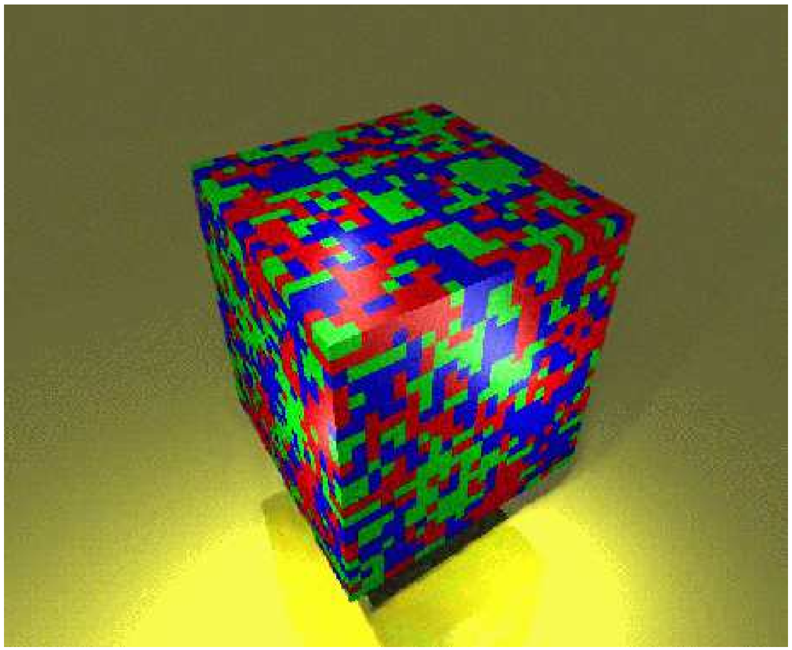
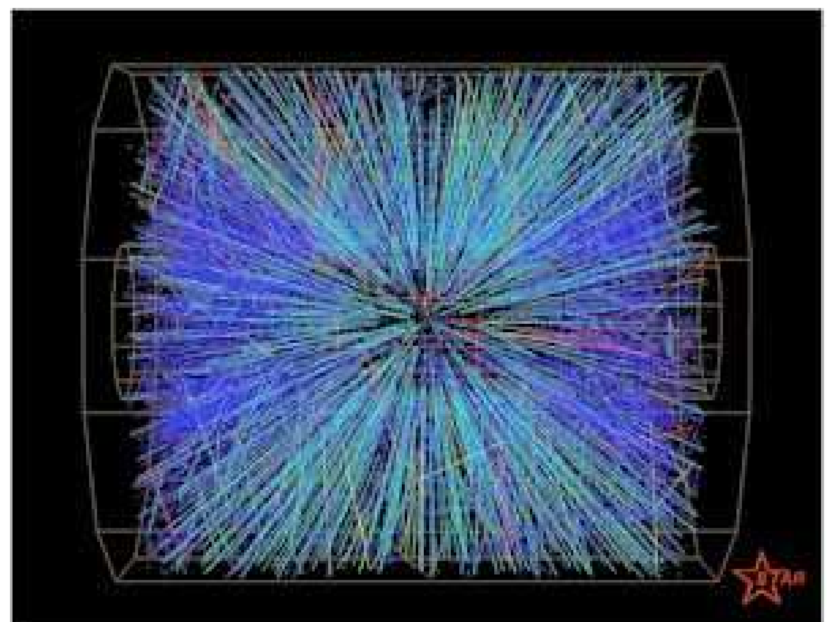
Lattice Gauge Theory and Heavy Ion Collisions





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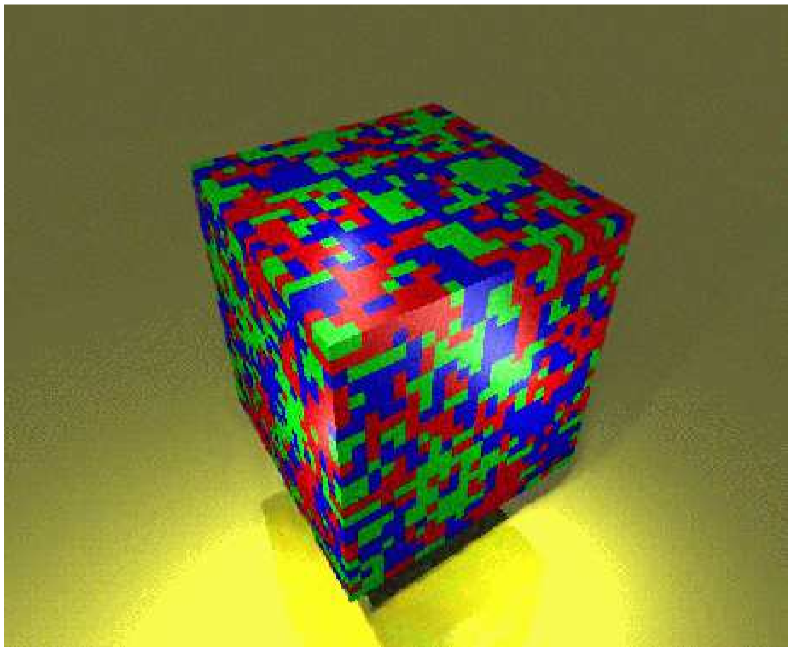
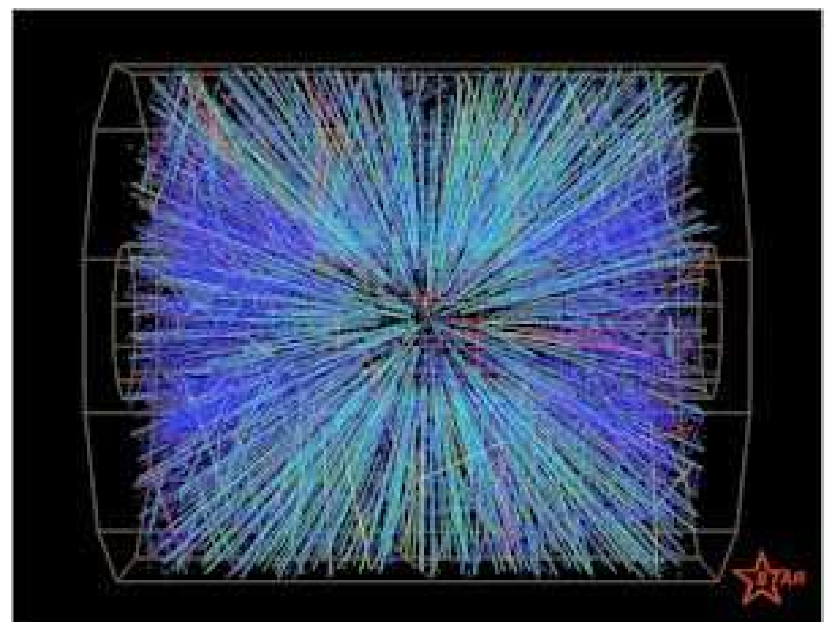


LGT:

- equilibrium thermodynamics of QCD;
- formulated in terms of basic degrees of freedom: quarks and gluons;
- observables expressed in terms of temperature and chemical potential



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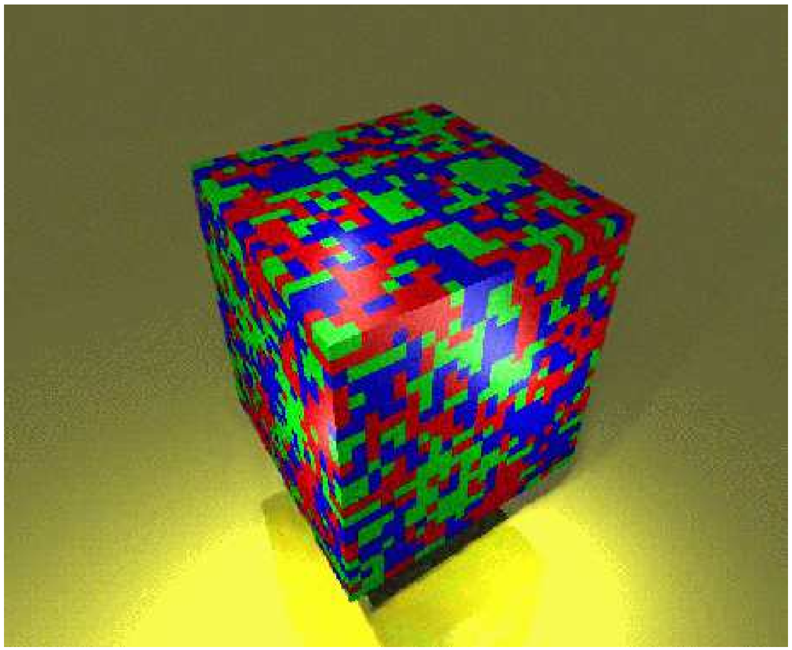

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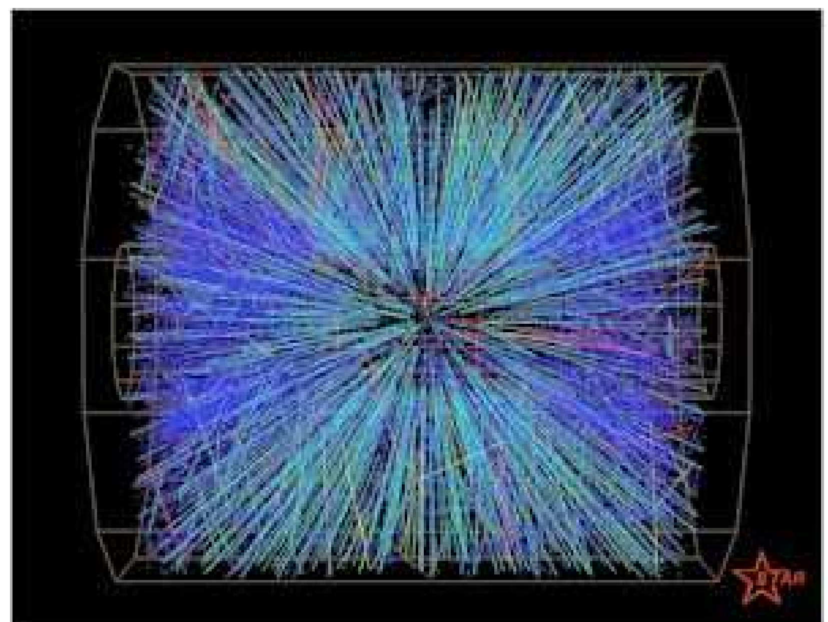
- evolution of a dense interacting medium described by QCD;
- observable properties in terms of hadrons, leptons and photons;
- observables parametrized in terms of energy and particle multiplicities



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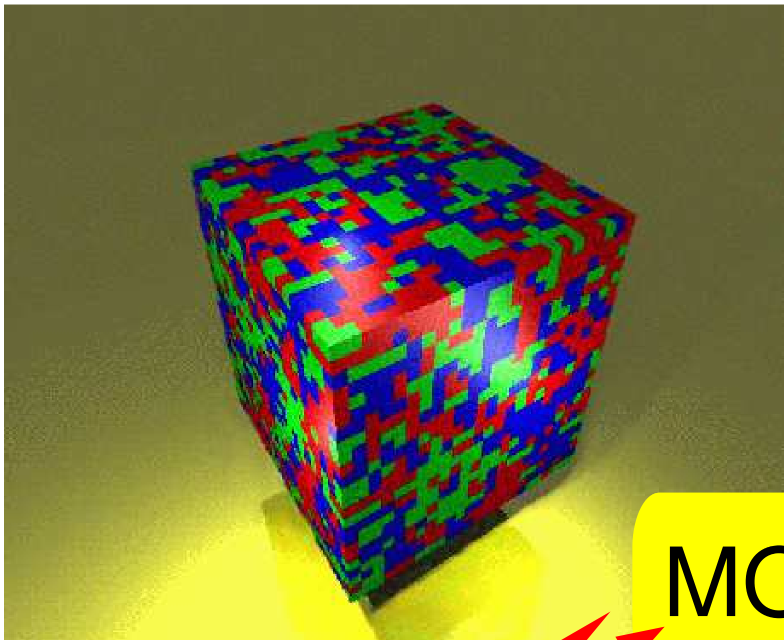

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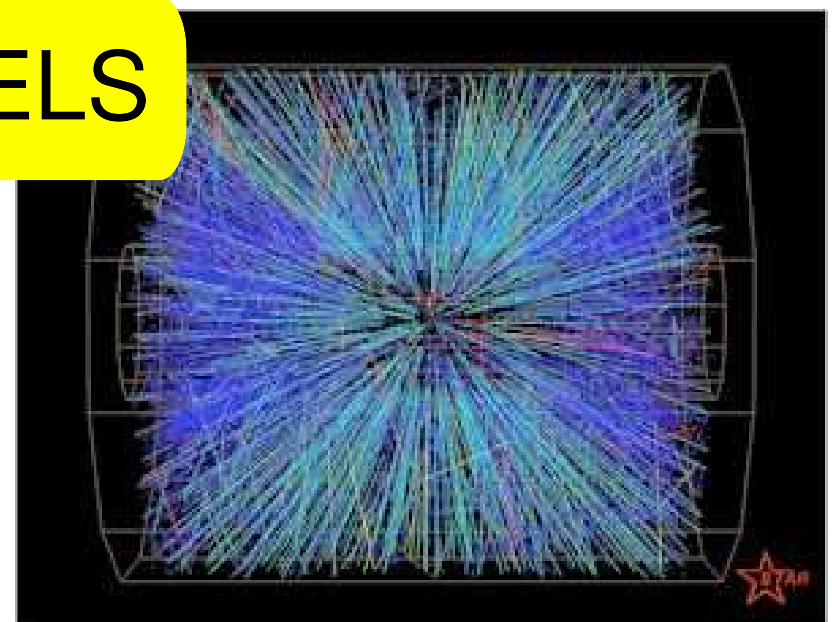
MODELS



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Lattice Gauge Theory and Heavy Ion Collisions

– exploring the properties of hot and dense matter

– testing QCD in extreme conditions

Quark gluon plasma

What are the properties of this new form of matter?

(equation of state, screening)

Critical behavior in dense matter

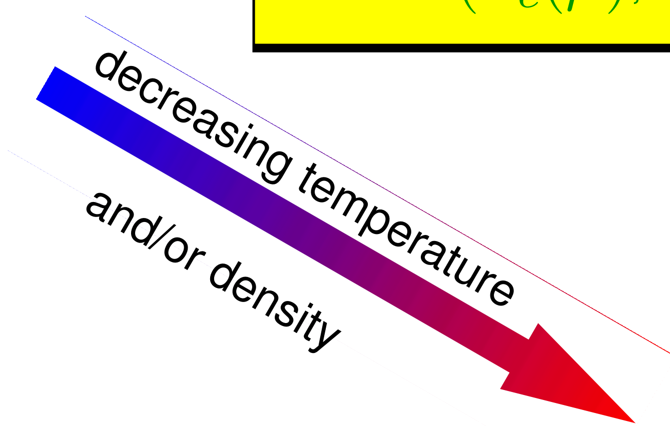
What are the critical parameter of the transition to the QGP?

$$(T_c(\mu), \epsilon_c)$$

Dense hadron gas

What happens to resonances in a dense hadronic gas?

$$(m_H(T,\mu), \Gamma(T,\mu))$$



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Outline:

- results from and limitations of current lattice calculations: T_c, EoS, phase diagram, fluctuations, dilepton rates, quarkonium

PHYSICAL REVIEW D VOLUME 8, NUMBER 12 15 DECEMBER 1973

Highly Excited Nuclear Matter*

G. F. Chapline, M. H. Johnson, E. Teller, and M. S. Weiss
Lawrence Livermore Laboratory, University of California, Livermore, California 94550

(Received 4 September 1973)

It is suggested that very hot and dense nuclear matter may be formed in a transient state in "head-on" collisions of very energetic heavy ions with medium and heavy nuclei. A study of the particles emitted in these collisions should give clues as to the nature of dense hot nuclear matter. Some simple models regarding the effects of meson and N^* production on the properties of dense hot nuclear matter are discussed.

What will be the effect of higher resonances? Models of the strong interactions based on the "bootstrap" idea lead to a density of states that increases exponentially with mass. This results from the fact that each new resonant state can combine with particles of lower or equal mass to make more resonant states.[8] In particular, the statistical bootstrap model leads to a density of states of the form[9,10]

$$N(m) = C m^{-3} e^{m/\theta_0}, \tag{3}$$

where θ_0, the "maximum temperature" of hadron matter, is about 174 MeV as determined from high-energy scattering experiments.[5] The param-

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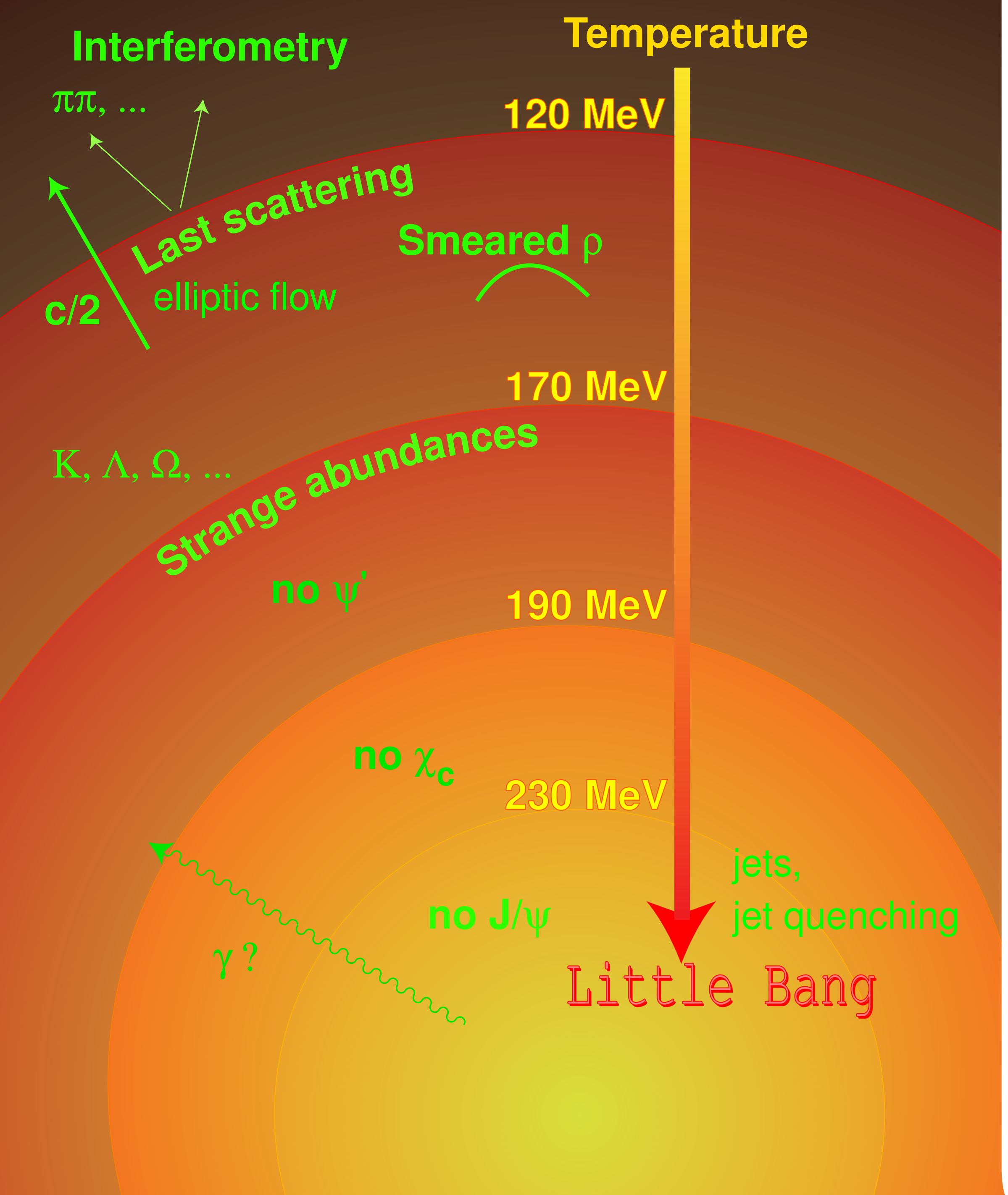
$$T_c \simeq 174\ MeV \quad (!!!)$$

