# Research Highlights

## • Exact Integral Equation for Physical Region Pion-Pion Scattering

An exact integral equation for pion–pion scattering involving only physical region partial waves was derived from axiomatic field theory [S.M. Roy, Physics Letters 36B, 353 (1971)]. This eqn. has proved to be valuable in the analysis of pion–pion data see e.g. T.P. Pool, Ph.D. Thesis, Groningen (1977) on "Roy's Equations for the Pion–Pion Scattering Amplitudes : Mathematical Properties and a Numerical Investigation"; A.C. Heemskerk, Ph.D. Thesis, Groningen (1978) on "Application of the N/D Method to the Roy Equation"; J. Gasser and G. Wanders, "one-channel Roy equations revisted", Eur. Phys. J. C10, 159 (1999); B. Ananthanarayanan, G. Colangelo, J. Gasser and H. Leutwyler, "Roy Equation Analysis of $\pi\pi$ scattering", Physics Reports 353, 207-279 (2001).

## • Unitarity Bounds on High Energy Cross Sections

The early results were summarised in the review article (S.M. Roy, Physics Reports 5C, No.3 (1972)). E.g.

Upper bound on particle–antiparticle total cross–section differences [S.M. Roy and V. Singh, Phys. Letters 32B, 50 (1970)]

$$|\sigma_{tot}^{\pi^- p} - \sigma_{tot}^{\pi^+ p}| \underset{s \to \infty}{\leq} \frac{\hbar}{m_\pi c} \sqrt{2\pi\sigma^{\pi^- p \to \pi^0 n}} \ell n(s/\sigma^{\pi^- p \to \pi^0 n}),$$

with $m_\pi$ = pion–mass. This bound has been tested first at the Serpukhov and then at the Fermilab accelerator. The left–hand side behaves as $s^{-(0.43\pm0.01)}$ and the right–hand side as $s^{-(0.57\pm0.03)}$ upto $p_{lab} = 200GeV/c$. The apparent violation of the bound (if the present experimental parametrizations continue to hold) raises important speculations.

## • Relativistic Collapse and Bosonic Chandrasekhar Mass

The $N$ particle Hamiltonian

$$\sum_{i=1}^{N} (\vec{p}_i^{\,2} + m^2)^{1/2} - \sum_{i<j\leq N} Gm^2/r_{ij}$$

implies a critical mass $M_{cr}$, a Bosonic analogue of the Chandrasekhar limit, beyond which there must be relativistic collapse. It was proved in [A. Martin and S.M. Roy, Phys. Lett. 233B, 407 (1989)] that

$$M_{cr} \geq 4[3\sqrt{3}Gm]^{-1} \sim 0.77(Gm)^{-1}.$$

This conflicts with the general relativistic estimate $M_{cr} \sim 0.633(Gm)^{-1}$ obtained from statistical Klein-Gordon field theory. A proper understanding of the discrepancy calls for further research on the general relativity effects.

## • Exponential Violation of Einstein-Bell Locality

Multiparticle Bell inequalities for general $n$ were obtained in, N.D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990) and S.M. Roy and V. Singh, Phys. Rev. Lett. 67, 2761 (1991) showing that quantum theory violates Einstein-Bell locality by a factor $2^{(n-1)/2}$ for a system of $n$ spin 1/2 particles. This has important consequences for quantum measurement theory, quantum-cryptography, and quantum computation.

## • Maximally Realistic Quantum Mechanics

Standard quantum mechanics (SQM) predicts probabilities of only one Complete Commuting Set (CCS) of observables in one experimental context and causal quantum mechanics are realistic extensions of SQM. It was shown that in $n$ space dimensions probabilities of $(n+1)$ CCS of quantum mechanics can be reproduced by one positive definite phase-space distribution of a causal quantum mechanics [S.M. Roy and V. Singh, Mod. Phys. Lett. A10, 709 (1995) ,S.M. Roy and V. Singh, Phys. Lett. A255,201 (1999)], and conjectured that a larger number of CCS cannot be so reproduced.

## • Phase Space Bell Inequalities and Quantum Marginal Theorems

The $N+1$ marginal problem in 2N-dimensional phase space has been solved.This proves a long standing conjecture. This was achieved by first deriving "phase space Bell inequalities" and then demonstrating that quantum mechanics violates them. Thus, they enable "experimental" tests of the orthodox-versus-hidden variable interpretations of quantum mechanics within the position-momentum sector, analogous to those performed within the spin sector. The new Bell inequalities are also useful in quantum information processing as quantitative tests of quantum entanglement. The most general positive phase space density which has the maximum number of marginals (N+1) coinciding with corresponding quantum probabilities of N+1 different (noncommuting) Complete Commuting Sets of observables has been constructed. These results open up new applications in continuous variable teleportation and in signal and image processing. [G. Auberson, G. Mahoux, S. M. Roy and V.Singh, Phys. Lett. A300,327 (2002); Journal of Mathematical Physics 44,2729-2747(2003); Journal of Mathematical Physics 45,4832-4854(2004)].

## • Quantum Anti-Zeno Paradox

An exact differential equation for continuous measurements of a Projection Operator with arbitrary time dependence is derived.The solution shows that generically, a quantum watched kettle is sure to change its state in a manner that depends on how it is watched but not on the system Hamiltonian. [A.P.Balachandran and S.M. Roy, Phys. Rev. Letters 84, 4019 (2000)].

## • Tests of Quantum Entanglement

Multipartite quantum separability inequalities exponentially stronger than Bell-Mermin-Roy-Singh local reality inequalities were derived. Their violation would signal quantum entanglement [ S. M. Roy, Phys. Rev. Letters 94, 010402 (2005)].

## • Preserving Quantum Coherence

We developed a bang-bang control type algorithm to preserve quantum states using unitary inverting pulses at chosen time intervals.In this algorithm the number of pulses required to keep the quantum system in the same subspace upto time $T$ with probability $\geq 1 - \epsilon$ increases only as $T \exp[sqrt \log(T^2/\epsilon]$, for large $T$ and small $\epsilon$, whereas it varies as $T^2/\epsilon$ for the quantum Zeno effect. [D. Dhar, L. K. Grover and S. M. Roy, Phys. Rev. letters,96,100405 (2006)]