# Scanning the phase diagram of QCD

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WHEPP XII Satellite Program: The Phase Diagram of QCD
VECC Kolkata India
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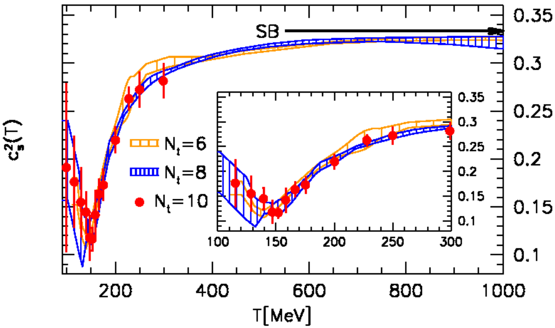
# Outline

# The QCD thermal cross-over

There is no phase transition in QCD at $\mu = 0$: gradual change from hadrons to quarks. Physically important: how fast does the fireball cool?




**Endrodi et al, arxiv:1007.2580**

Crucial question: what are the dof from 130 MeV$\leq T \leq$ 200 MeV?

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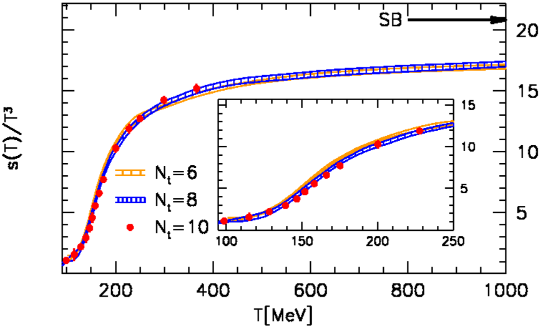


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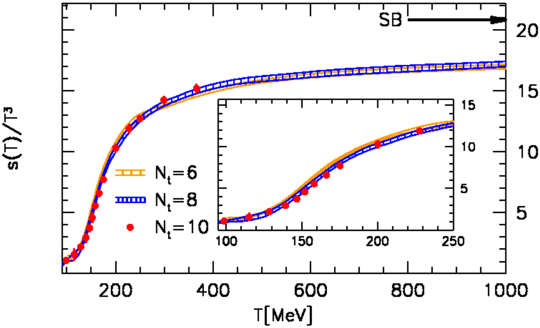


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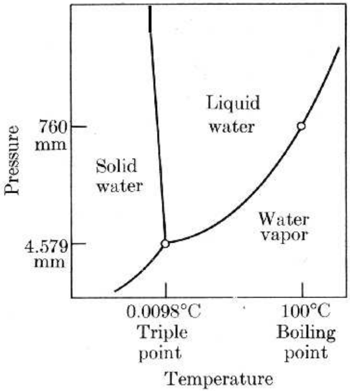
# The nature of a cross over




Phase diagram: map of the singularities of the free energy.
No singularity: blank.

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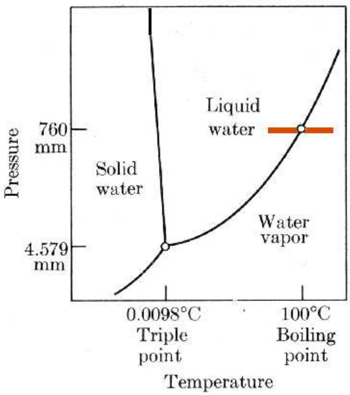


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First order: latent heat; second order: divergent susceptibilities or specific heat; cross over: neither.

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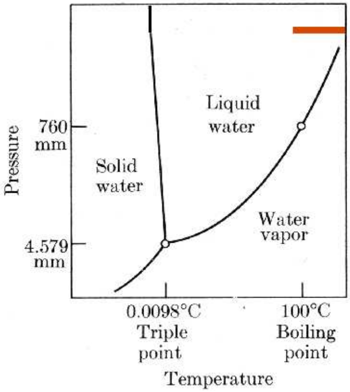


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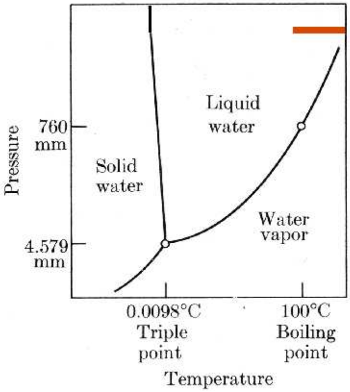