Towards A New State of Matter

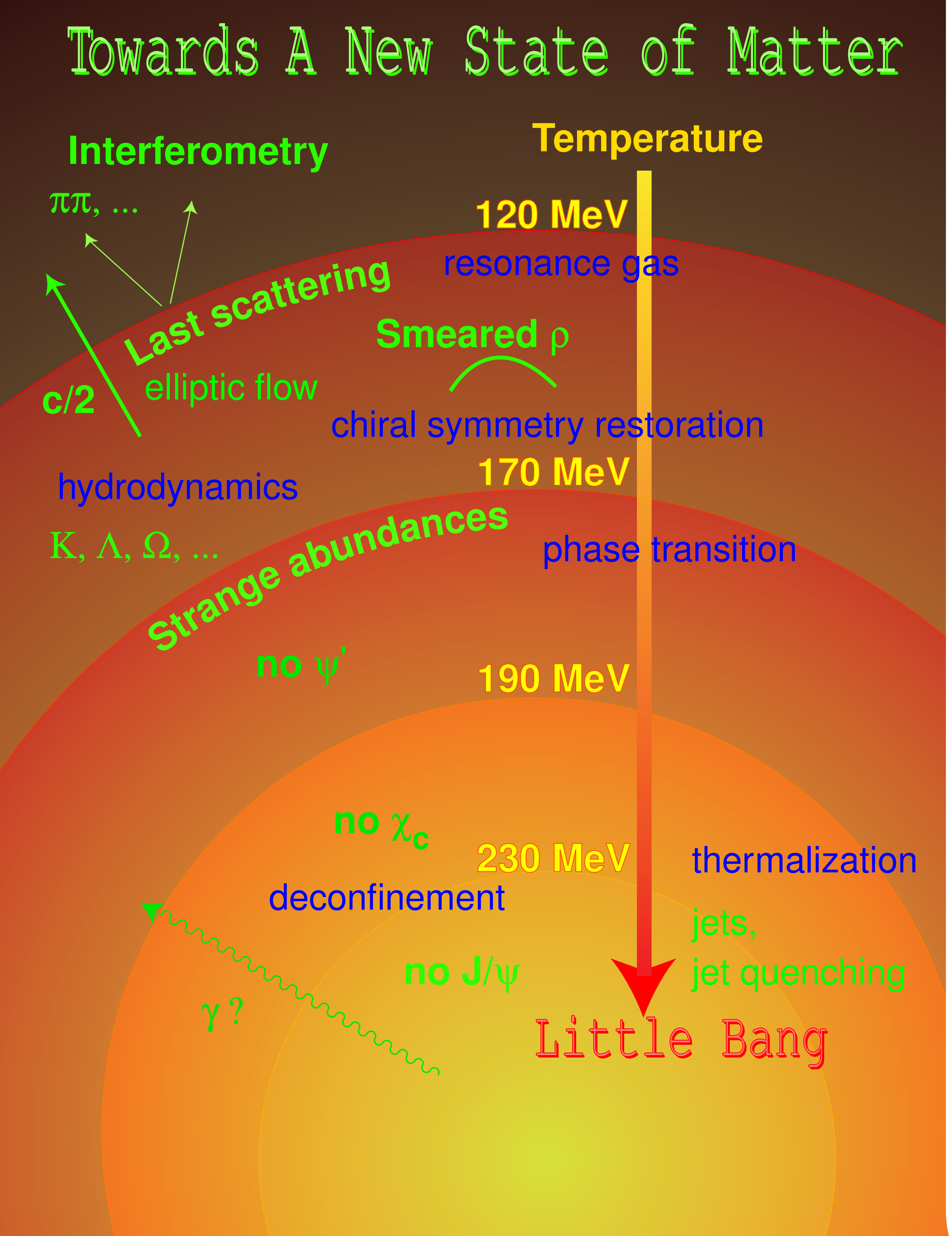


Where lattice calculations do/will contribute to the

development of theoretical concepts

and the

analysis of experimental observables



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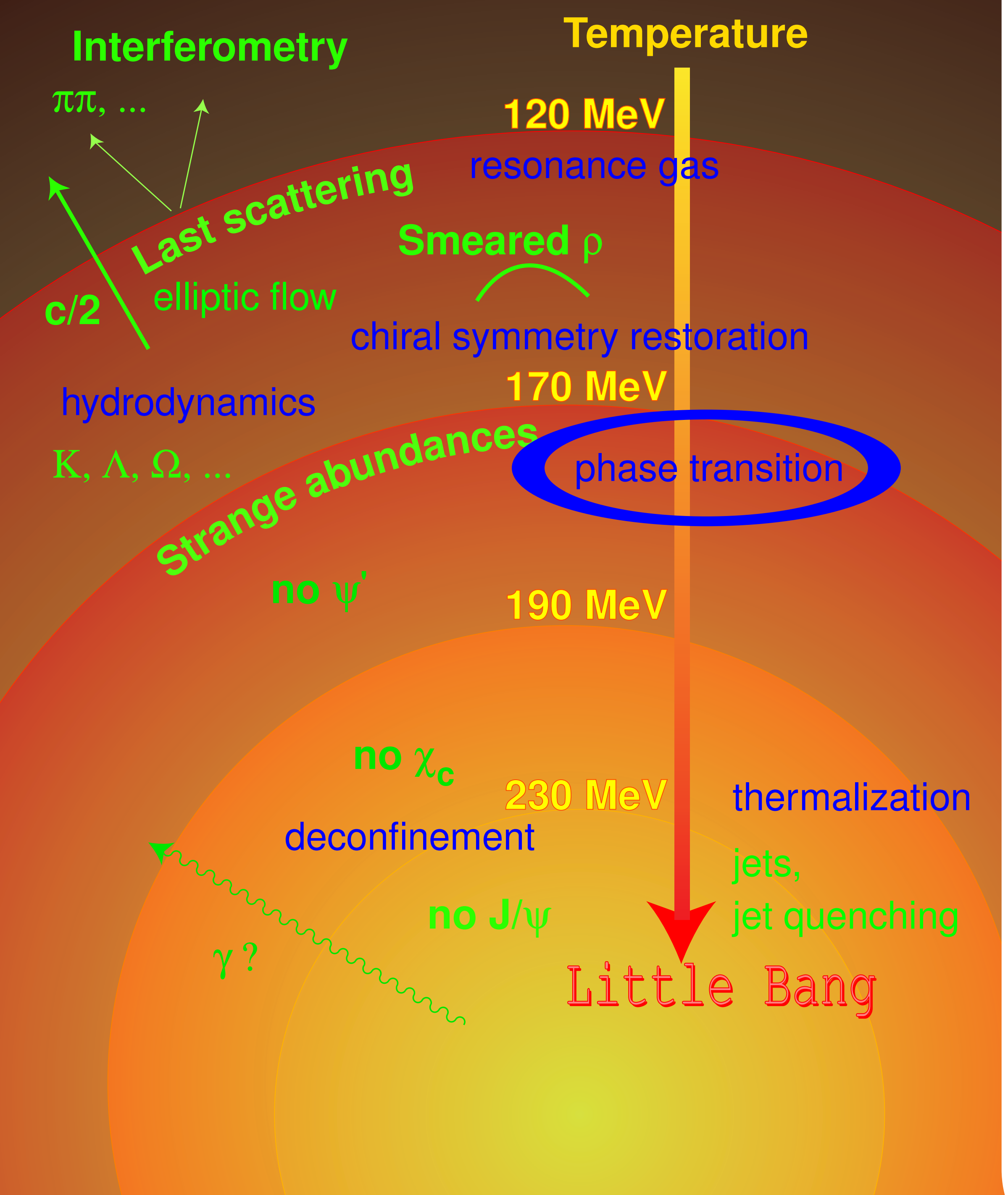

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$T_c,\ \epsilon_c$

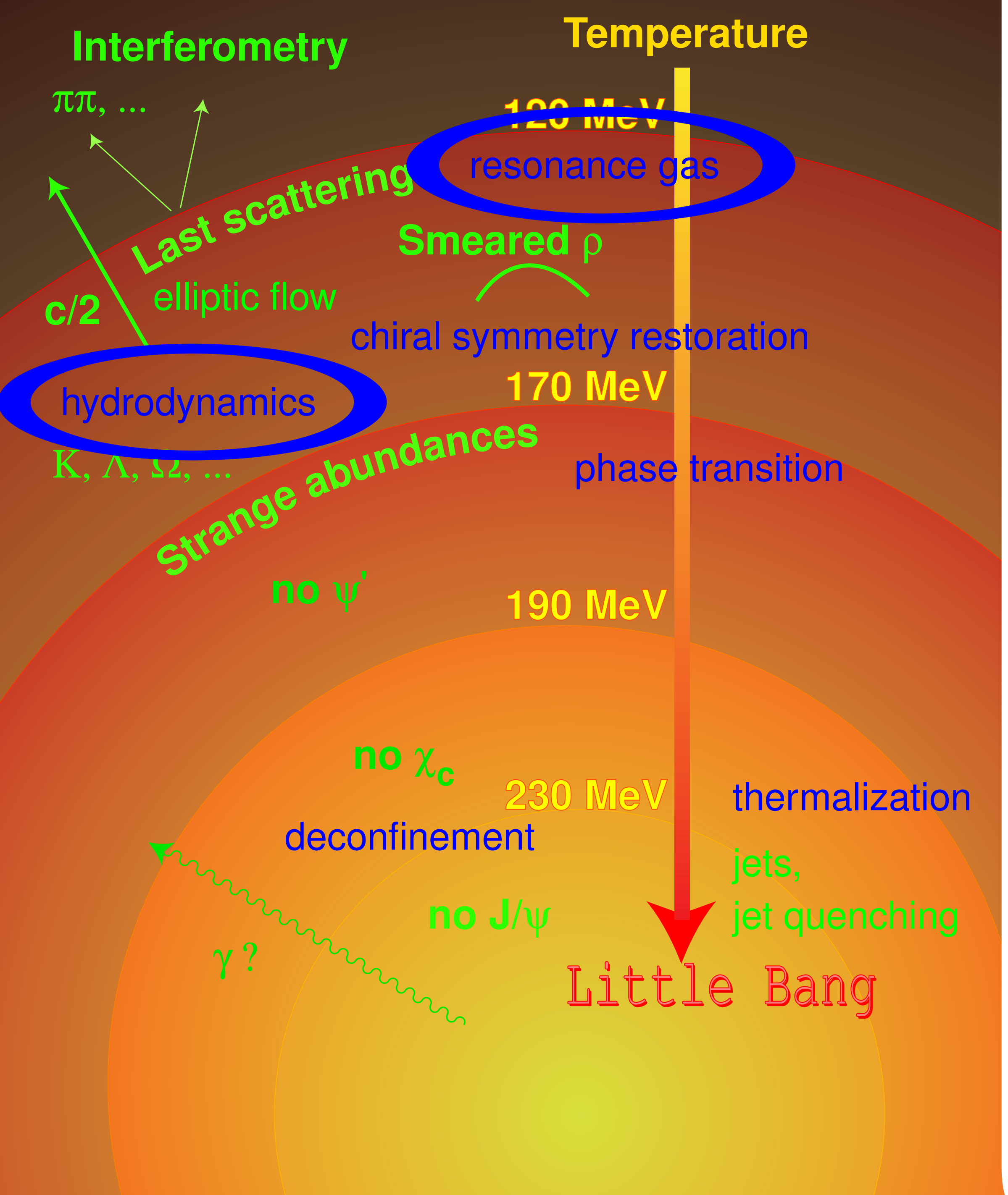
phase diagram in the $(T,\ \mu_B)$-plane;

$\mu \simeq 0$: RHIC (LHC)

$\mu > 0$: SPS (GSI future)

chiral critical point

Towards A New State of Matter



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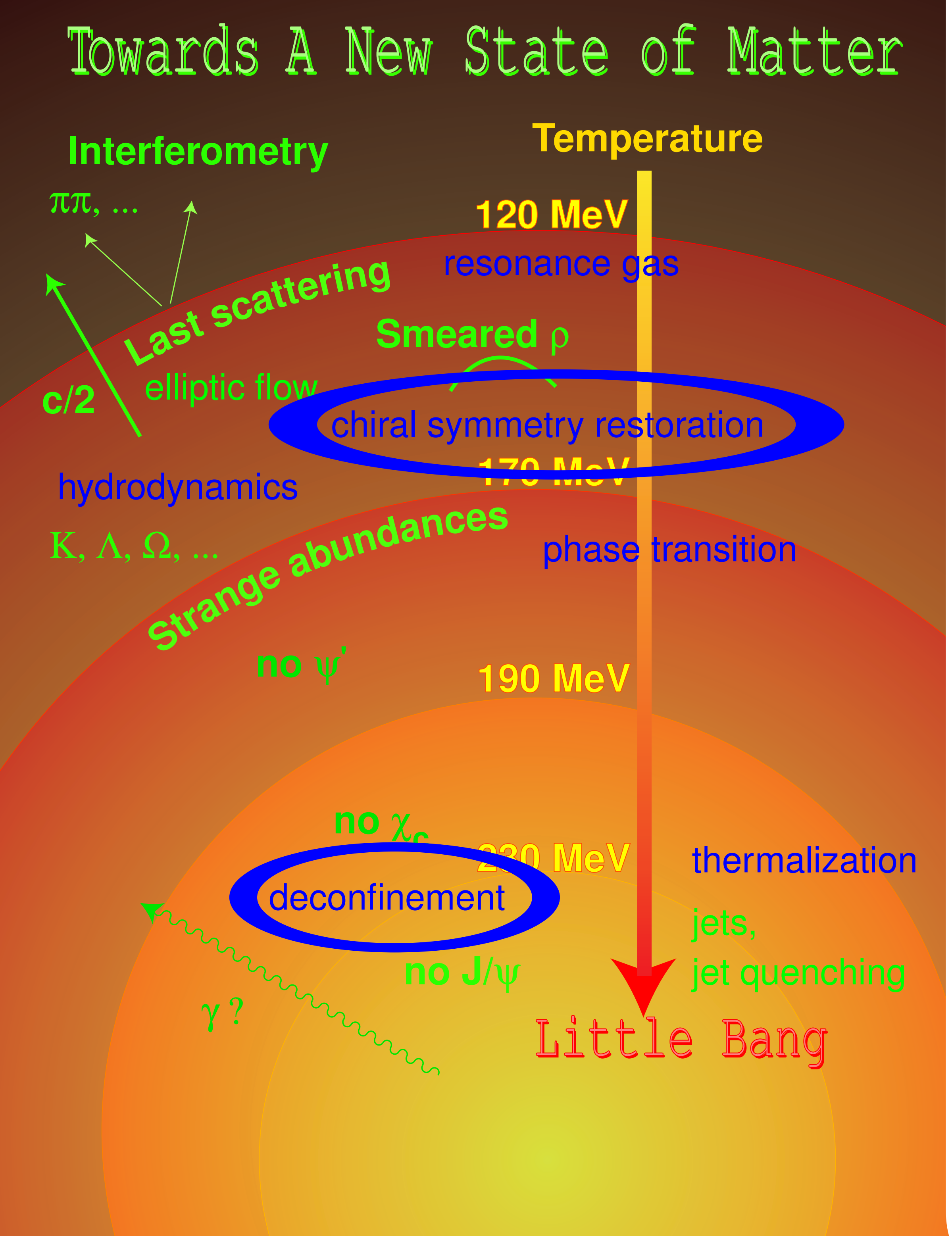
EoS

energy density, pressure, velocity of sound,...; susceptibilities (baryon number fluctuations);

strangeness contribution



Where lattice calculations do/will contribute to the

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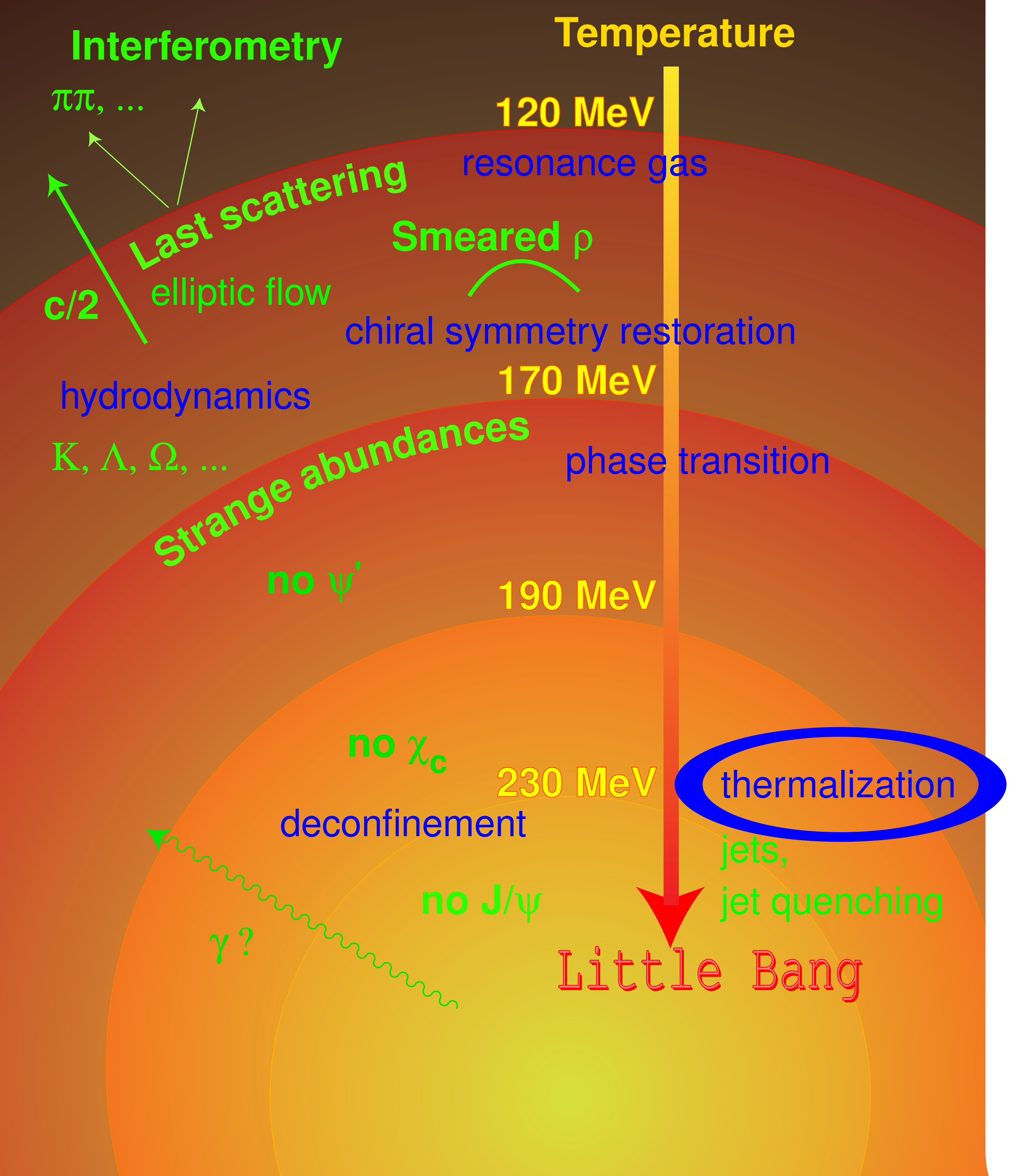
$$In - medium$$
$$hadron\,properties$$

heavy quark potential, screening;
charmonium spectroscopy;
light quark bound states;

thermal dilepton rates

Towards A New State of Matter



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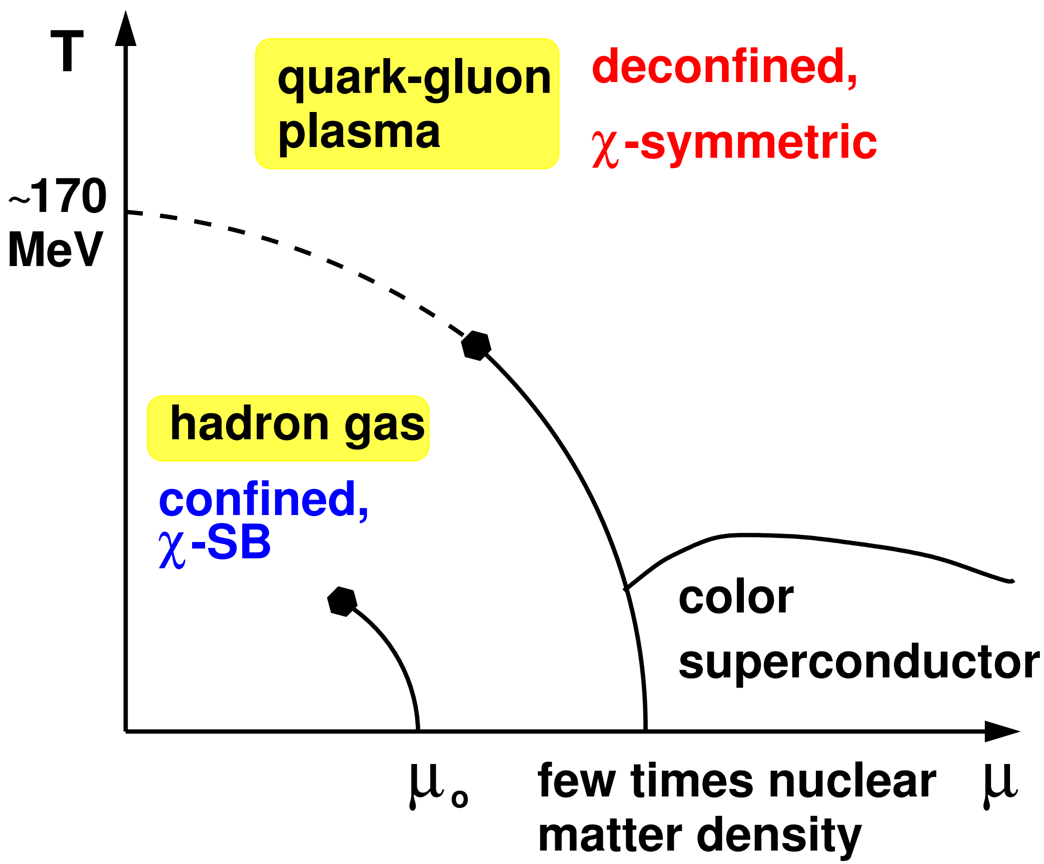
short vs. long distance physics

running coupling constant;
transport coefficients (??)

