Lambda point phenomena

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1. Introduction
2. Learning from a model of QCD
3. The non-linear QNS in QCD
4. Summary

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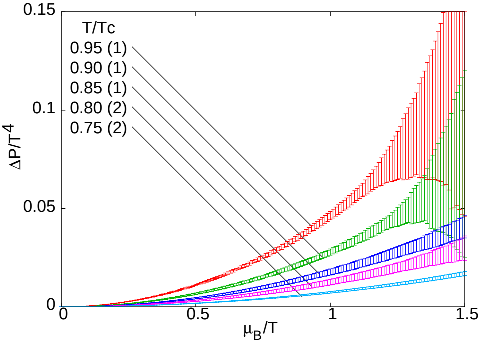
Context



SG, Karthik, Majumdar, 1405.2206

Resummed series expansion of pressure. How to check whether there is good control of the series coefficients, *i.e.*, non-linear QNS? One answer by Datta, Gavai, Gupta (Lattice 2013). Another answer in this talk.

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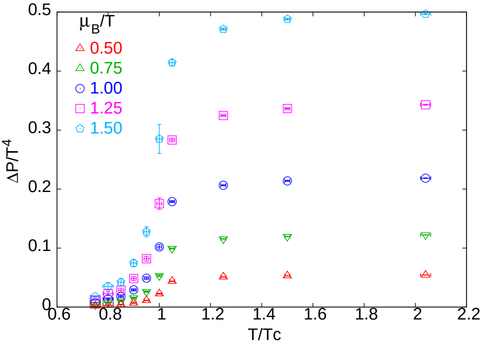

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Critical behaviour

The free energy of a system may be decomposed into a regular and a singular part,

$$F(T,m) = F_r(T,m) + F_s(T,m),$$

where the regular part, F_r, does not resolve anything singular at the critical point. The singular part, F_s has a scaling form

$$F_s(T,m) = t^{2-\alpha}\Phi(\tau), \quad \text{where } t = \left|1 - \frac{T}{T_c}\right|, \quad \tau = \frac{t}{m^{1/\beta\delta}}.$$

As a result,

$$c_V = \frac{\partial^2 F}{\partial T^2} \simeq t^{-\alpha},$$

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and the specific heat diverges at T_c with a critical exponent α as long as $\alpha > 0$.

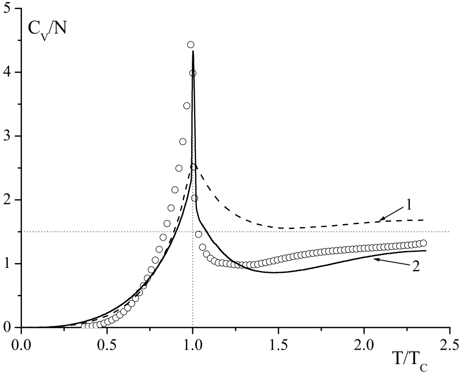
O(N) critical exponents in 3D

		β	δ	α	
O(∞)		1/2	5	-1	[Antonenko et al, 1995]
O(4)	chiral QCD	0.380	4.86	-0.2268	[Engels et al, 2000]
O(3)	?	0.365	4.79	-0.115	[Zinn-Justin et al, 1977]
O(2)	liquid He	0.349	4.78	-0.0172	[Engels et al, 2000]
O(1)	liquid-gas	0.325	4.8	0.11	[Zinn-Justin et al, 1977]
MFT		1/2	3	0	

Specific heat exponent, α, always negative for $N > 1$.

But, for liquid He, experiments show a peak in c_V at T_c. Why?

The λ point of liquid Helium



Vakarchuk, Pastukhov, Prytula, arxiv:1110.3941

When α is negative, contribution of F_s to $c_V(T_c) = 0$. So the value of $c_V(T_c) = 0$ is entirely due to the regular part. There is a cusp at T_c: non-anayIticity, must be due to F_s.