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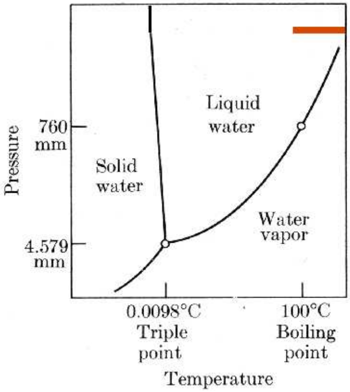


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For lattice gauge theory this is the Polyakov loop susceptibility. Its peak position is a (non-unique but definite) measure of the cross over

$$T_c \simeq 175 MeV$$

# Outline

# Thermodynamics and beyond

## Thermodynamics

Only extensive quantities treated in classical thermodyanamics. Twentieth century: micro-scale measurements begin, theory of fluctuations began. Only second moments treated in Landau and Lifschitz.

## Fluctuations

In heavy-ion collisions the number of particles $\ll N_A$. Theory of fluctuations must be extended: systematic finite size scaling theory. Closely related to nanophysics.

## What is gained

Thermodynamics forgets microscopic physics. Fluctuations keep track of macroscopic and microscopic physics simultaneously.

## Standardizing notation

Take a random variable $B$, with a probability distribution $P(B)$. The generating function is

$$Z(z) = \left\langle \mathrm{e}^{Bz} \right\rangle = \int dB \mathrm{e}^{Bz} P(B).$$

Moments are the Madhava-Maclaurin expansion coefficients of $Z(z)$—

$$Z(z) = \sum_n \langle B^n \rangle \frac{z^n}{n!}, \qquad \langle B^n \rangle = \left. \frac{d^n Z}{dz^n} \right|_{z=0}.$$

The characteristic function, $F(z) = \log Z(z)$. Cumulants are the expansion coefficients of $F(z)$—

$$F(z) = \sum_n [B^n] \frac{z^n}{n!}, \qquad [B^n] = \left. \frac{d^n F}{dz^n} \right|_{z=0}.$$

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# The shape variables

Standard shape variables for $P(B)$ are the cumulants:

$$[B], \quad [B^2], \quad [B^3] \quad [B^4], \cdots$$

Older texts have other shape variables: $\mu = [B]$, $\sigma^2 = [B^2]$, and

$$S = \frac{[B^3]}{[B^2]^{3/2}}, \qquad K = \frac{[B^4]}{[B^2]^2} - 3, \cdots$$

In the heavy-ion context ratios of cumulants are useful:

$$m_0 = \frac{[B^2]}{[B]}, \quad m_1 = \frac{[B^3]}{[B^2]}, \quad m_2 = \frac{[B^4]}{[B^2]}, \quad m_3 = \frac{[B^4]}{[B^3]}, \cdots$$

# Madhava-Maclaurin series method from Mumbai

Series expansion of pressure ($t = T/T_c$ and $z = \mu_B/T$):

$$\frac{1}{T}^4 P(t,z) = \frac{P(T)}{T^4} + \frac{\chi^{(2)}(T)}{T^2}\frac{z^2}{2!} + \chi^{(4)}(T)\frac{z^4}{4!} + T^2\chi^{(6)}(T)\frac{z^6}{6!} + \cdots ,$$

**Gavai, SG (2003)**

Derivatives give the successive “susceptibilities”:

$$\chi^{(1)}(t,z) = \frac{\chi^{(2)}}{T^2} z + \chi^{(4)}\frac{z^3}{3!} + T^2\chi^{(6)}\frac{z^5}{5!} + \cdots ,$$

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Series diverge at the critical point: can be used to estimate the position of the critical point:

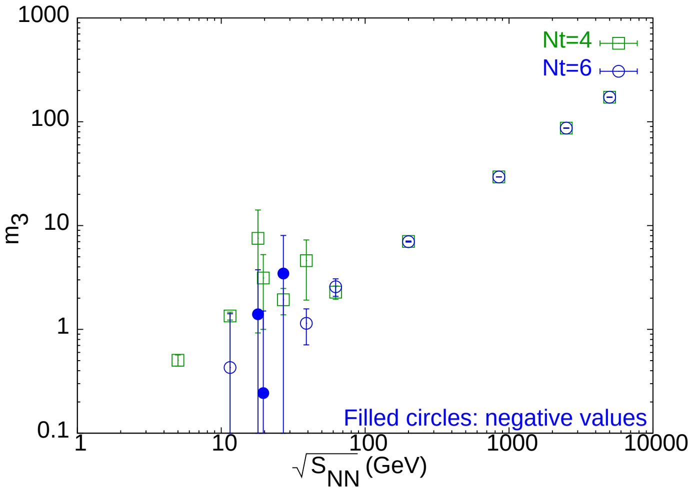
$$z_* = 1.8 \pm 0.1 \qquad \text{lattice cutoff 1.2 GeV}$$