Critical behavior in hot and dense matter: QCD phase diagram

crossover vs.

phase transition



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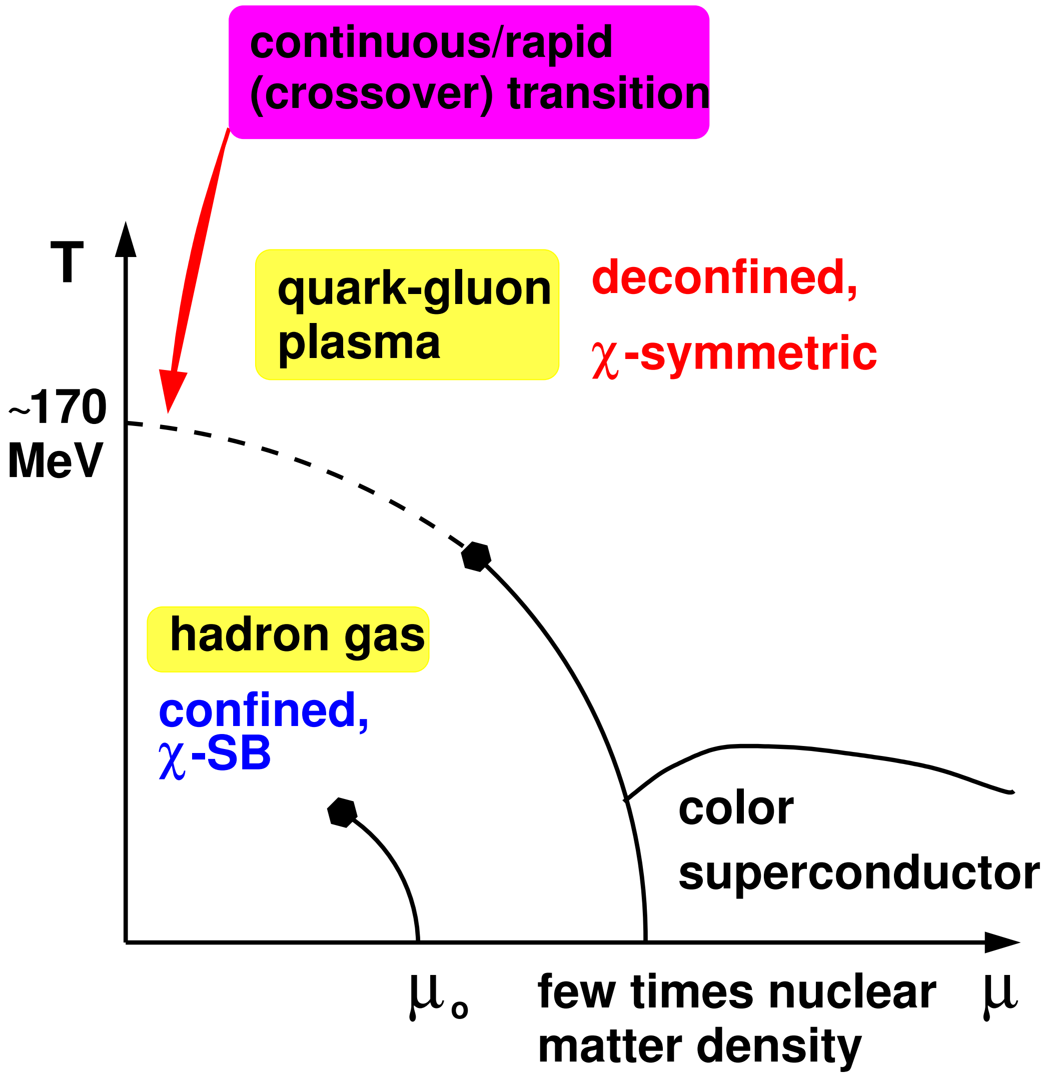
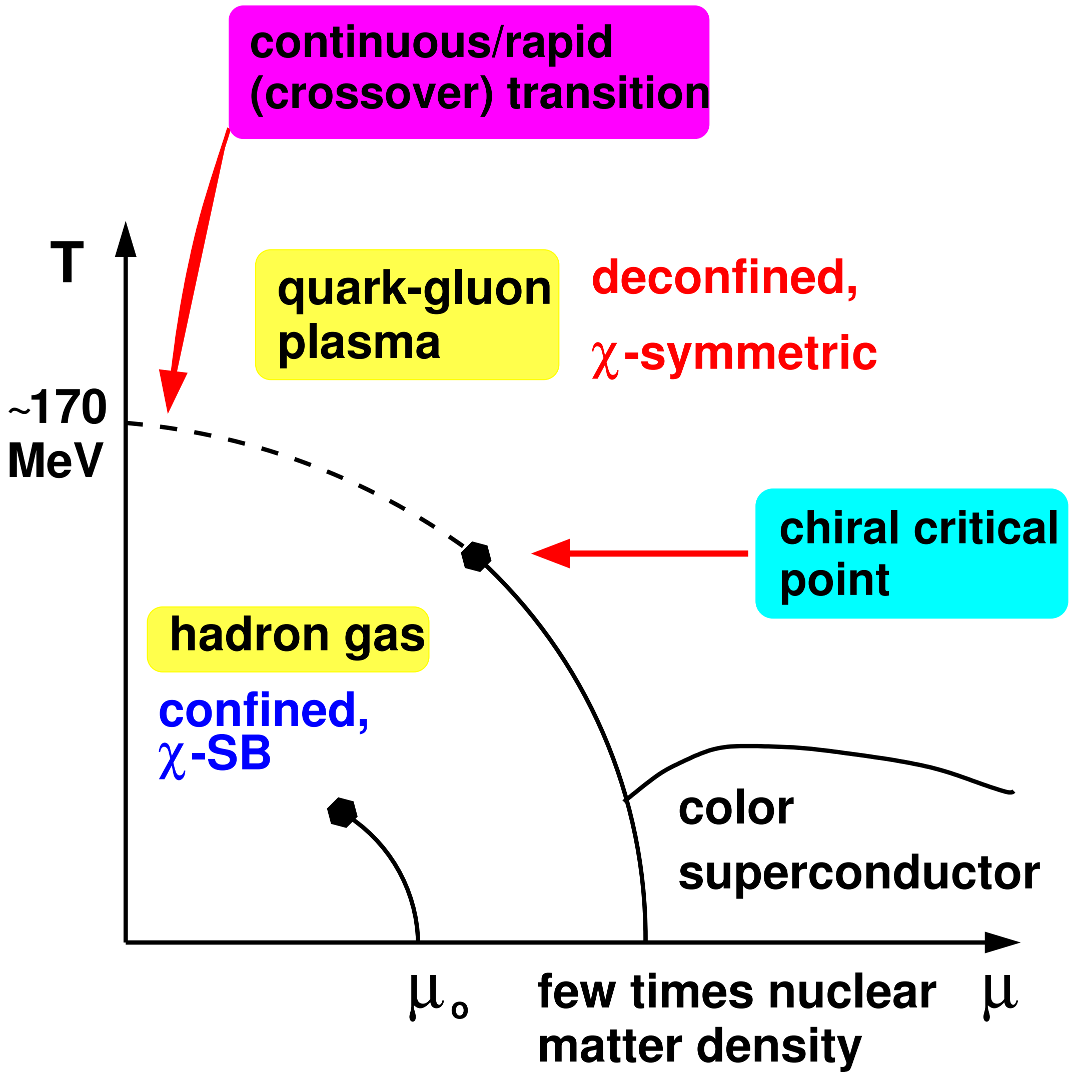


continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170\, MeV$$
$$\epsilon_c \simeq 0.7\, GeV/fm^3$$

Critical behavior in hot and dense matter: QCD phase diagram



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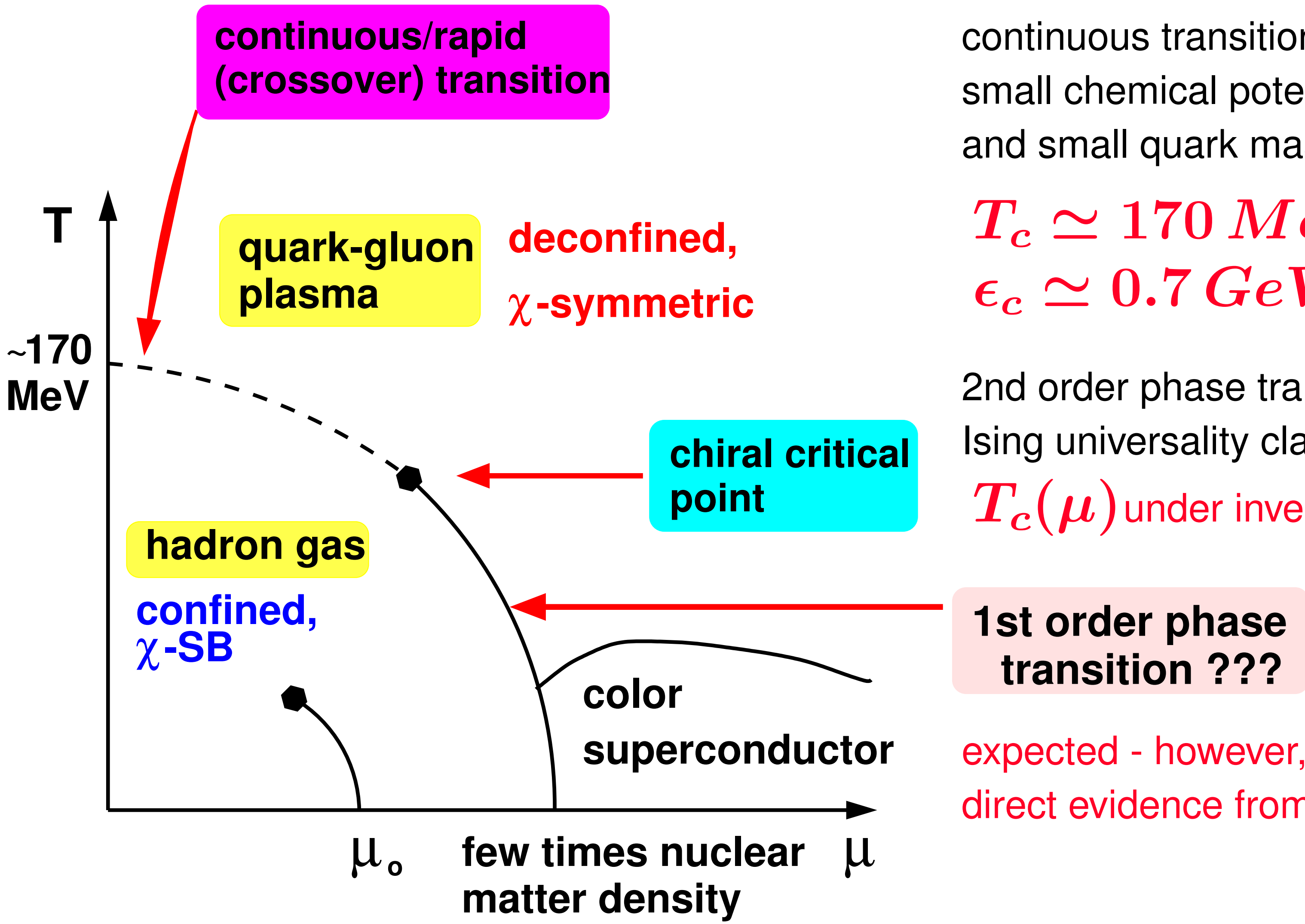
2nd order phase transition;
Ising universality class
$T_c(\mu)$ under investigation

Critical behavior in hot and dense matter: QCD phase diagram



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expected - however, so far no direct evidence from lattice QCD

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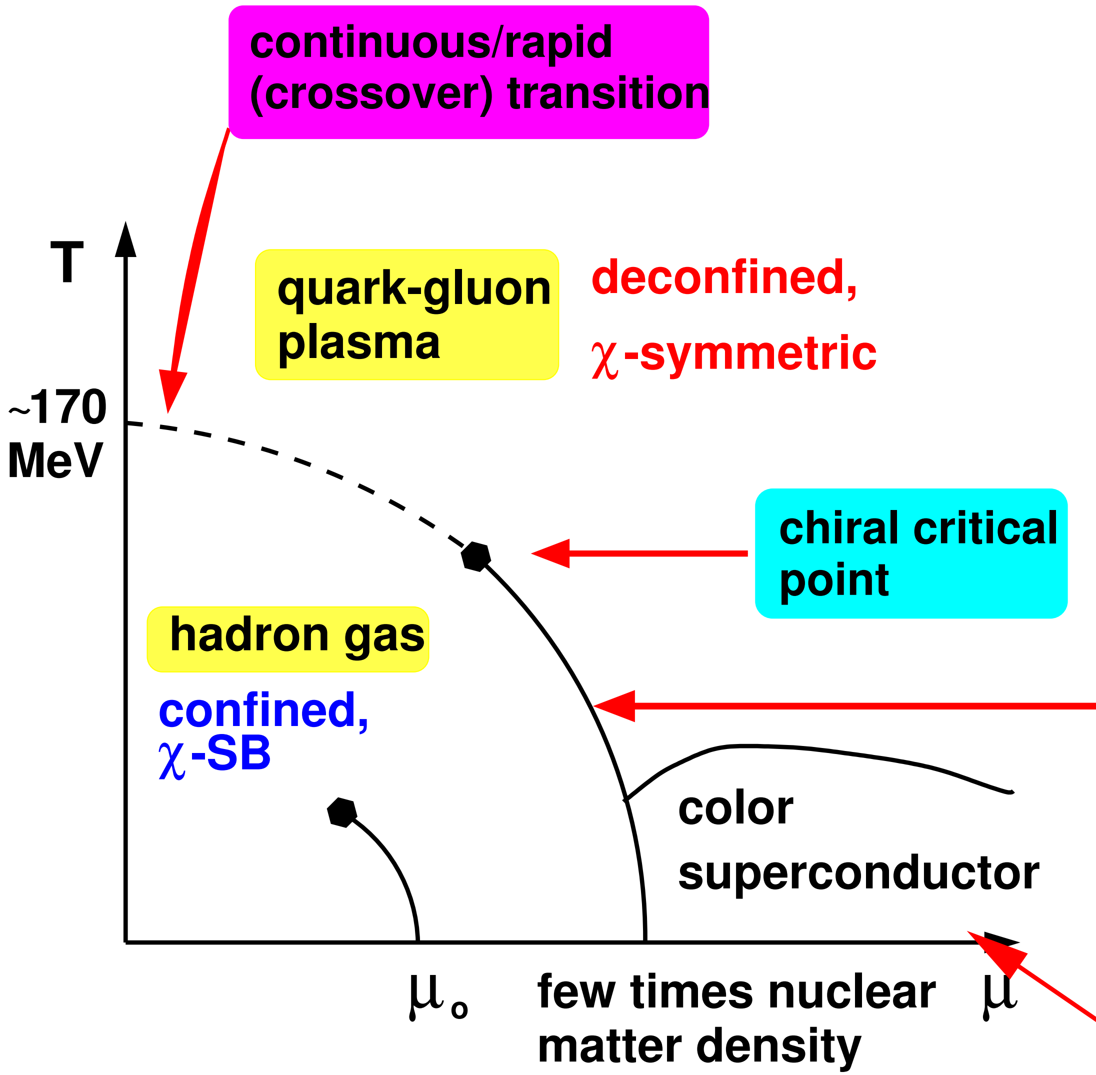

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2nd order phase transition; Ising universality class

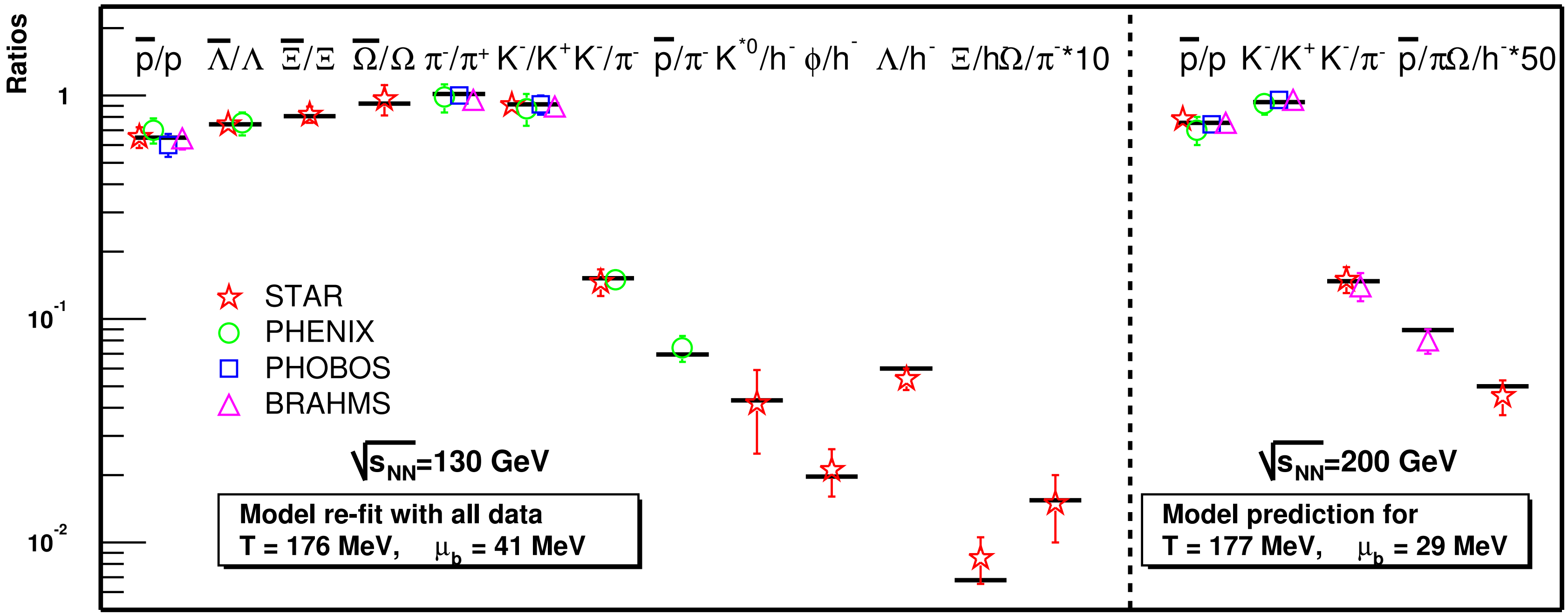
$T_c(\mu)$ under investigation

1st order phase transition ???

expected - however, so far no direct evidence from lattice QCD

attractive 1–gluon exchange => qq–condensates

Particle ratios and freeze out conditions

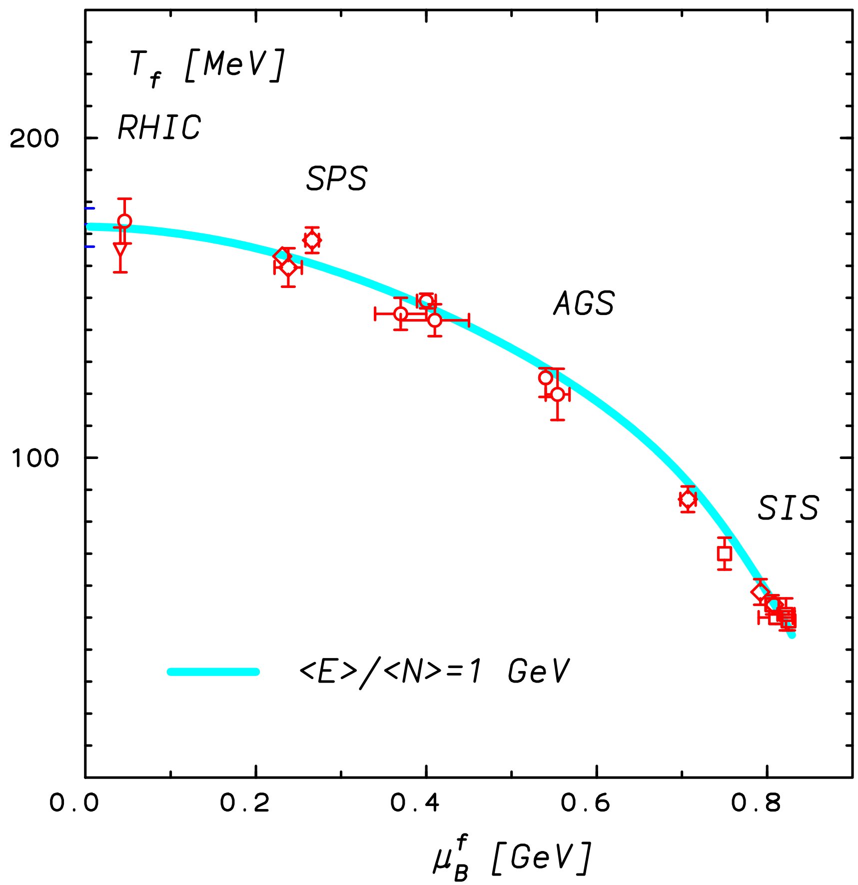


resonance gas: $Z(T,V,\mu_i) \;=\; \mathrm{Tr}e^{-\beta(H-\sum_i \mu_i Q_i)}$

describes observed particle ratios and
freeze out conditions

P. Braun-Munzinger, D. Magestro, K. Redlich,
J. Stachel, Phys. Lett. B518 (2001) 41

Particle ratios and freeze out conditions



- Is the freeze out temperature the critical temperature of the QCD transition?

- Which role do resonances play for the occurence of the transition to the QGP?

