τ is the mean-free time and ω is a typical frequency. **Fick's law replaced** by
$$\tau_R \frac{d\vec{j}}{dt} + \vec{j} = -D\nabla n$$
where n is the density, $\vec{j}$ the corresponding current and D the diffusion constant. This restores relativistic causality.
- Second order expansion needed for causal relativistic hydrodynamics in general: **replace Navier-Stokes** theory.
- Analysis of viscous flow in second-order theory has just begun. Transport coefficients computed in weak coupling in QCD and strong coupling in supersymmetric models.

Equation of state

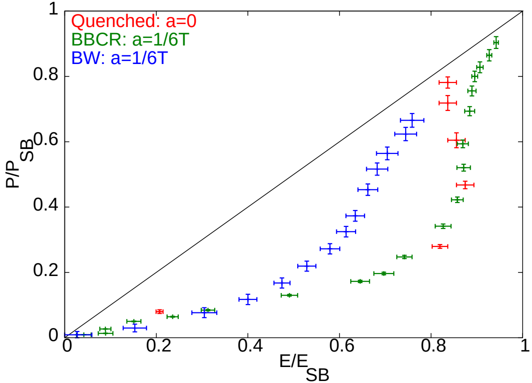
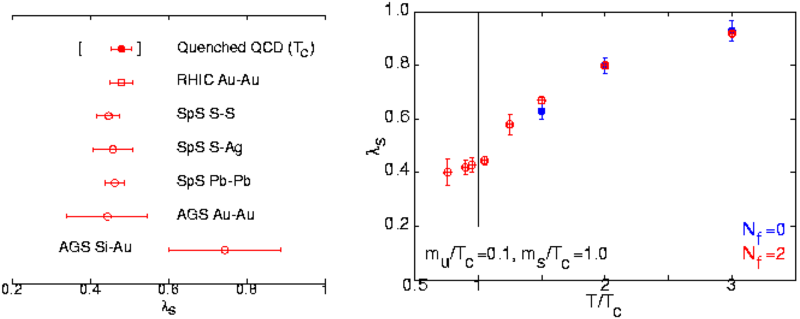


Figure: Diagonal line is $E = 3P$, *i.e.*, conformal symmetric. Supersymmetric theories lie 3/4 of the way up this line. Weak-coupling computations agree with lattice to extremely good accuracy for $T > 3T_c$. Pure gauge theory SB value scales as N_c^2, hence large-N_c limit is expected to (approximately) scale on this figure.

Chemistry: strangeness abundance



The Wroblewski parameter is the ratio of primary produced strange to non-strange quark pairs, and can be deduced from the abundances of various particles resulting from heavy-ion collisions. Lattice computations agree with experimental results.

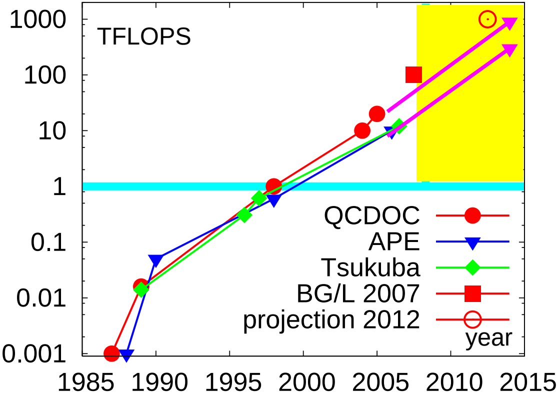
Credits

- **Phase diagram**: Collins, Perry, Wilczek, Rajagopal, Alford, Stephanov, Son, Creutz, SG, Berges, Halasz, de Forcrand, Philipsen, ...
- **Lattice thermodynamics**: Karsch, Fodor, Katz, Gavai, SG, Hands, Philipsen, de Forcrand, Laermann, Ejiri, Schmidt, Datta, Lombardo, Gupta, Stoltz, Mukherjee, Petersson, Cheng, ...
- **Heavy-Ion physics**: McLerran, Baym, Satz, Kajantie, Blaizot, Gyulassy, ...
- **Applications**: Ollitrault, Muronga, Heinz, Baier, Wiedemann, Bhalerao, SG, Cleymans, Redlich, Braun-Munzinger, Stachel, Braaten, Pisarski, Yaffe, Laine, Schroeder, Bodeker, Arnold, Moore, Son, Kovtun, Starinets, ...

Summary

- Long distance physics of QCD needs lattice methods. Computing demands are heavy: current state of the art is **100 Tflops**. In India (ILGTI) the largest sustained speeds are a few hundred Gflops.
- Educated guesses about the topology of the phase diagram are now being supplemented with lattice computations. These have to face the **fermion sign problem**. Some progress in this direction. First results on the critical end point of QCD obtained.
- Relativistic heavy-ion collisions probe the phase diagram of QCD. A clear 15 year program of future experiment**s**; strong Indian participation.
- Many theoretical approaches needed to connect QCD to experiment. Lattice results for chemical abundances and equation of state. Weak coupling theory for EOS at high T and transport. Non-perturbative models for transport.

Karsch's Law


TFLOPS
1000
100
10
1
0.1
0.01
0.001
1985
1990
1995
2000
2005
2010
2015
year
QCDOC
APE
Tsukuba
BG/L 2007
projection 2012

Critical end point

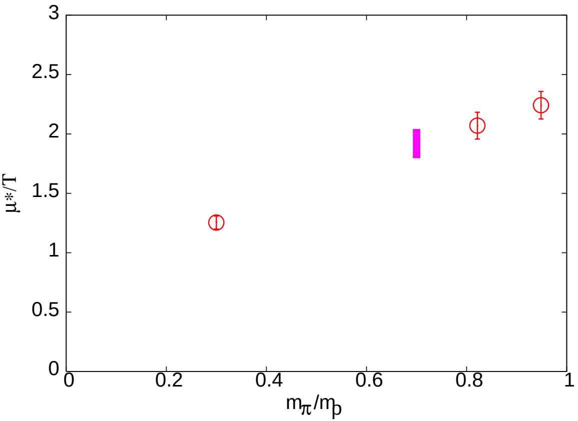


Figure: Gavai and Gupta, Nucl. Phys. A 785, 18, 2007. Bar is estimated from data in Allton *et al.*, Phys. Rev. D 71, 054508, 2005.

$SU(3)$ equation of state

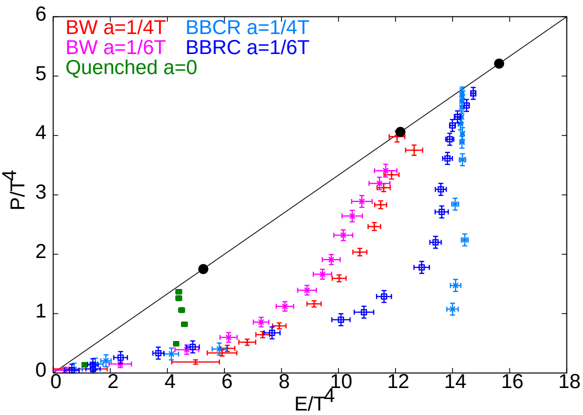


Figure: Quenched data from Gavai *et al.*, hep-lat/0506015; $N_f = 2+1$ data from Aoki *et al.*, hep-lat/0510084; Cheng *et al.*, arXiv:0710.0354 Peak $(E-3P)/T^4$ at $N_t = 8$ (HotQCD) drops by 20% from $N_t = 6$; no change for $T > 1.5T_c$.