**Gavai, SG (2008)**

Also tested for 3d Ising Model

**Moore, York (2011)**

# Lattice predictions along the freezeout curve

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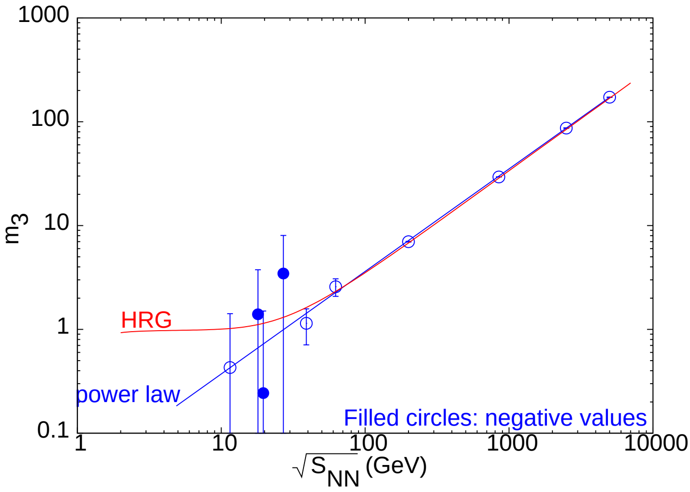


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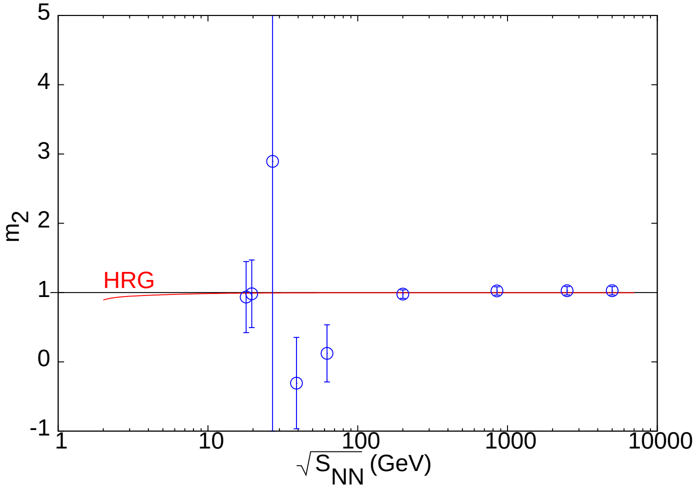


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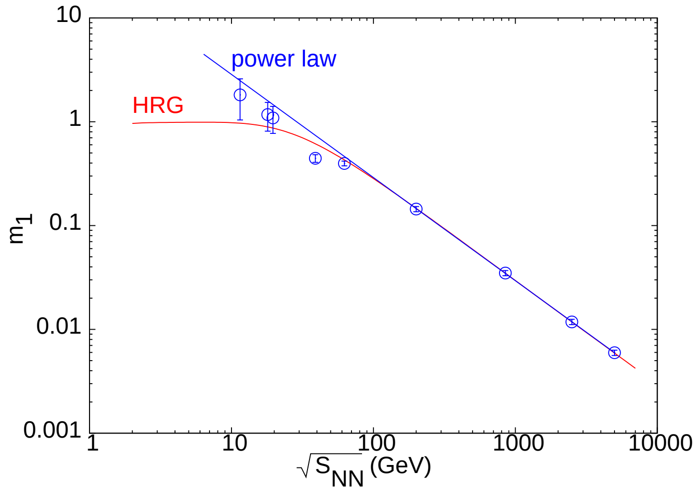


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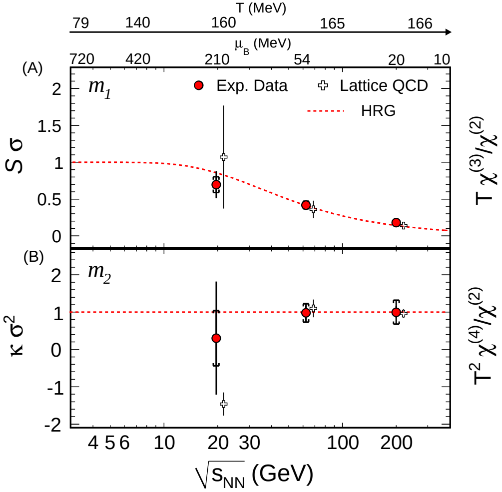
# Heavy-ion collisions