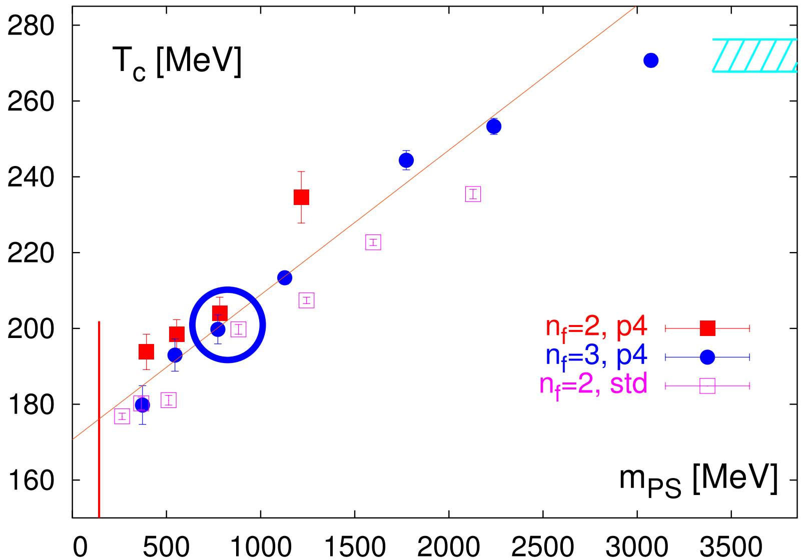
- ... and what about deconfinement and chiral symmetry restoration?

resonance gas

describes observed particle ratios and freeze out conditions

$$\ln\ Z(T,V,\mu_B,..)\ =\ \sum_{m_i} \ln\ Z_i(T,V,\mu_B,..)$$

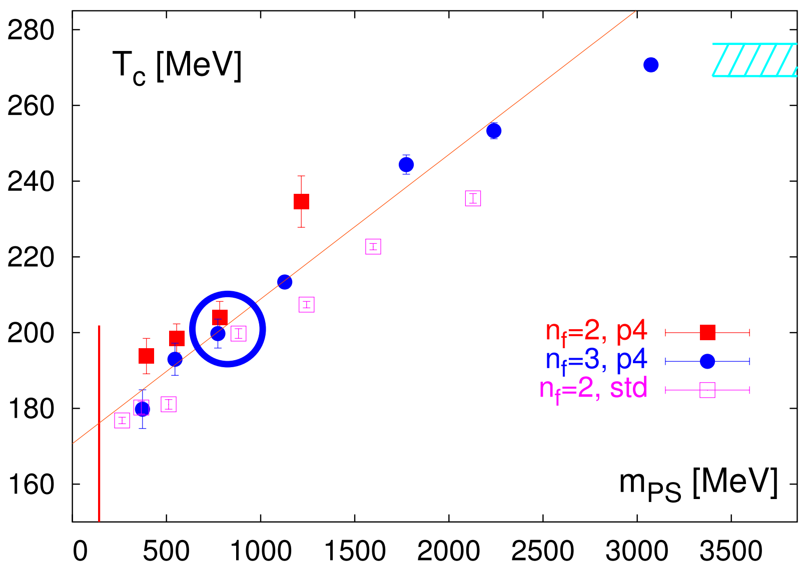
The QCD (phase) transition

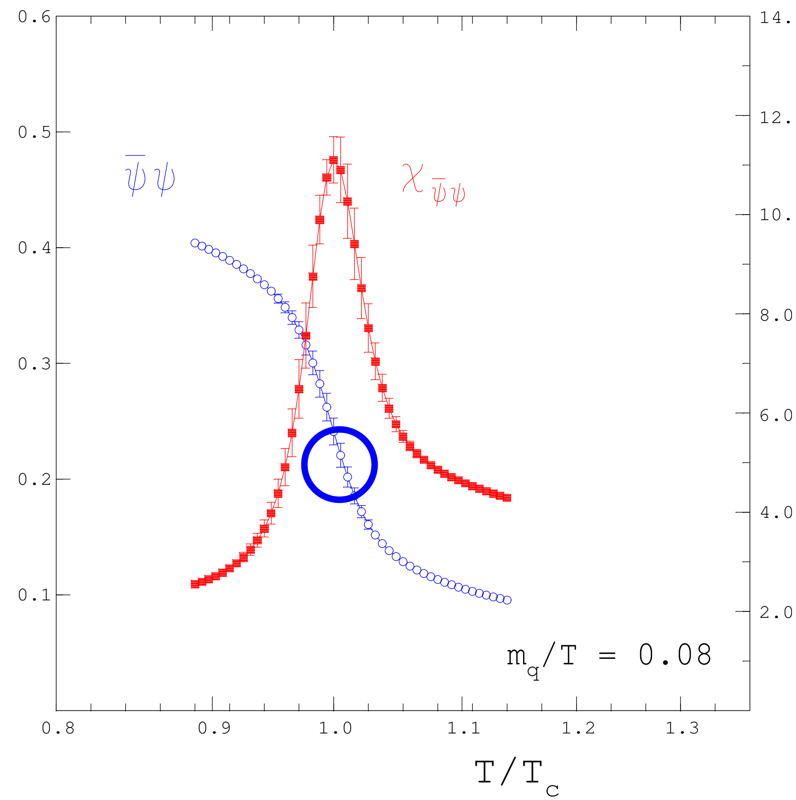


QCD transition in a world with

heavy pions: $m_\pi \simeq 770\ MeV$

The QCD (phase) transition



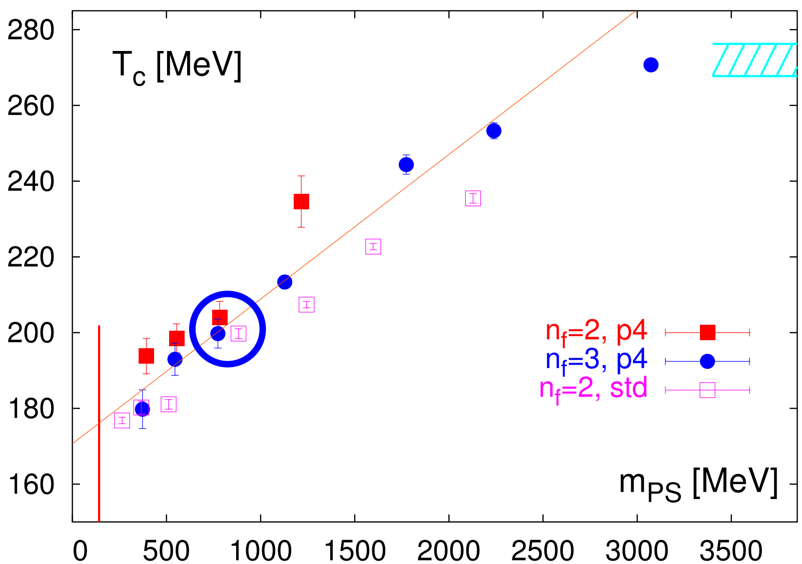


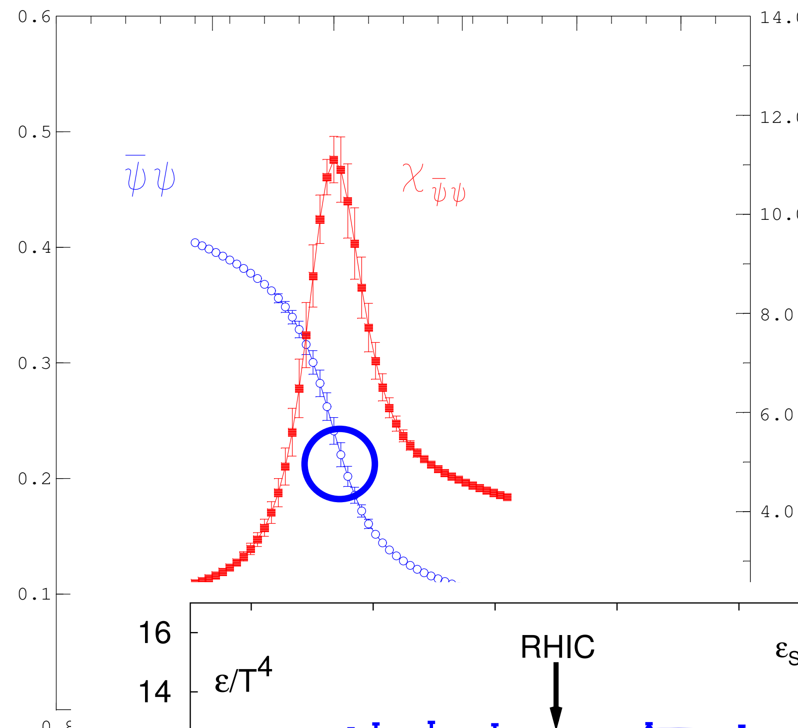
chiral symmetry restoration

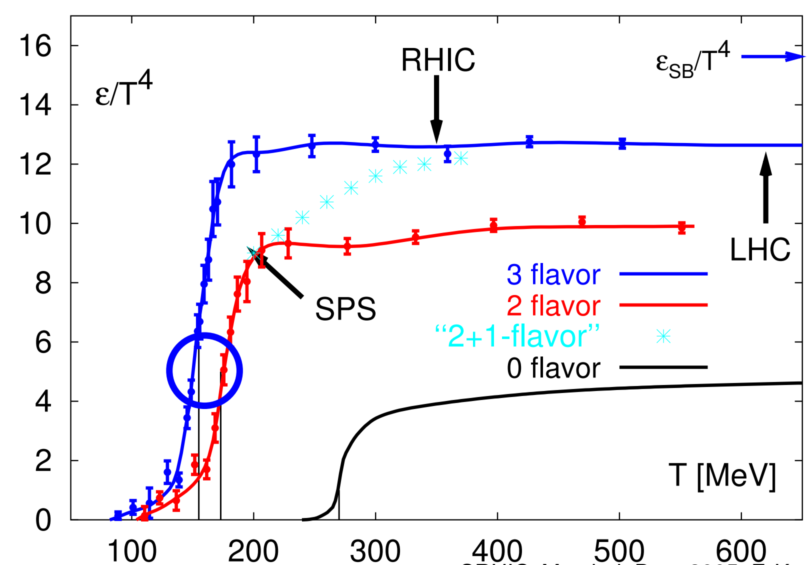
– rapid drop of $\langle\bar{\psi}\psi\rangle$

however: $\langle\bar{\psi}\psi\rangle(T_c) \ > \ 0$

The QCD (phase) transition







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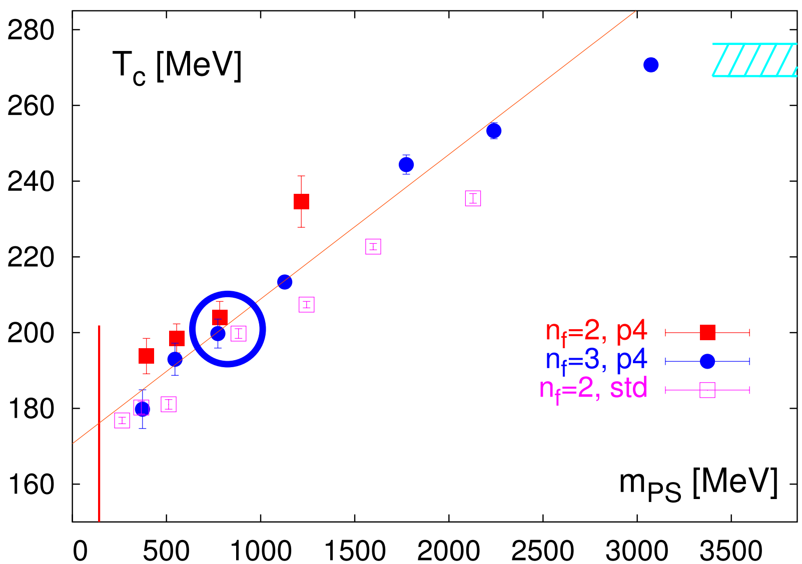
deconfinement

– rapid increase of d.o.f.

however: "no rigorous confinement"

The QCD (phase) transition



heavy quark free energy:

$F_{\bar{q}q}(\infty, T) < 0$ for all T

string breaking $\Leftrightarrow$

no rigorous confinement

chiral symmetry restoration

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however: $\langle\bar{\psi}\psi\rangle(T_c) \;>\; 0$

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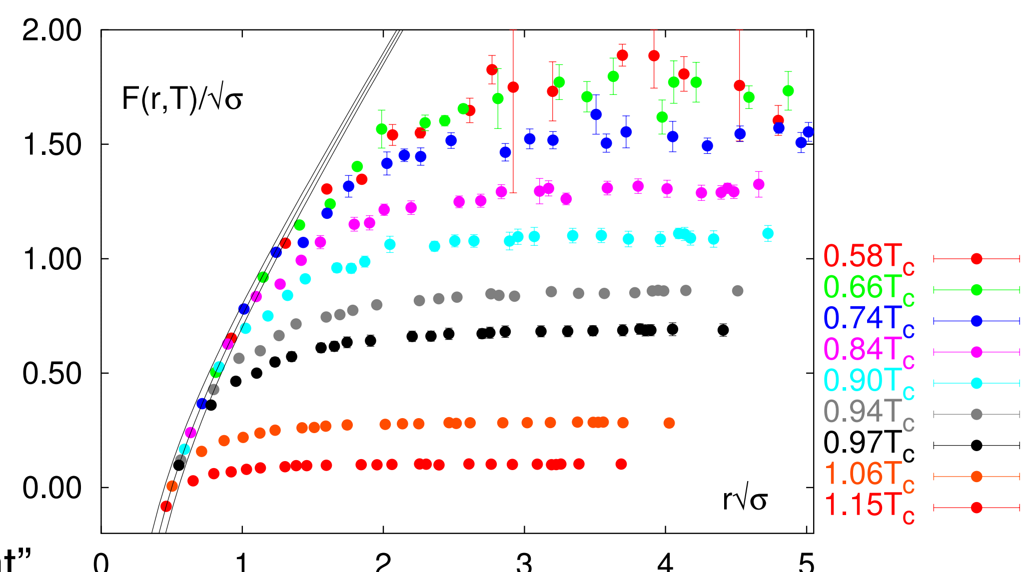
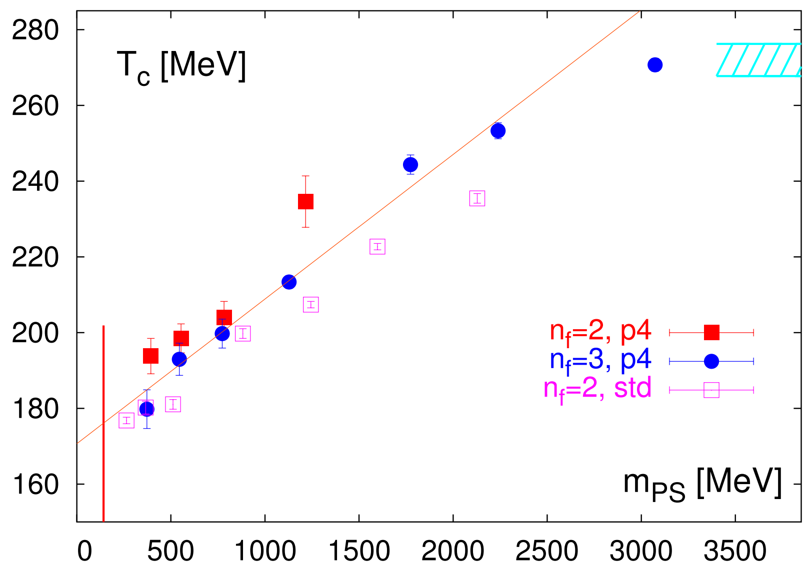


Critical temperature, equation of state
and the resonance gas

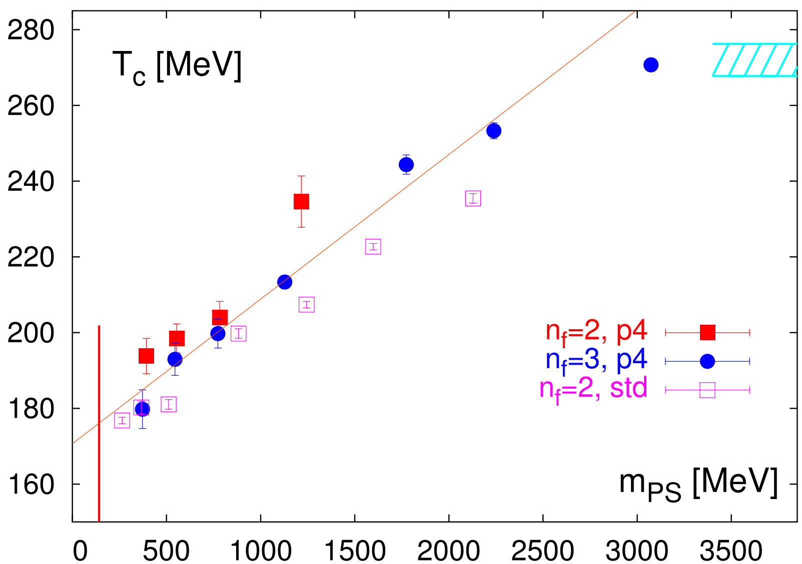


What triggers the transition

to the QCD plasma phase?

- chiral symmetry apparently not needed for transition to the QGP to occur
- strictly confining potential not needed for transition to the QGP to occur

Critical temperature, equation of state and the resonance gas



$m_{PS} \simeq 140\ MeV:\quad T_c \simeq 170\ MeV$

$m_{GB} \simeq 1.5\ GeV:\quad T_c \simeq 270\ MeV$
$(m_{PS} = \infty)$

lightest masses apparently do not control the transition

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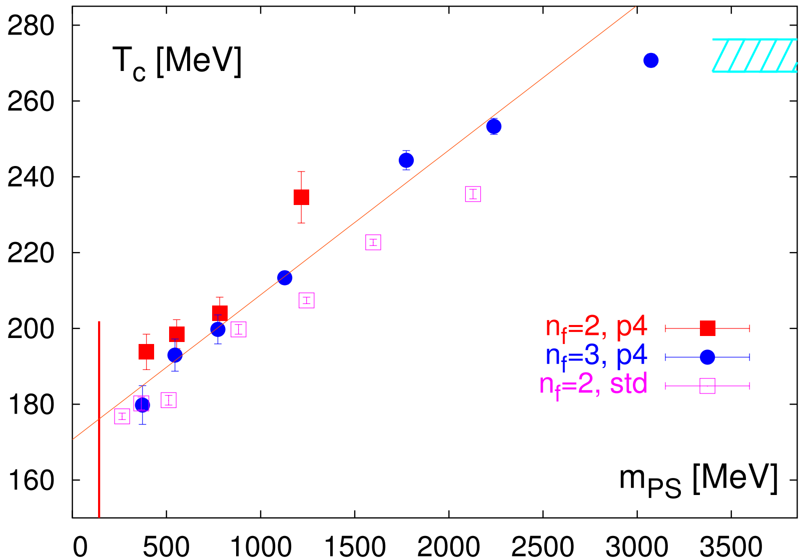

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$\Leftarrow$ understood in terms of an exponentially rising energy spectrum for string fluctuations:

$$\frac{T_c}{\sqrt{\sigma}} \simeq \sqrt{\frac{3}{(d-2)\pi}}$$

$\Rightarrow$ resonance gas

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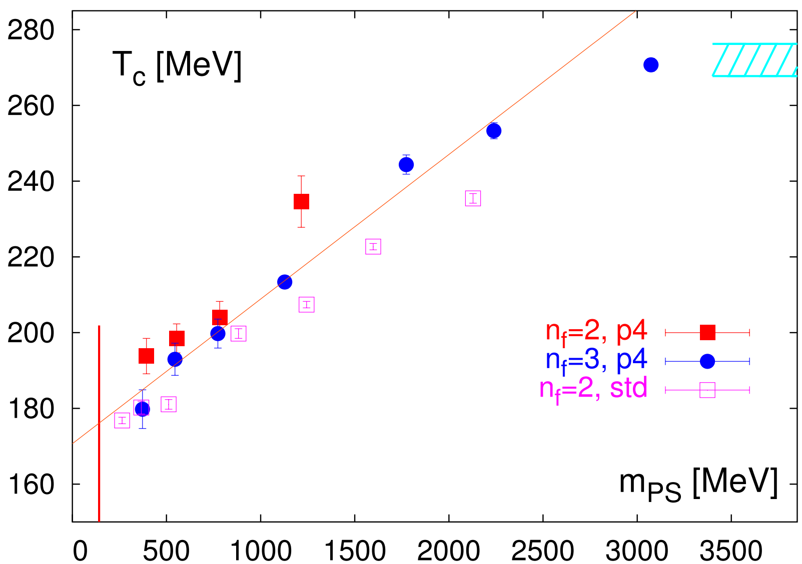

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$\Rightarrow$ resonance gas

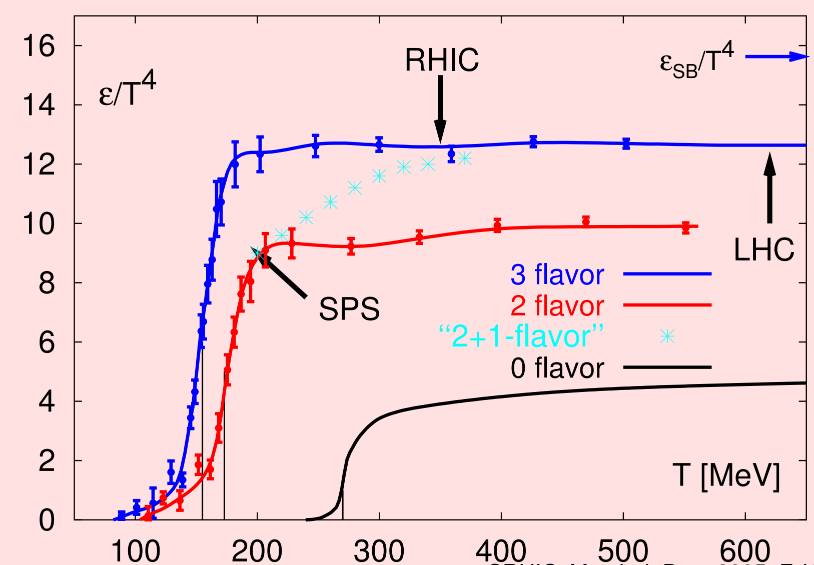
$$n_f = 2: \quad \epsilon_c \simeq (6 \pm 2)\, T_c^4$$
$$\simeq (0.3 - 1.3)\, GeV/fm^3$$
$$n_f = 0: \quad \epsilon_c \simeq (0.5 - 1)\, T_c^4$$
$$\simeq (0.3 - 0.7)\, GeV/fm^3$$

change in ϵ_c/T_c^4 compensated by shift in T_c

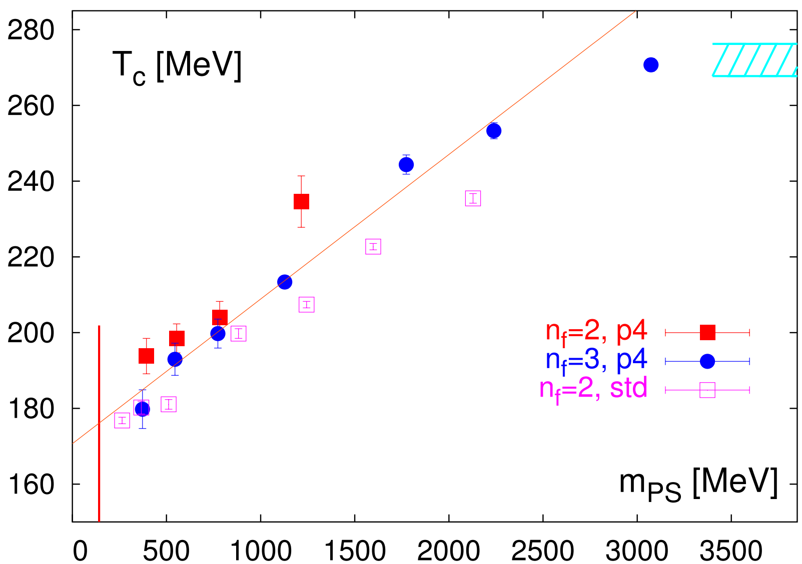
transition sets in at similar energy densities