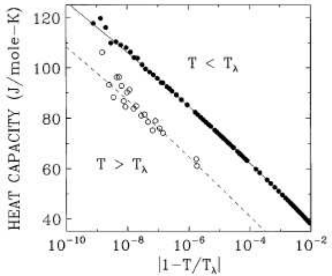
Critical indices from the λ point

When $|T - T_c|$ is small enough, contribution of regular part can be taken to be a constant. Then

$$c_V = A + t^{-\alpha}(B + Ct^{\Delta}),$$

where Δ is a possible sub-leading exponent. A is constrained to be positive, so B must be negative.

By taking explicit derivatives, it can be shown that it is possible to do this with $\Phi(\tau) > 0$. So the internal energy need not be negative.



Lipa et al, PRL 76, 944 (1996)

Space shuttle based experiment measured c_p for $|T - T_c| \leq 2$ nK. Found $\alpha = -0.01285(38)$.

Weirdness as N$\to \infty$

In the limit $N \to \infty$, it is seen that $\alpha = 1$. As a result, one expects

$$c_V \simeq A_r + B_s t + \cdots ,$$

with $A_r > 0$ and $B_s < 0$. The Taylor expansion of the regular term can also give a linear term, $B_r t$. So one has

$$c_V \simeq A_r + (B_r + B_s)t + \cdots .$$

Since the regular part may depend on non-universal terms, it may be possible to have O(N$\to \infty$) models which have no λ point.

It may be possible to have O(N$\to \infty$) models which have a specific heat minimum at c_V.

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The NJL model

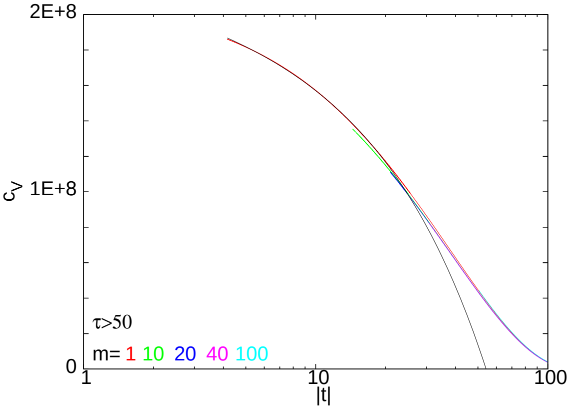
P/NJL models have the O(4) symmetry of QCD in the chiral limit. The quark mass m is the analogue of the magnetic field. Then

$$c_V(T,m) = A + t^{-\alpha}\Psi(\tau), \quad \text{where } t = \left|1 - \frac{T}{T_c}\right|, \quad \tau = \frac{t}{m^{1/\beta\delta}}.$$

T_c and other non-universal features change between NJL and PNJL, but universal critical features remain the same.

1. No singularities if the $T \to T_c$ at fixed m, *i.e.*, $t \to 0$ and $\tau \to 0$.
2. λ point can be seen only when $m \to 0$ before $T \to T_c$. Must take $\tau \to \infty$ first and then $t \to 0$.

The specific heat



Data collapse successful when small τ is removed. What controls how large τ should be? Combinations of F_π and $\langle\overline{\psi}\psi\rangle$.

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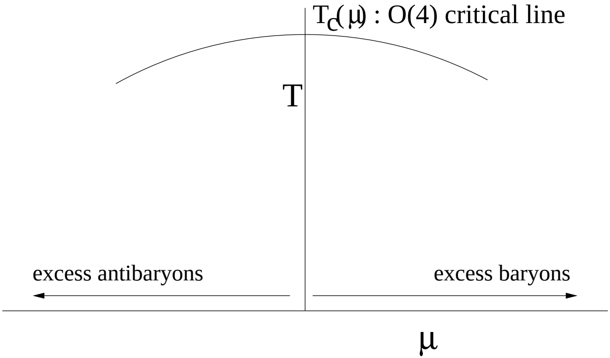
The phase diagram of chiral QCD



CP symmetry implies symmetry $\mu \leftrightarrow -\mu$. As a result, the critical line is even in μ—

$$T_c(\mu) = T_c + \frac{1}{2}\kappa\mu^2 + \cdots$$

The quark number susceptibilities

Then, for chiral QCD, if one assumes that the scaling function depends on μ only through $T_c(\mu)$, one finds