**Gavai, SG (2010); STAR (2010); GLMRX, Science (2011)**

# Outline

# Two earlier suggestions

If the critical point is far from the freezeout curve over a certain range of energy, then $m_1$ decreases with increasing $\sqrt{S_{NN}}$ (since $z$ decreases) and $m_3$ increases. Using these two measurements and comparing with lattice predictions, it is possible to estimate the freezeout conditions: $T/T_c$ and $\mu_B/T$. This method is independent of the usual one in which hadron yields are interpreted through a resonance gas picture [15]. Comparison of the two methods then allows us to estimate $T_c$ by inverting the argument of the previous paragraph. Mutual agreement of the values of $T_c$

so derived at different $\sqrt{S_{NN}}$ would constitute the first firm experimental proof of thermalization. If this proof holds then one also obtains the simplest and most direct measurement of $T_c$ found till now. Since such a thermometric measurement can be made reliably with data at large $\sqrt{S_{NN}}$, where $\mu_B$ is small, it would remain a valid measurement whether or not a critical point is found in the low energy scan at RHIC.

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## The first strategy

Use the chemical freezeout curve and the agreement of data and prediction along it to measure

$$T_c = 175^{+1}_{-7}\ \mathrm{MeV}.$$

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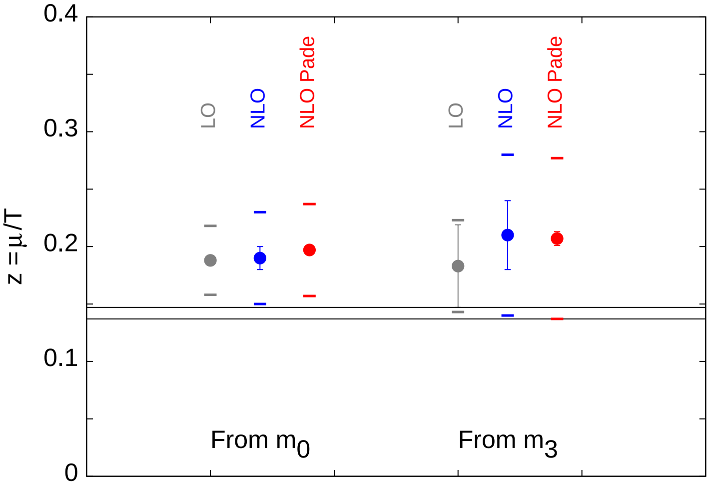
# The second strategy

Because of the critical divergence of $\chi^{(2)}(t,z)$, near the critical point the ratios of shape variables have poles as a function of $z = \mu/T$.

$$
\begin{aligned}
m_0 &= \frac{[B^2]}{[B]} = \frac{\chi^{(2)}(t,z)/T^2}{\chi^{(1)}(t,z)/T^3} = \frac{1+\mathcal{O}\left(\frac{z}{z_*}\right)^2}{z\left[1-3\left(\frac{z}{z_*}\right)\right]} \\
m_3 &= \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}(t,z)}{\chi^{(3)}(t,z)/T} = \frac{1+\mathcal{O}\left(\frac{z}{z_*}\right)^2}{z\left[1-10\left(\frac{z}{z_*}\right)\right]}
\end{aligned}
$$

Match lattice predictions and data (including statistical and systematic errors) assuming knowledge of $z_*$ to get estimates of freeze-in conditions.

# The second strategy: $\mu$metry



# Outline

# Critical point from the top RHIC energy

As before

$$m_0 = \frac{[B^2]}{[B]} = \frac{\chi^{(2)}(t,z)/T^2}{\chi^{(1)}(t,z)/T^3} = \frac{1+\mathcal{O}\left(\frac{z}{z_*}\right)^2}{z\left[1-3\left(\frac{z}{z_*}\right)\right]}$$

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Now fit $m_0$ and $m_3$ simultaneously to get both $z$ and $z_*$. Since $z_*$ is the position of the critical point: high energy data already gives information on the critical point!

From the highest RHIC energy using both statistical and systematic errors:

$$\frac{\mu^E}{T^E} \geq 1.7$$

# Three signs of the critical point

At the critical point $\xi \to \infty$.