$\Rightarrow$ percolation

energy density for 0, 2 and 3-flavor QCD



Critical temperature, equation of state and the resonance gas



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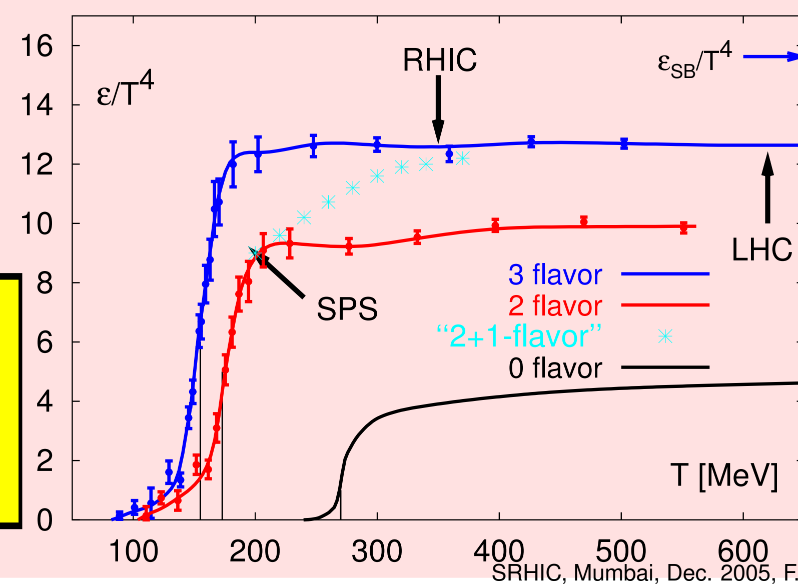
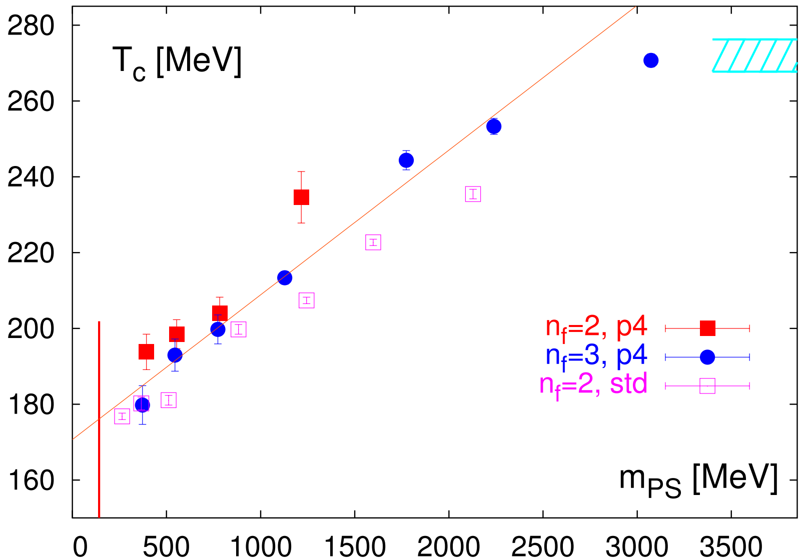
energy density for 0, 2 and 3-flavor QCD



a pion gas would only give rise to about 20% of the total energy density of a non-interacting hadron gas at T_c

Critical temperature, equation of state
and the resonance gas



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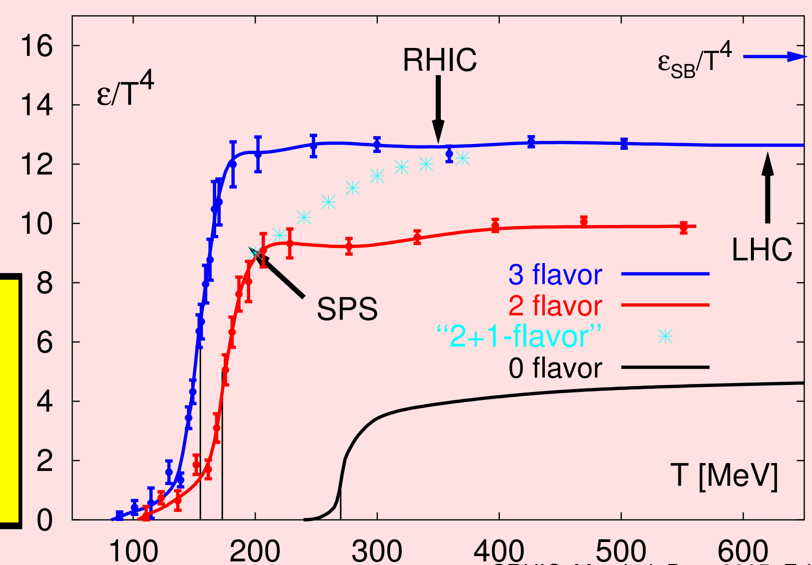
energy density for 0, 2 and 3-flavor QCD



the 20 lightest hadrons contribute only 50% of the total energy density of a non-interacting hadron gas at T_c

Critical temperature, equation of state and the resonance gas

$$\mathbf{Hagedorn\,spectrum:}\quad \rho(m_H) \sim c\, m_H^a\, \mathrm{e}^{m_H/T_H}$$

$$\ln Z(T,\mu_B) \; = \; \int \mathrm{d}m_H\; \rho(m_H)\;\ln Z_{m_H}(T,\mu_B)$$

- $\int \;\Rightarrow\; \sum \;\sim$ 1000 exp. known resonance d.o.f.

Critical temperature, equation of state and the resonance gas

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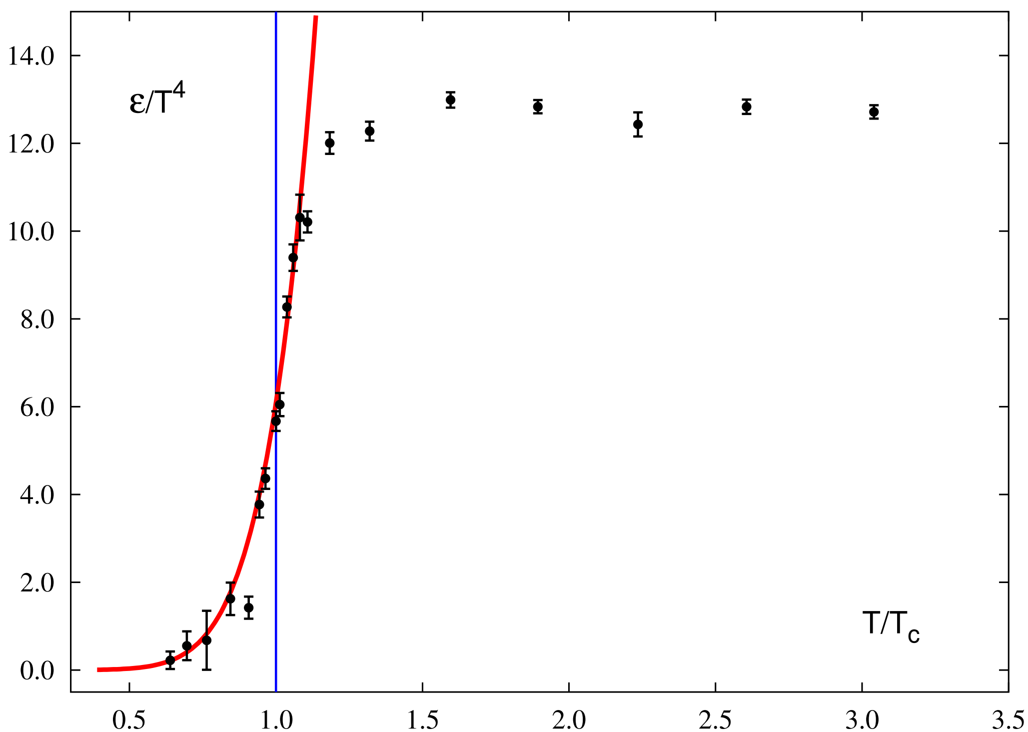
$$\ln Z(T,\mu_B) = \int \mathrm{d}m_H\, \rho(m_H)\, \ln Z_{m_H}(T,\mu_B)$$



resonance gas:

$\sim$ 1000 exp. known resonance d.o.f.

vs.

lattice calculation:

(2+1)-flavor QCD, $m_q/T = 0.4$

resonances give large contribution at T_c

Critical temperature, equation of state and the resonance gas

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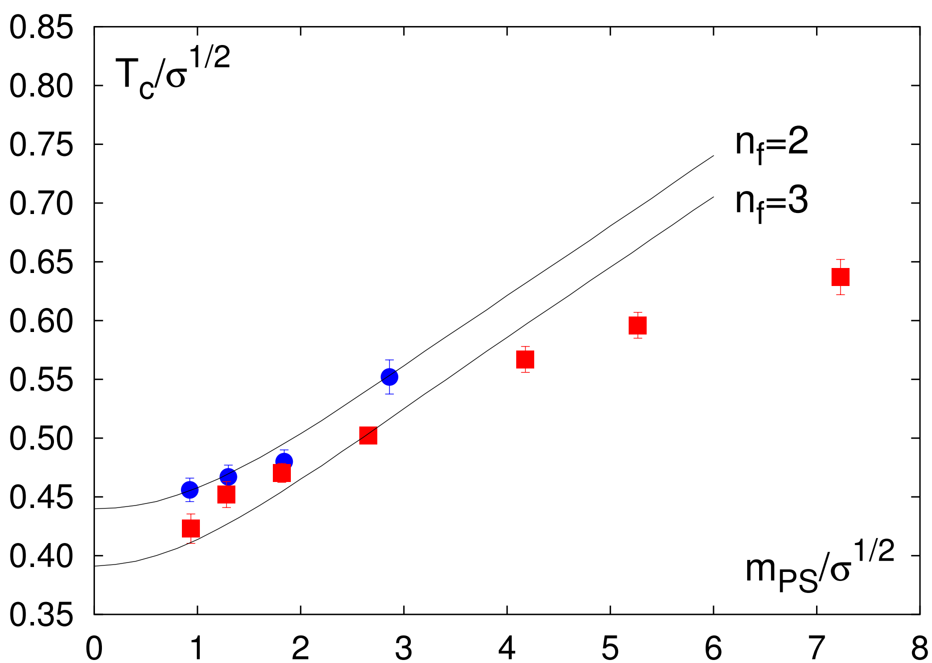

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resonances give large contribution at T_c

and explain quark mass dependence of T_c

FK, K. Redlich, A. Tawfik, hep-ph/0303108

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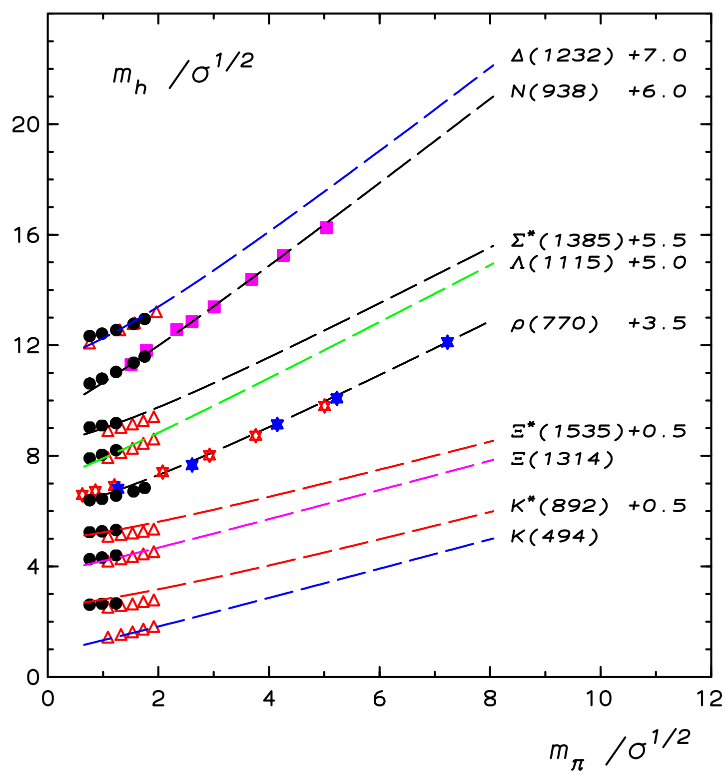
$\Uparrow$



hadron	$m_q \sim 0$	$m_q \to \infty$
pion	$m_\pi \sim \sqrt{m_q}$	$m_\pi \sim 2m_q$
rho	$m_\rho \sim 770\ \mathrm{MeV}\ +\ c_\rho m_q$	$m_\rho \sim 2m_q$
...higher meson resonances...		
nucleon	$m_N \sim 940\ \mathrm{MeV}\ +\ c_N m_q$	$m_N \sim 3m_q$
...higher baryon resonances...		

adjust hadron spectrum to conditions realized on the lattice

Critical temperature, equation of state and the resonance gas

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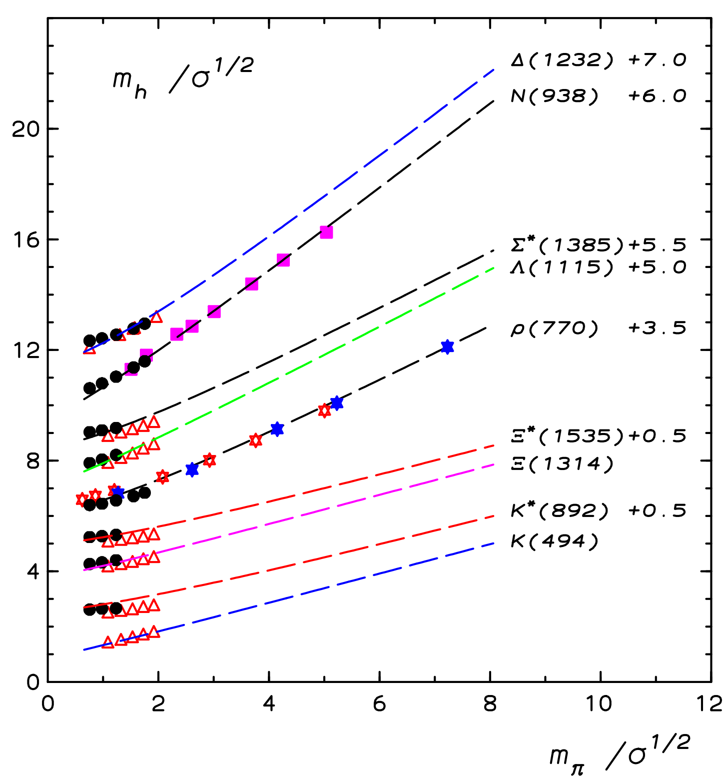
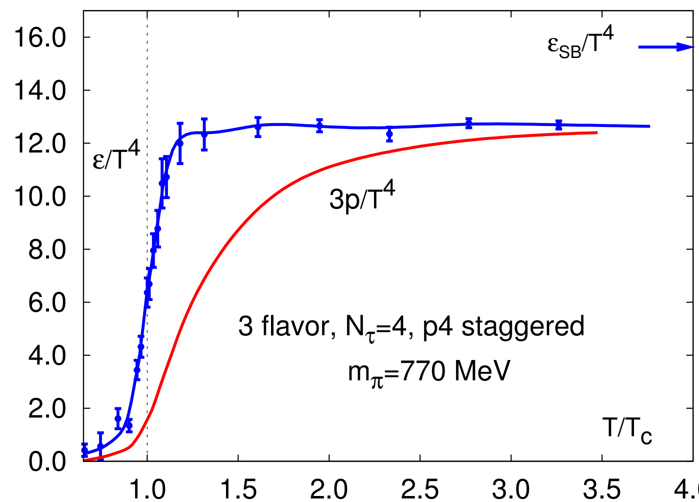


Future:

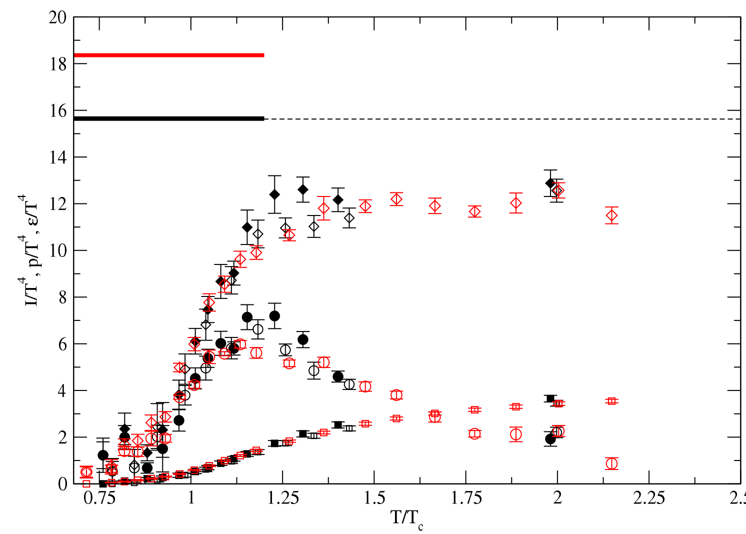
need lattice calculations with realistic quark masses in order to

- perform more direct comparisons
- check (un)importance of light pions
- finally determine T_c and ϵ_c

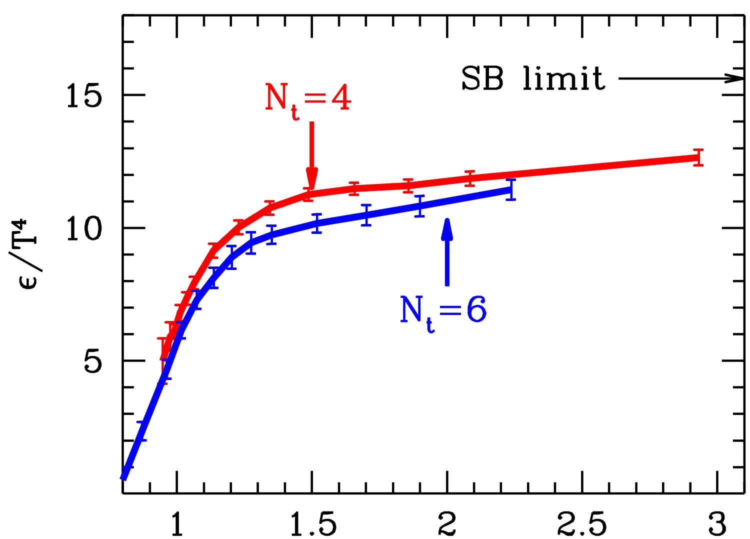
recent results on QCD EoS



old Bielefeld result, 2001
improved staggered (p4), $N_\tau = 4$
3-flavor, $m_\pi \simeq 770$ MeV



MILC-collaboration, hep-lat/0509053
$\mathcal{O}(a^2)$ improved staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \gtrsim 250$ MeV



$\epsilon_c/T_c^4 \simeq 6$ insensitive to m_π and a^{-1}
HOWEVER: thermodynamic limit?? $TV^{1/3} \simeq 2$
cut-off effects?? $\Rightarrow$ improved actions

Y. Aoki et al., hep-lat/0510084
standard staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \rightarrow 140$ MeV (extrap.)
ϵ/T^4 rescaled with $(\epsilon_{SB}/T^4)(N_\tau)$

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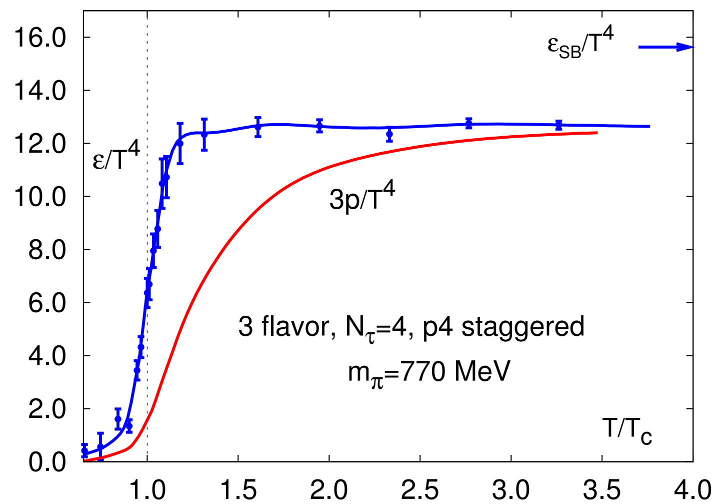

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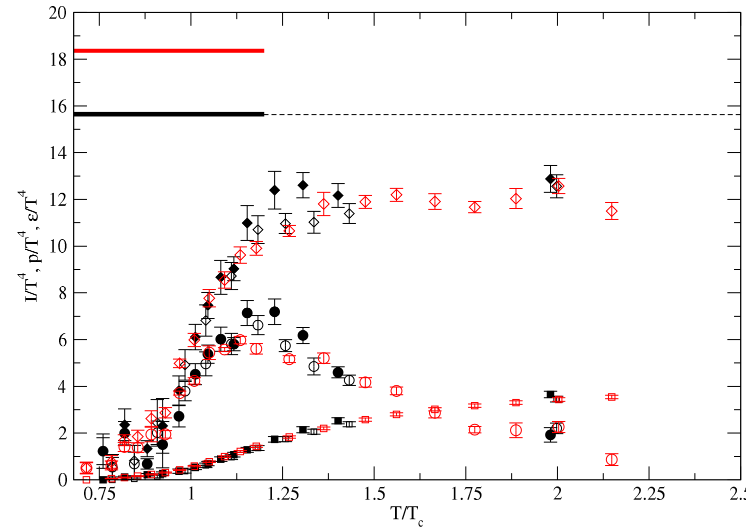

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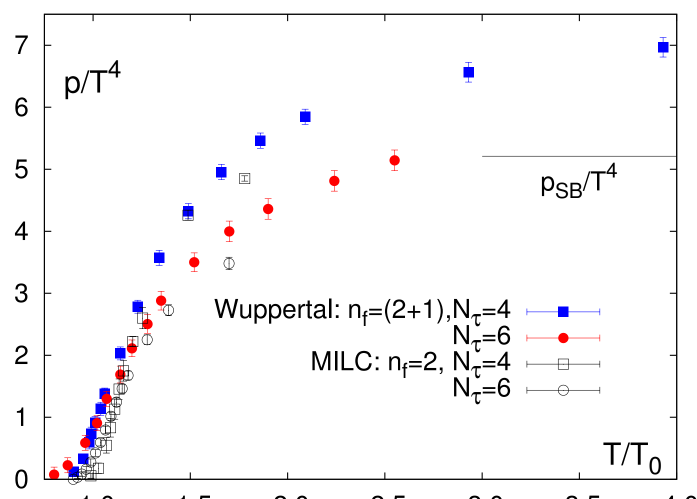

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Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\; \mathrm{e}^{-S_E(V,T,\mu)}$$

$$= \int \mathcal{D}\mathcal{A}\; \left[det\ M(\mu)\right]^f\; \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$complex fermion determinant;

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⇑complex fermion determinant;

ways to circumvent this problem:

- reweighting: works well on small lattices; requires exact evaluation of $detM$
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ;
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
  R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
  M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505

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recent progress;

- reweighting: larger lattices; smaller quark mass;
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searches for the CCP:

μ_B sensitive to V (and m_q)
$\mu_B \sim 360$ MeV

no clear-cut evidence

$\mu_B \sim 180$ MeV
(might be ~ 230 MeV)?

no evidence

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still an open question

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⇑complex fermion determinant;

⇓Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu)$$

$$\equiv \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2 \left(\frac{\mu}{T}\right)^2 + c_4 \left(\frac{\mu}{T}\right)^4 + \mathcal{O}((\mu/T)^6)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$

quark number density $$\frac{n_q}{T^3} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$$

quark number susceptibility $$\frac{\chi_q}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$$

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty}\left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

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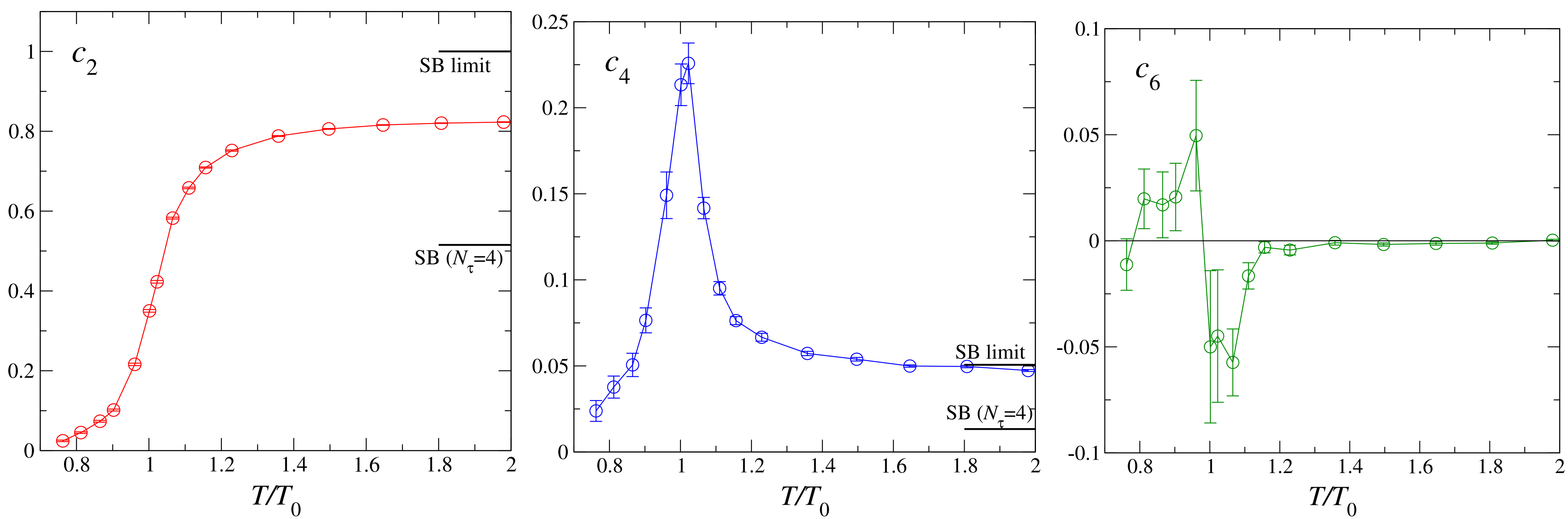

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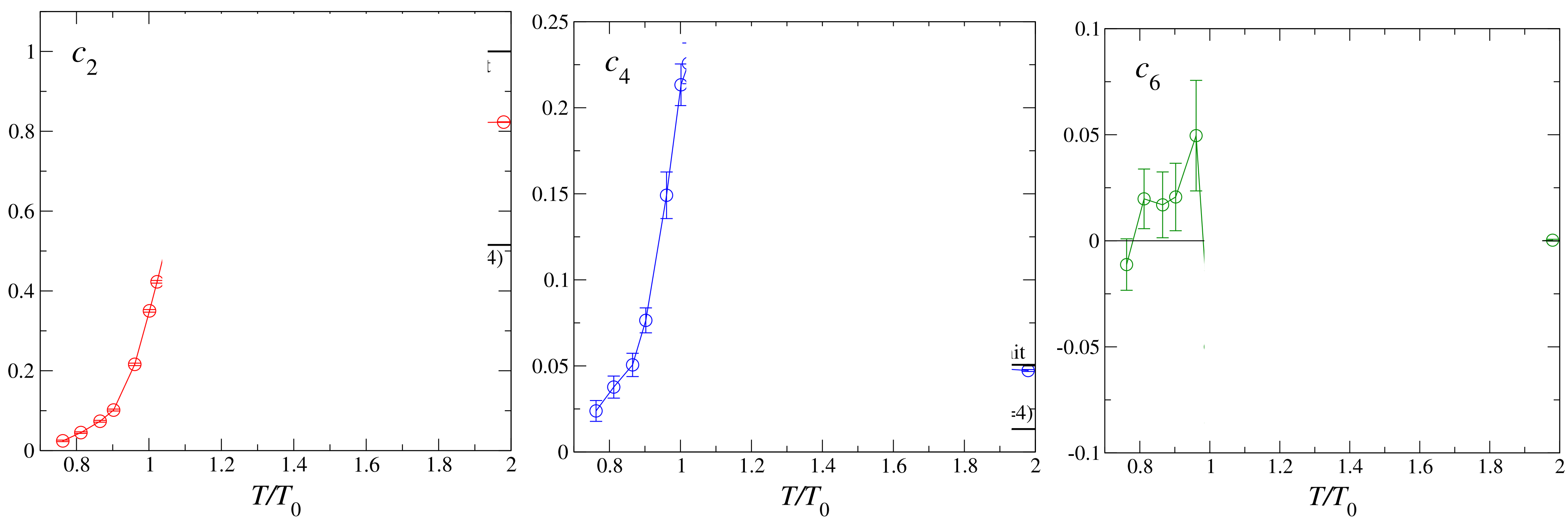

$c_n > 0$ for all n and $T \lesssim 0.95\ T_c$ $\Leftrightarrow$ singularity for real μ (positive μ^2)

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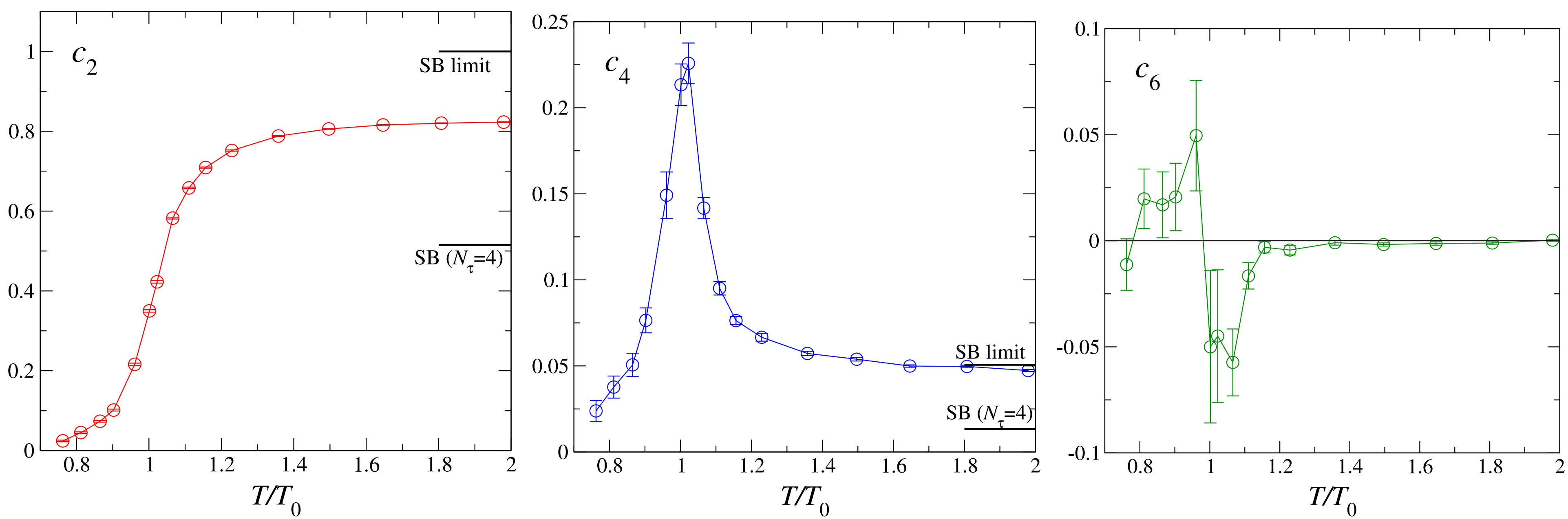
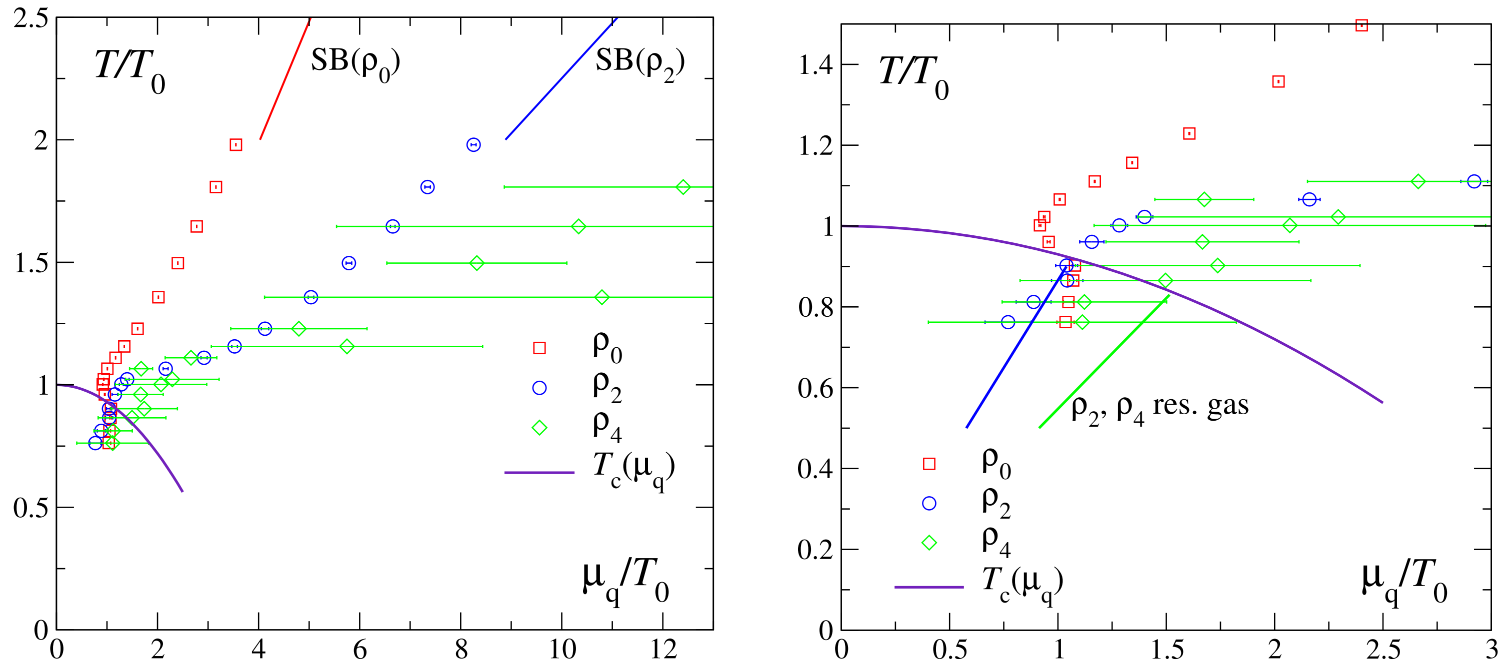


irregular sign of c_n for $T \gtrsim T_c$ $\Leftrightarrow$ singularity in complex plane

Radius of convergence: lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$



$T < T_0$: $\rho_n \simeq 1.0$ for all n $\Rightarrow$ $\mu_B^{crit} \simeq 500$ MeV

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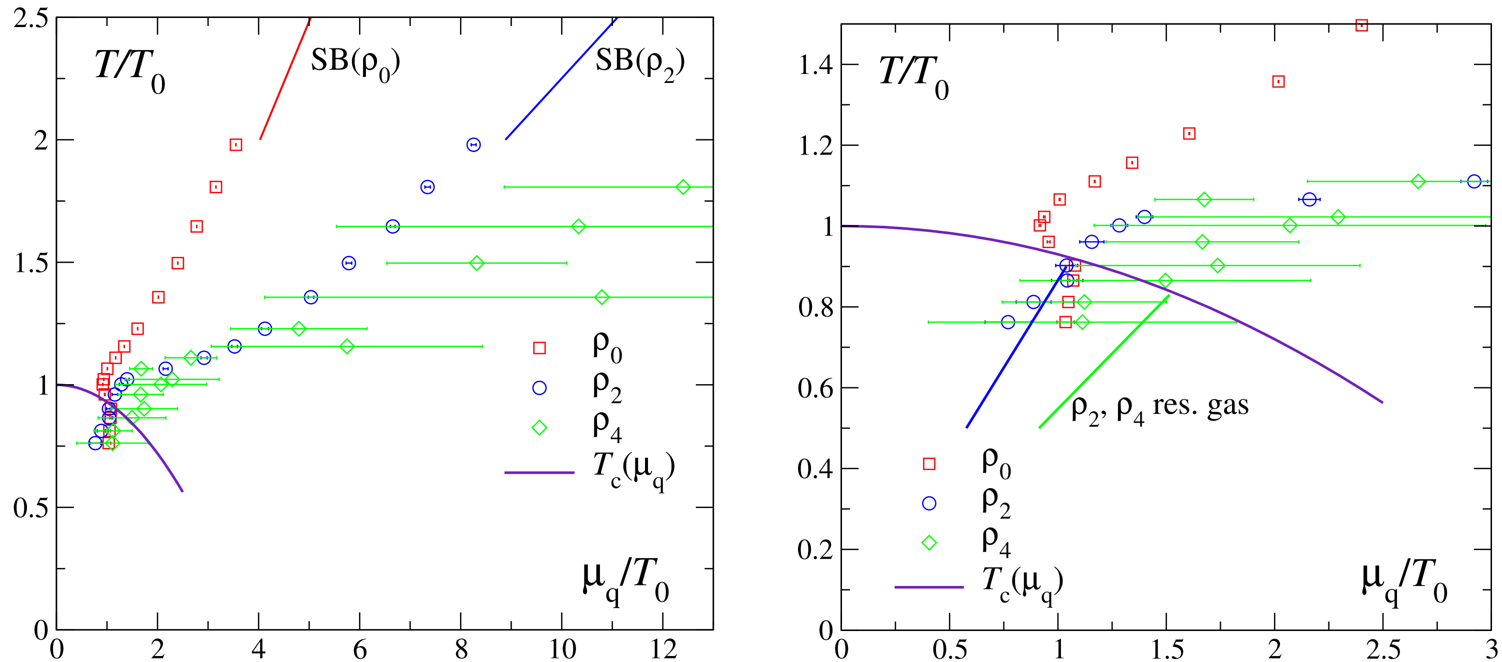

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HRG analytic, LGT consistent with HRG $\Rightarrow$ infinite radius of convergence not yet ruled out

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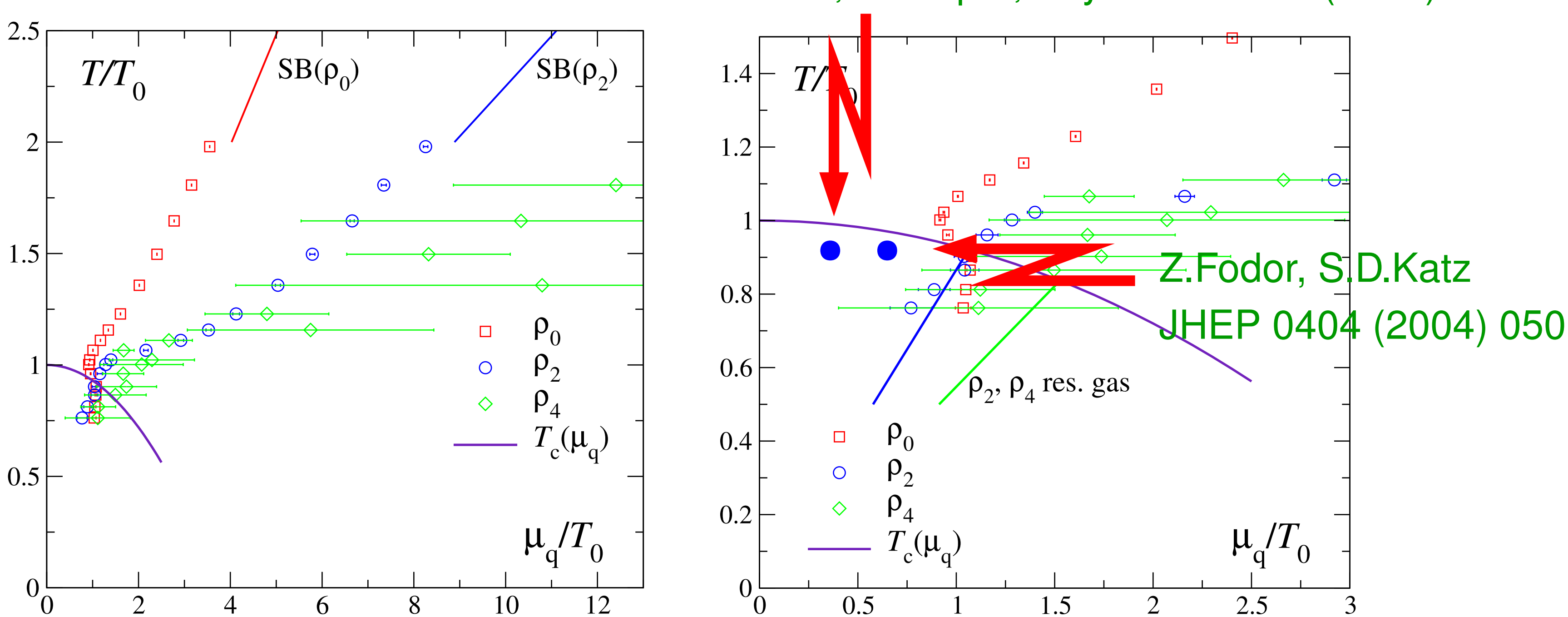
R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014



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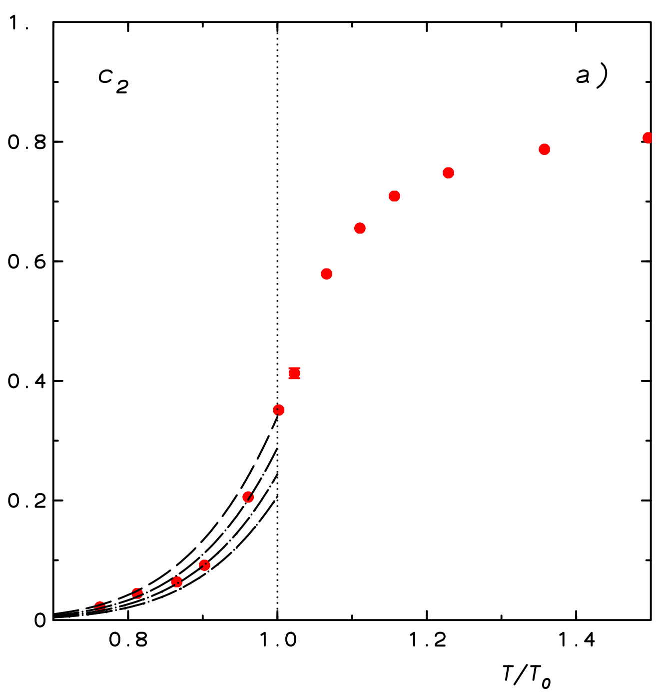
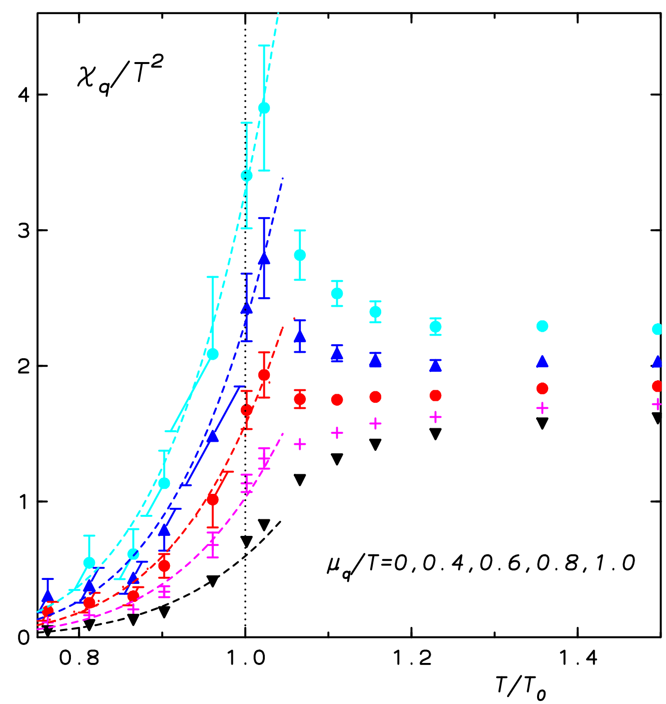
Resonance gas:
spectrum dependent consequences

- "fit" with modified spectrum $m_H(m_\pi) = m_H(0) + A \left(\frac{m_\pi}{m_H(0)}\right)^2$
  $\Rightarrow$ tests factorization

$$\frac{\chi_q}{T^2} = 9F(T)\cosh(3\mu_q/T) \sim c_2(T)\left(2 + 12\,\frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)\right)$$