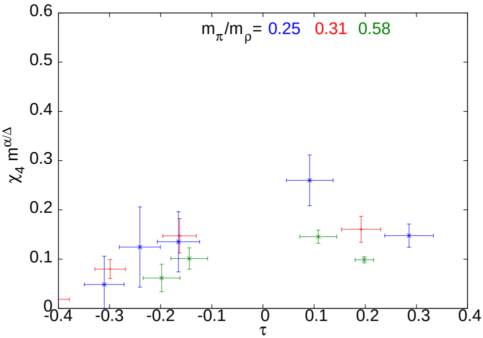
$$\left.\frac{\partial}{\partial t} g(t,\tau)\right|_{\mu=0} = \frac{2T_c}{\kappa}\frac{\partial^2}{\partial\mu^2} g(t,\tau)\bigg|_{\mu=0}$$

when the derivatives are applied to scaling functions. As a result, $\chi_4 \propto c_V$ in the chiral limit.

Caveat

The more complicated possibility of a new scaling variable $m(\mu)$, has not been ruled out. We use the simpler alternative for now.

Scaling via data collapse



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$N_f = 2$ QCD with $N_t = 4$, and O(4) exponents. Largest pion mass may be outside the scaling regime.

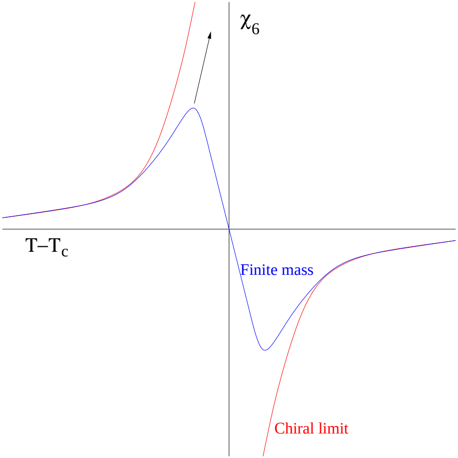
The QCD third order phase transition

$\alpha < 0$ but $1 + \alpha > 0$, so derivative of c_V is singular and universal. So QCD in the chiral limit has a third order phase transition!

Ehrenfest vs Wilson

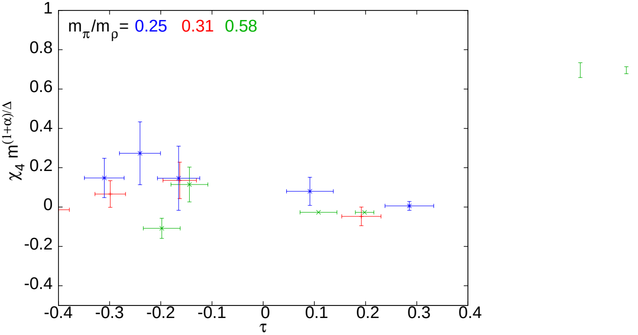
A century ago Ehrenfest proposed a classification of phase transitions depending on which order of derivative of F diverged. In the Wilsonian approach a transition is either critical or first order. If it is critical then the values of critical exponents determine which orders of derivatives diverge.

The scaling of χ_6 with m



Critical exponent: $1 + \alpha$, so regular contribution negligible for $m = 0$. Scaling analysis by data collapse possible.

Scaling via data collapse



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$N_f = 2$ QCD with $N_t = 4$, and O(4) exponents. Quality similar to χ_4.

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1. Negative specific heat exponent in QCD, as for liquid He or BES, leads to c_V with subtle interplay of singular and regular behaviour.
2. The shape of the phase diagram of QCD makes it possible to relate QNS (in limit $m \to 0$) to derivatives of free energy with respect to T. Jacobian measured by various lattice groups. More precision may be expected in future.
3. Scaling analysis of QNS measured in $N_t = 4$ simulations presented. Data collapse shows that scaling regime may be reached for $m_\pi/m_\rho < 0.35$. Range of data collapse similar for χ_4 and χ_6.
4. HRG has no singular contribution, and so must miss QCD values of NLS.