## 1: CLT fails

Scaling $[B^n] \simeq V$ fails: fluctuations remains out of thermal equilibrium. Signals of out-of-equilibrium physics in other signals.

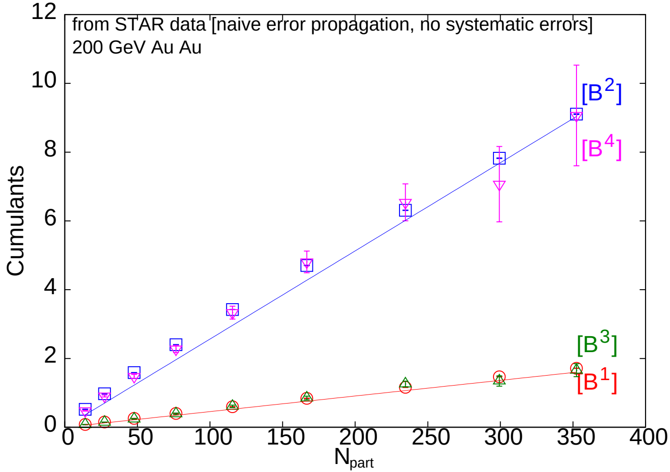
## 2: Non-monotonic variation

At least some of the cumulant ratios $m_0$, $m_1$, $m_2$ and $m_3$ will not vary monotonically with $\sqrt{S}$. If no critical point then $m_{0,3} \propto 1/z$ and $m_1 \propto z$.

## 3: Lack of agreement with QCD

Away from the critical point agreement with QCD observed. In the critical region no agreement.

# Evolution of shape

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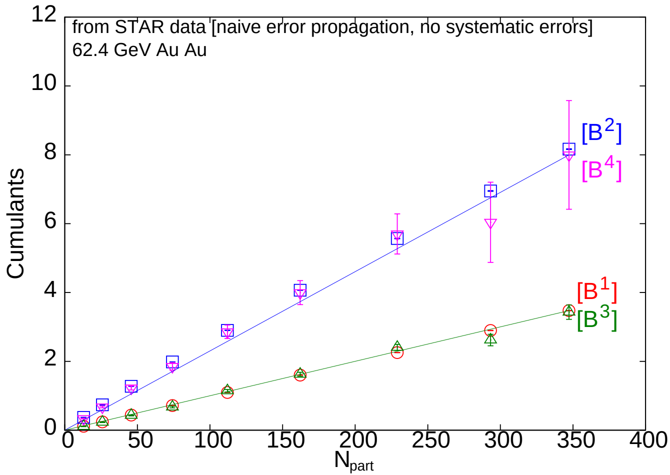


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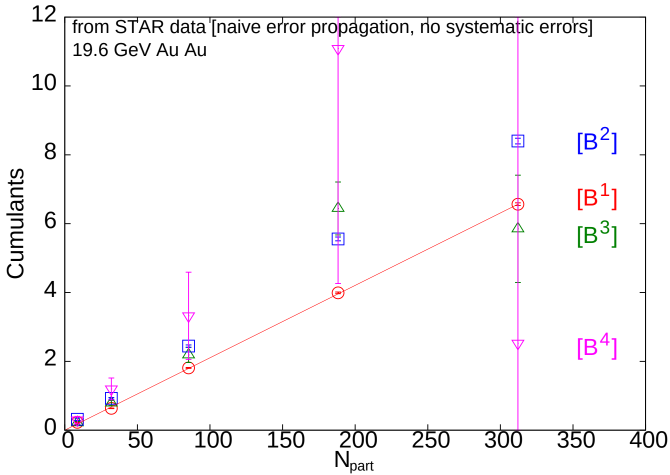


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# Outline

# Questions for investigation

1. Agreement between experiment and lattice allows us to go beyond old paradigms. For example: direct implication of high energy data on the critical point (if it exists).
2. Examine the BES critically: is thermalization lost in the fireball at some $\sqrt{S}$? If so, is this due to a long thermalization time or a short fireball liefetime? Long thermalization time is interesting: failure of CLT and non-Poisson fluctuations.
3. Resolve the physics of a cross over. Equation of state shows a gradual change [Schmidt]; QCD cross-over is broad; its physics is not just a single number. Implication for the degrees of freedom?
4. Meson-like correlators show little change in the cross-over region [Nikhil, Padmanath]. Baryon-like correlators change even before $T_c$ in quenched QCD [Padmanath]: probably therefore in unquenched QCD.