$A = 0.9,\ 1.0,\ 1.1,\ 1.2$



$A = 1.0$

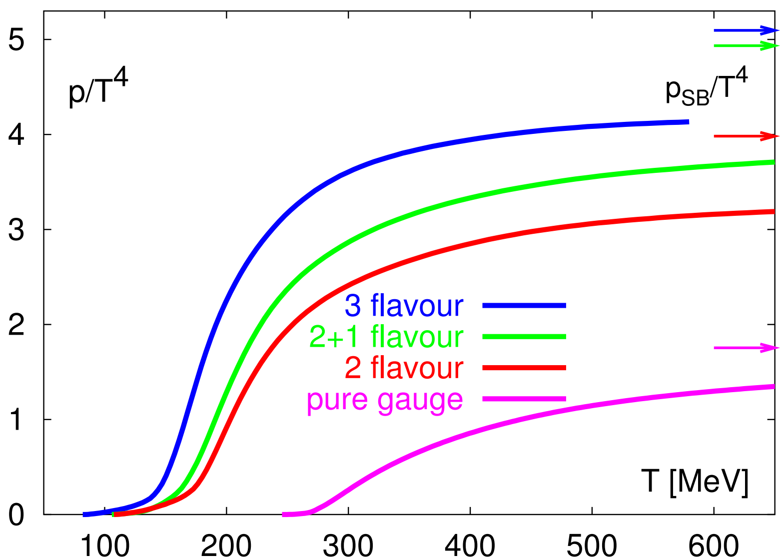
The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

$\mu_q = 0, \quad 16^3 \times 4$ lattice

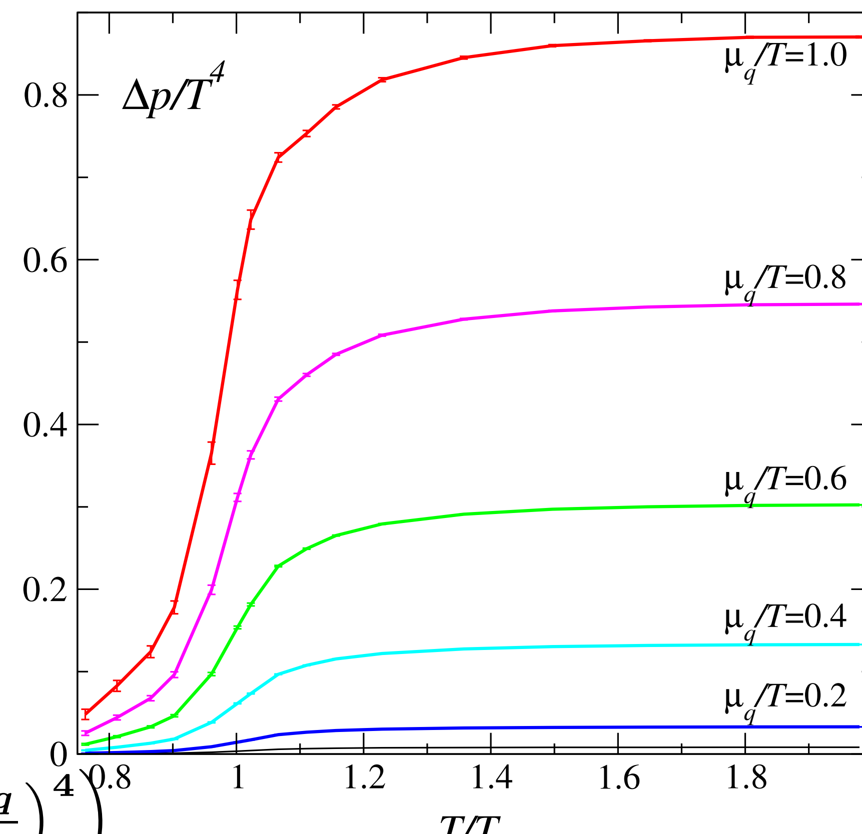
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$



contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$



high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$

similar F. Csikor et al., JHEP 0311 (2003) 070

SPS: $\mu_q/T \lesssim 0.6$ RHIC: $\mu_q/T \lesssim 0.1$

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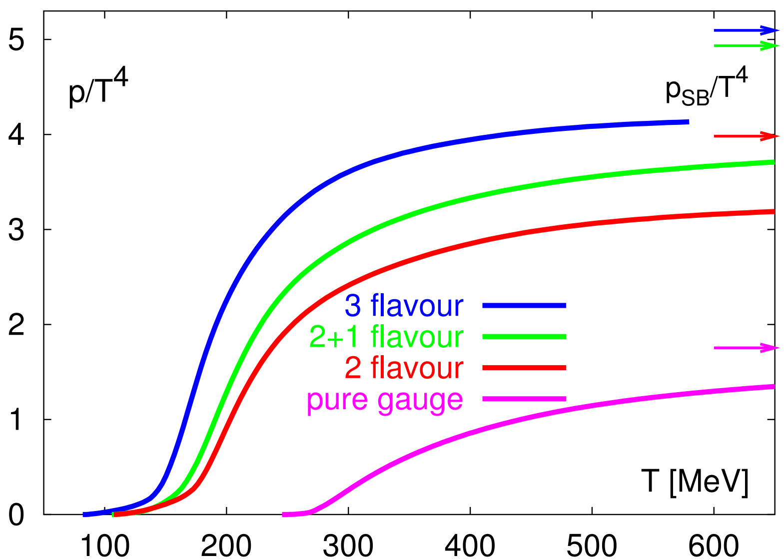
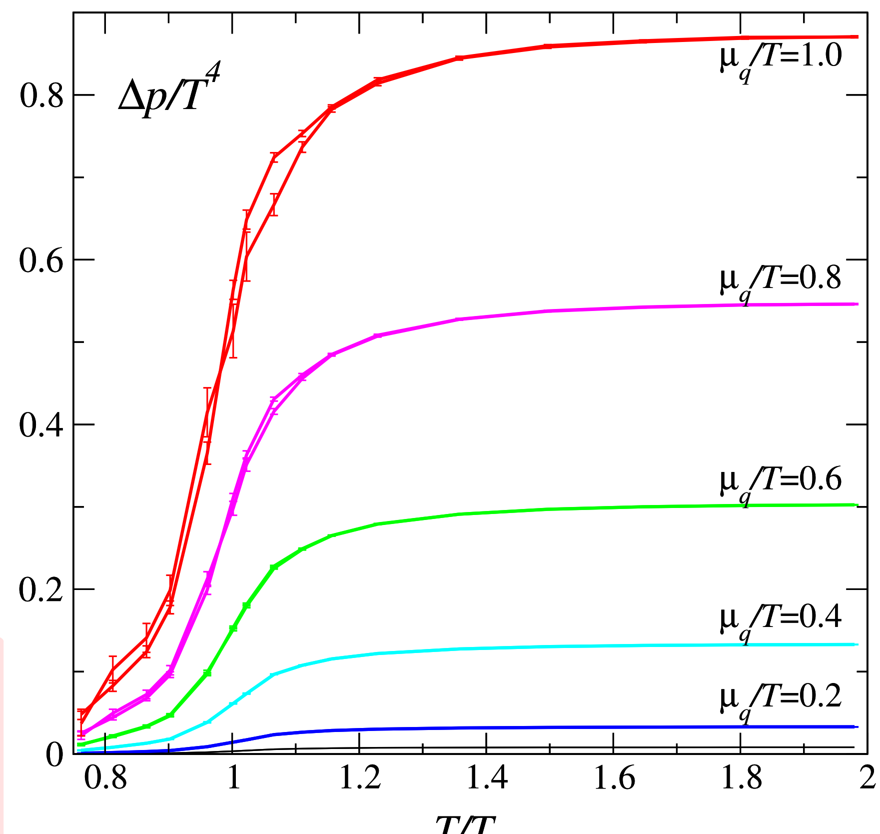


PRD71 (2005) 054508

contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$



pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

SPS: $\mu_q/T \lesssim 0.6$ RHIC: $\mu_q/T \lesssim 0.1$

Energy and Entropy density for $\mu_q > 0$

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, in preparation

Thermodynamics: (NB: continuum $\hat{m} \equiv m_q$
lattice $\hat{m} \equiv m_q a$, implicit T-dependence)

- pressure $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T,\mu_q) = \sum_{n=0}^{\infty} c_n(T,\hat{m}) \left(\frac{\mu_q}{T}\right)^n$$

- energy density from ”interaction measure”

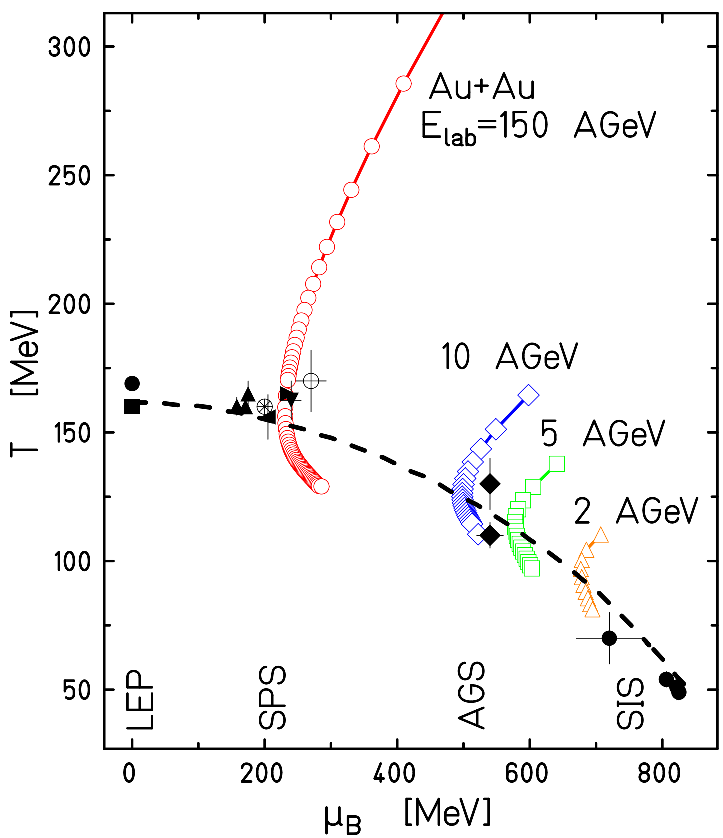
$$\frac{\epsilon - 3p}{T^4} = \sum_{n=0}^{\infty} c_n'(T,\hat{m}) \left(\frac{\mu_q}{T}\right)^n \quad , \quad c_n'(T,\hat{m}) \equiv T \frac{\mathrm{d}c_n(T,\hat{m})}{\mathrm{d}T}$$

- entropy density

$$\frac{s}{T^3} \equiv \frac{\epsilon + p - \mu_q n_q}{T^4} = \sum_{n=0}^{\infty} \left((4-n)c_n(T,\hat{m}) + c_n'(T,\hat{m})\right) \left(\frac{\mu_q}{T}\right)^n$$

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram



for example:

isentropic expansion,
"mixed phase model":
V.D. Toneev, J. Cleymans, E.G. Nikonov, K. Redlich, A.A. Shanenko, J. Phys. G27 (2001) 827

EoS on HIC trajectories

- dense matter created in a HI-collision expands and cools at fixed entropy and baryon number
  $\Rightarrow$ lines of constant S/N_B in the QCD phase diagram
  - high T: ideal gas

$$\frac{S}{N_B} = 3\,\frac{\frac{32\pi^2}{45 n_f} + \frac{7\pi^2}{15} + \left(\frac{\mu_q}{T}\right)^2}{\frac{\mu_q}{T} + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^3}$$

$S/N_B =$ constant $\Leftrightarrow$ μ_q/T constant

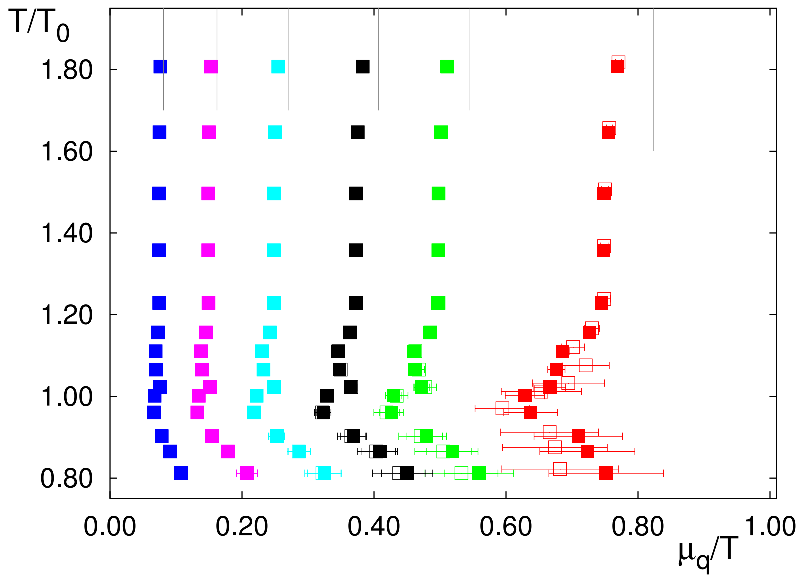
  - low T: nucleon + pion gas

$T \rightarrow 0$: $\quad \mu_q/T \sim c/T$

Lines of constant S/N_B

$S/N_B =$ 300 150 90 60 45 30

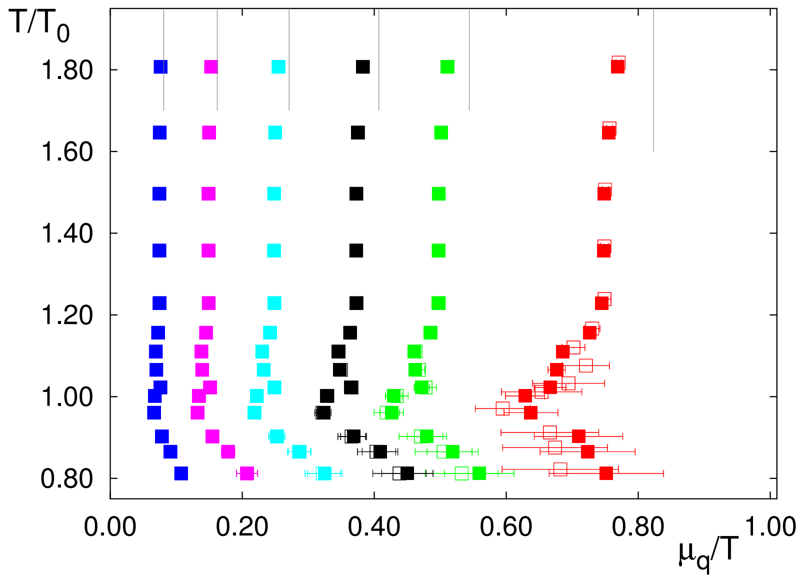
S. Ejiri, FK, E. Laermann, C. Schmidt, in prep.



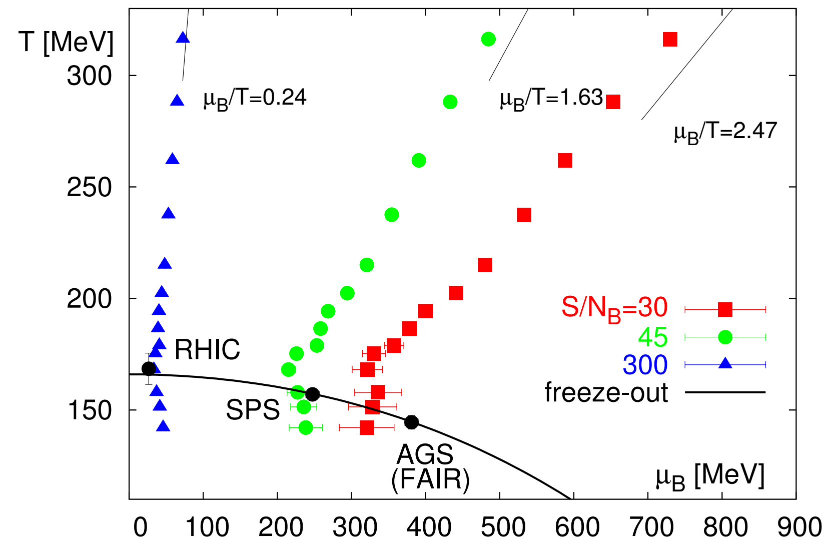
- isentropic trajectories close to ideal gas behavior for $T > T_c$
- trajectories bend towards larger μ_q for $T < T_c$
- $\mathcal{O}(\mu_q^6)$ correction (open sym.) is small for $\mu_q/T \lesssim 0.8$ (despite large errors)

Lines of constant S/N_B

$S/N_B =$ 300 150 90 60 45 30

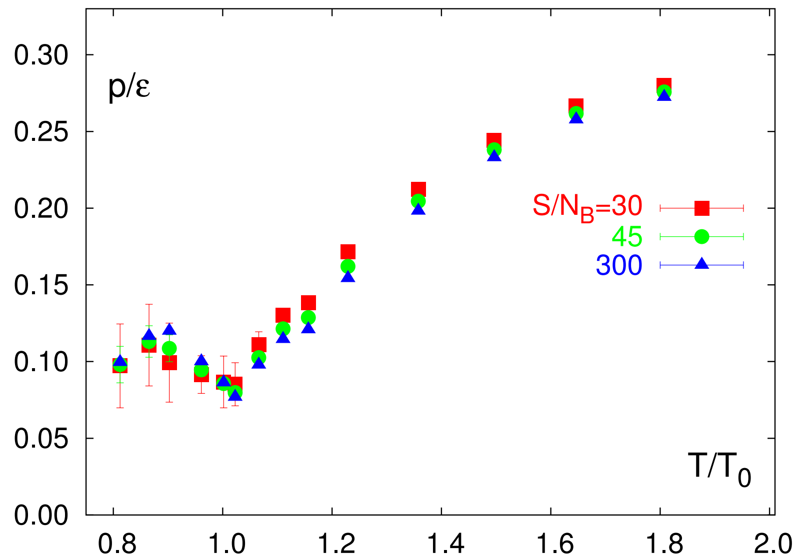


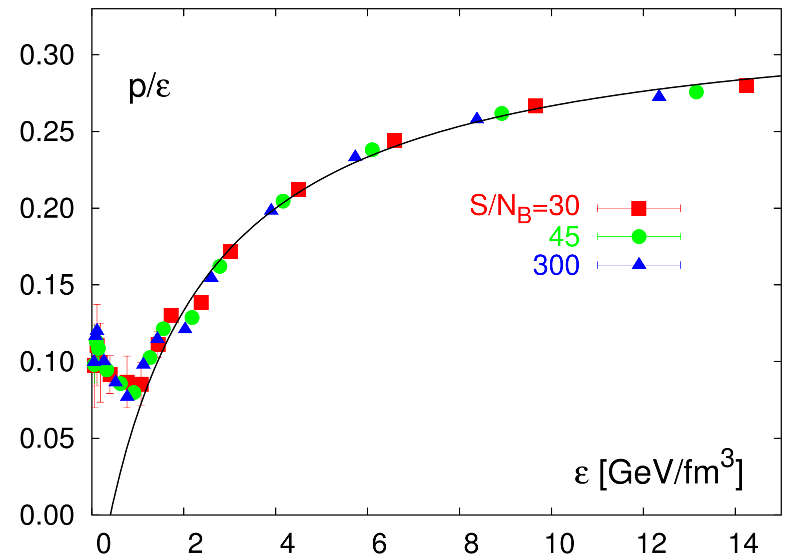
S. Ejiri, FK, E. Laermann, C. Schmidt, in prep.



- isentropic trajectories close to ideal gas behavior for $T > T_c$
- trajectories bend towards larger μ_q for $T < T_c$
- $\mathcal{O}(\mu_q^6)$ correction (open sym.) is small for $\mu_q/T \lesssim 0.8$ (despite large errors)
- RHIC corresponds to $S/N_B \simeq 300 \simeq \infty$
- SPS corresponds to $S/N_B \simeq 45$
- FAIR will operate at $S/N_B \simeq 30$ or $\mu_q/T \lesssim 0.9$

Isentropic Equation of State: p/ϵ



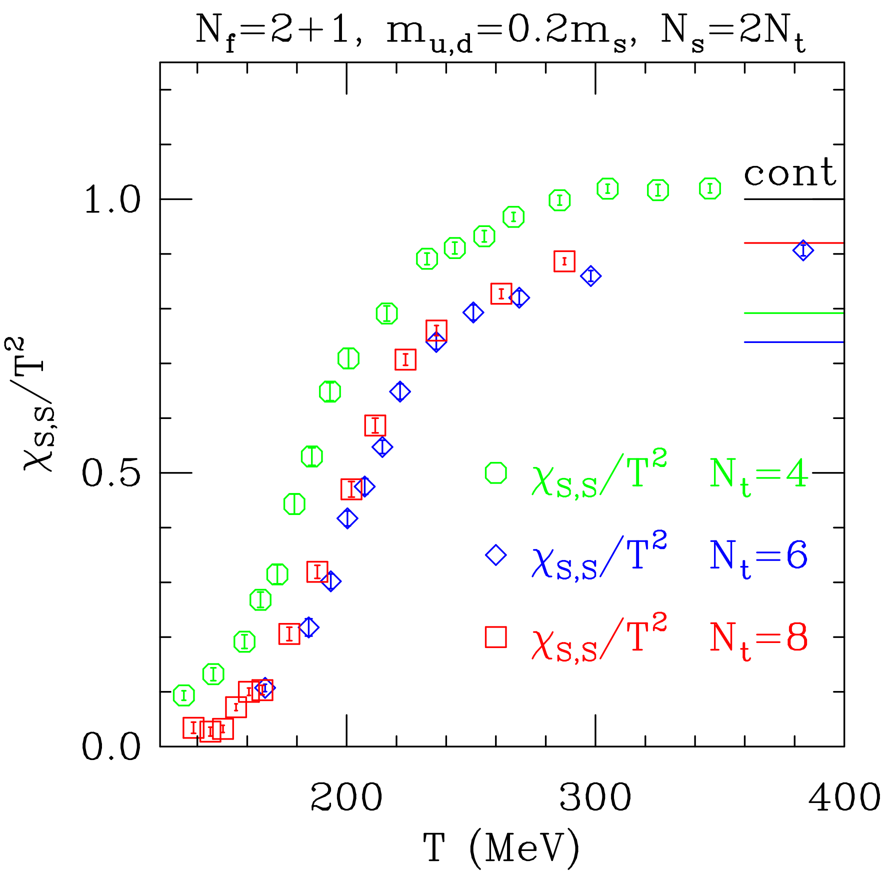


- p/ϵ vs. ϵ shows almost no dependence on S/N_B
- softest point: $p/\epsilon \simeq 0.075$
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\epsilon}\right)$$

Fluctuations of the baryon number density ($\mu \geq 0$)

baryon number density fluctuations:
(MILC coll., hep-lat/0405029)
$\mu = 0$



$$\frac{\chi_q}{T^3} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

$$= \frac{9\,T}{V} \left(\langle N_B^2 \rangle - \langle N_B \rangle^2 \right)$$

susceptibilities = integrated correlation functions
= integrated spectral functions

to be studied in event-by-event fluctuations

recent papers:

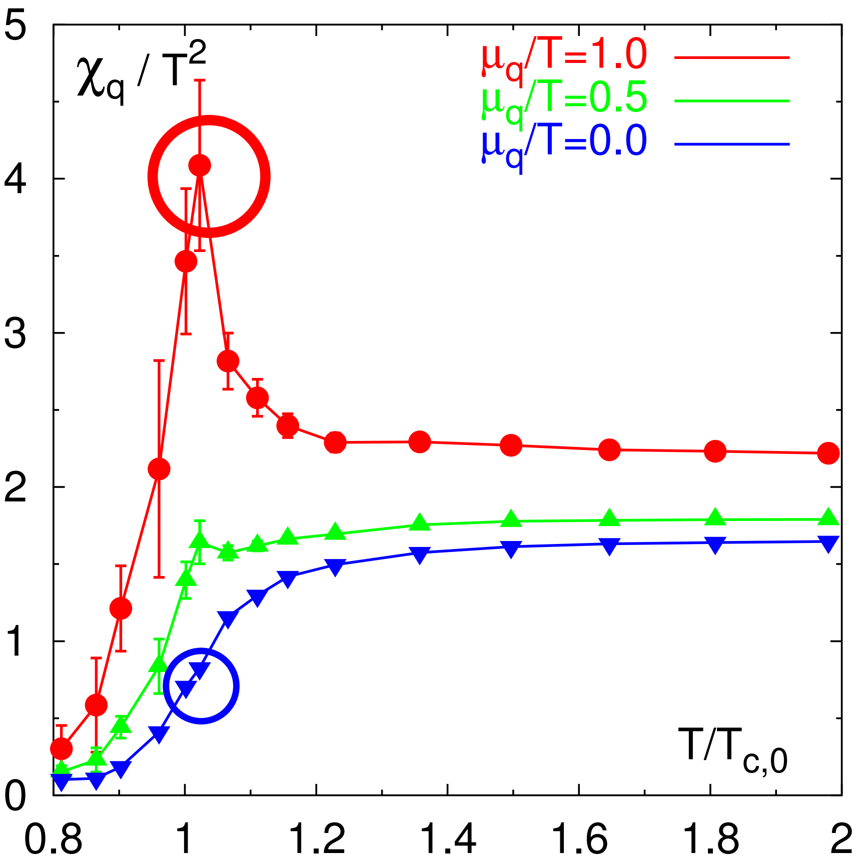
V. Koch, E.M. Majumder, J. Randrup, nucl-th/0505052
S. Ejiri, FK, K. Redlich, hep-ph/05090521
R.V. Gavai, S. Gupta, hep-lat/0510044

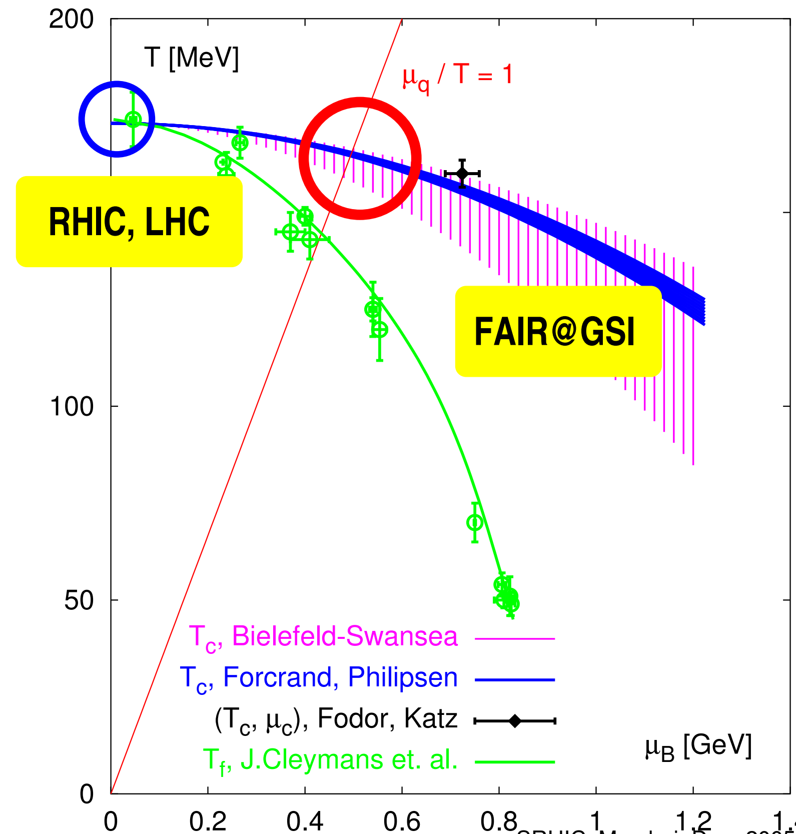
Fluctuations of the baryon number density ($\mu \geq 0$)

baryon number density fluctuations:
(Bielefeld-Swansea, PRD68 (2003) 014507)
$\mu \geq 0, n_f = 2$

$$\frac{\chi_q}{T^3} = \left(\frac{\mathrm{d}^2}{\mathrm{d}(\mu/T)^2}\frac{p}{T^4}\right)_{T\ \mathrm{fixed}}$$

$$= \frac{9\,T}{V}\left(\langle N_B^2\rangle - \langle N_B\rangle^2\right)$$





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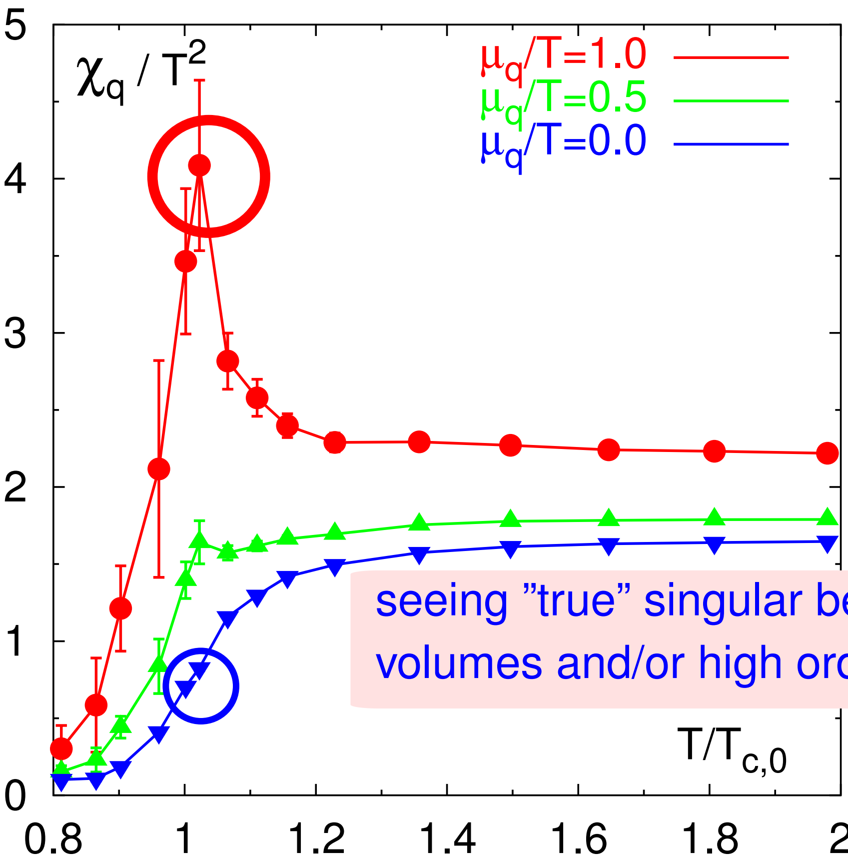
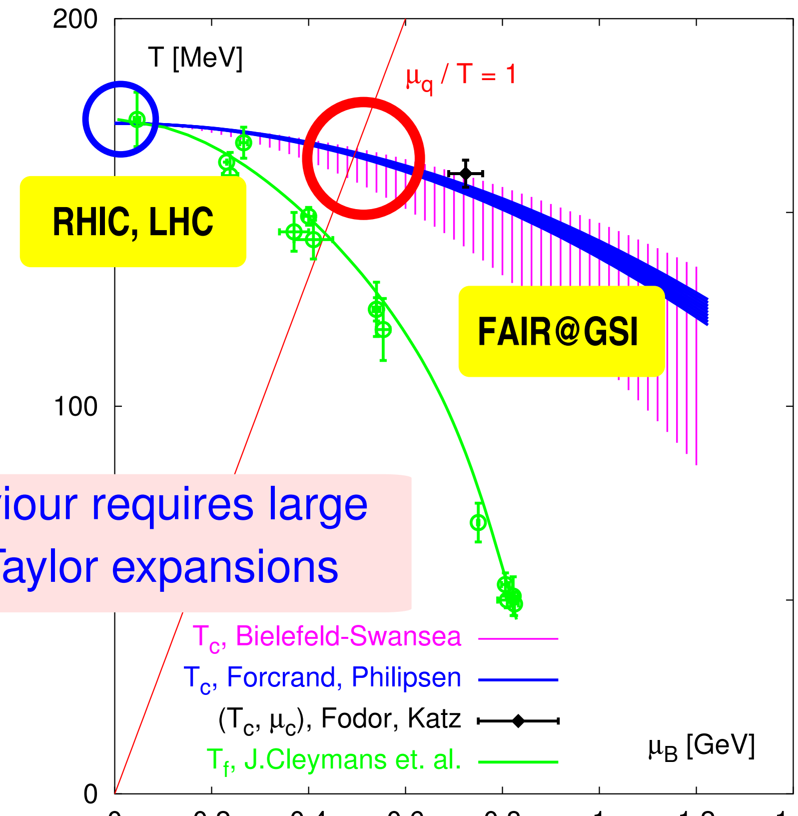


seeing "true" singular behaviour requires large volumes and/or high order Taylor expansions

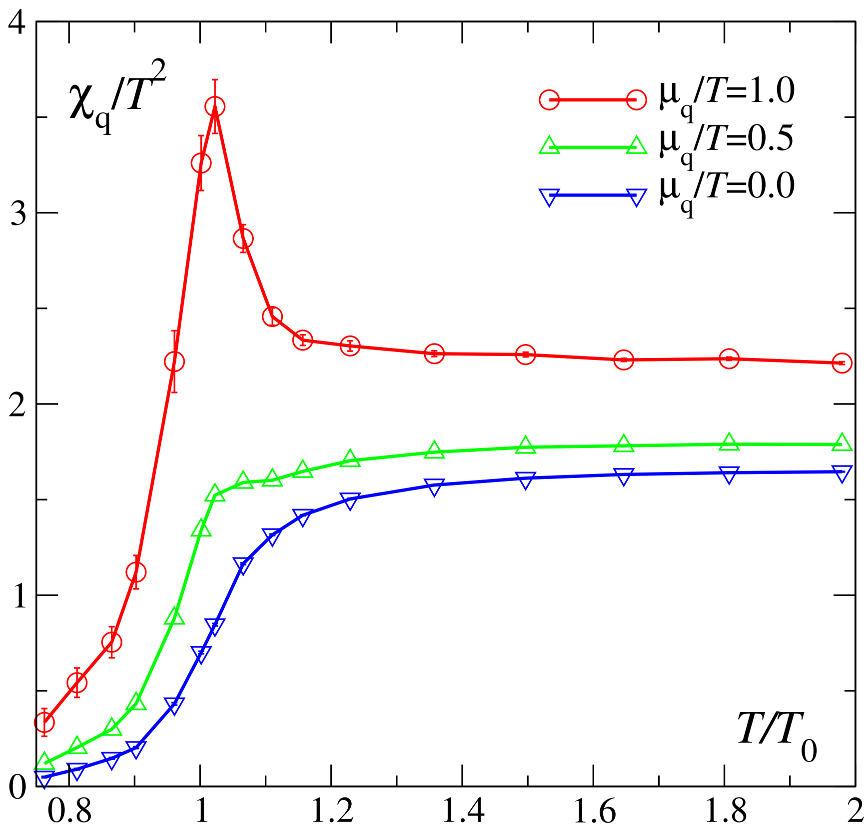


Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations:
$$\frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu_q/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

up to $\mathcal{O}\left((\mu_q/T)^2\right)$

$$= \frac{1}{VT^3} \left(\langle N_q^2 \rangle - \langle N_q \rangle^2 \right)$$



high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations: $\frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu_q/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$

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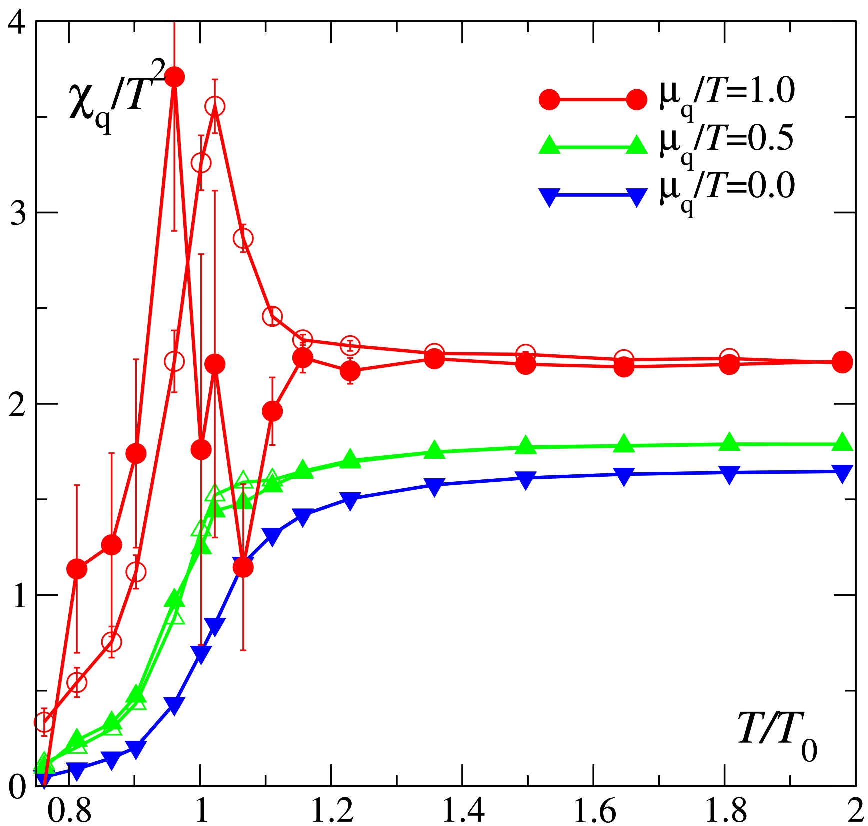

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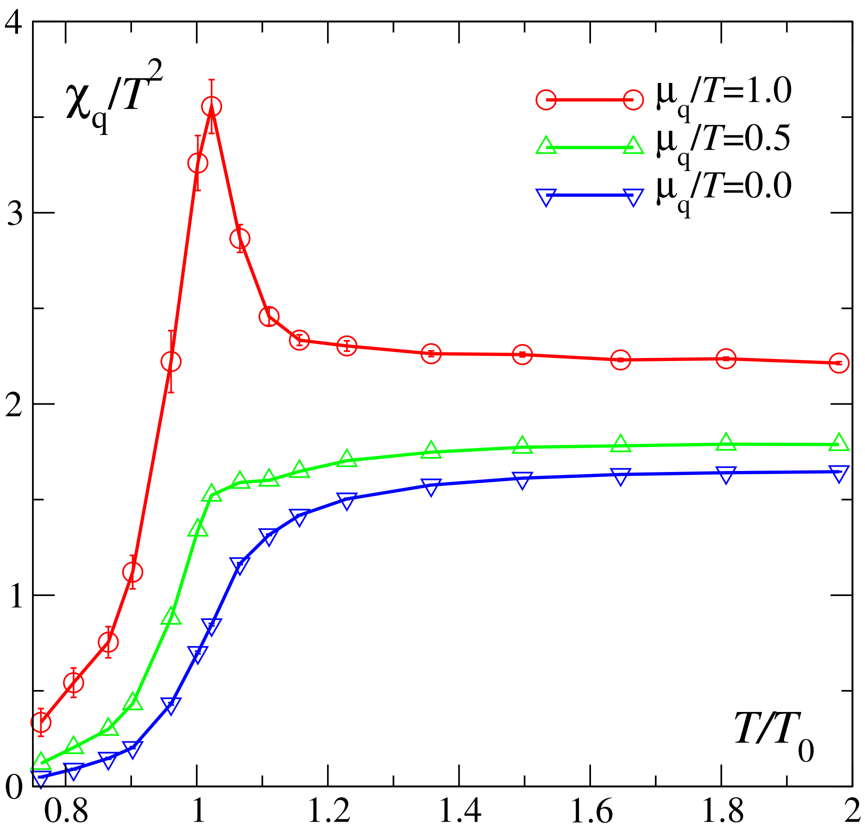


high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2} \left(\frac{\mu_q}{T} \right)^2$$

larger density fluctuations for $\mu_q > 0$;

coming closer to the chiral critical point?

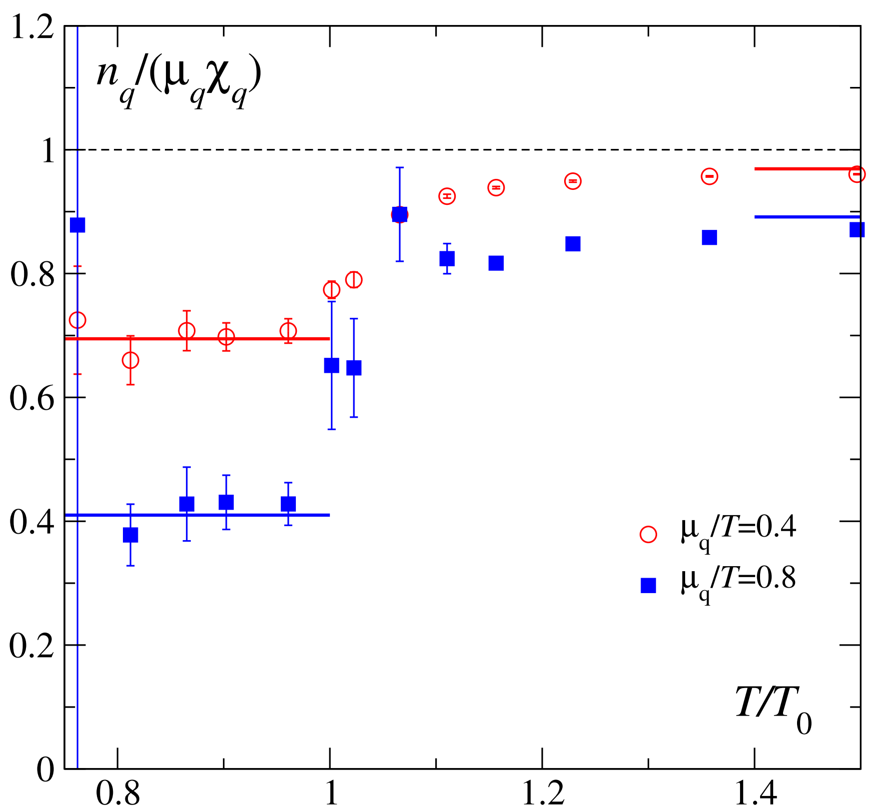
$$\left(\frac{\partial p}{\partial n_q} \right)_T = \frac{n_q}{\chi_q}$$

$\Rightarrow$ χ_q will diverge on chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T\ \mathrm{fixed}}$$



high-T, massless limit: polynomial in (μ_q/T)

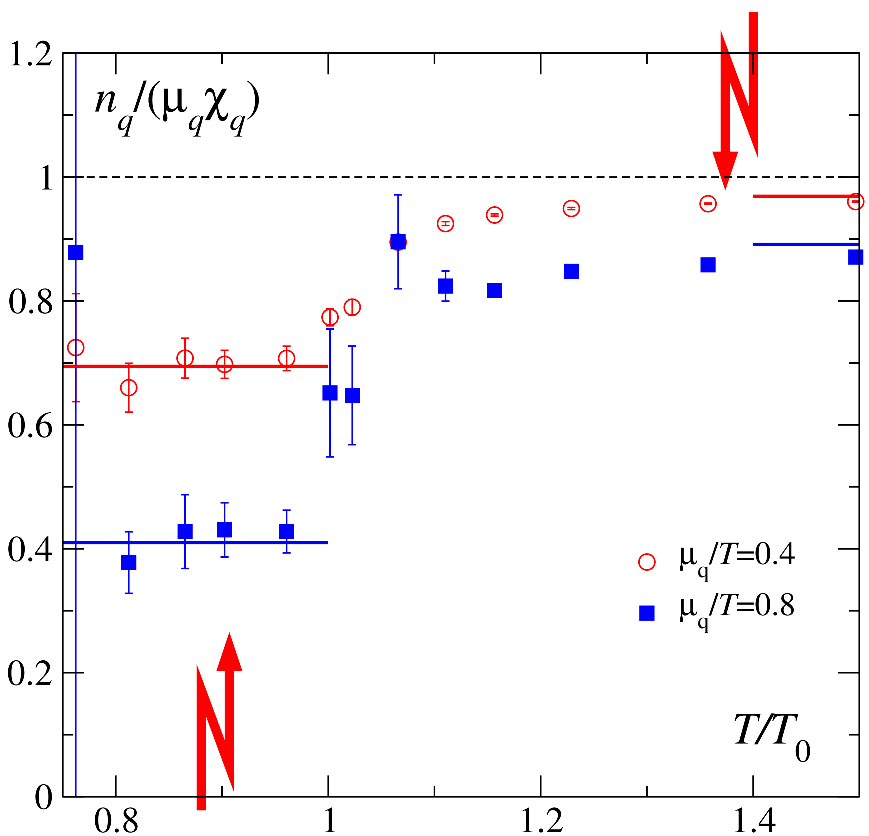
$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T \text{ fixed}}$$

ideal $q\bar{q}$ gas



resonance gas

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

large density fluctuations for $\mu_q > 0,\ T < T_c$

"saturated" by fluctuations in a

hadron resonance gas

expect: $\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q} = 0$

at chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T \text{ fixed}}$$

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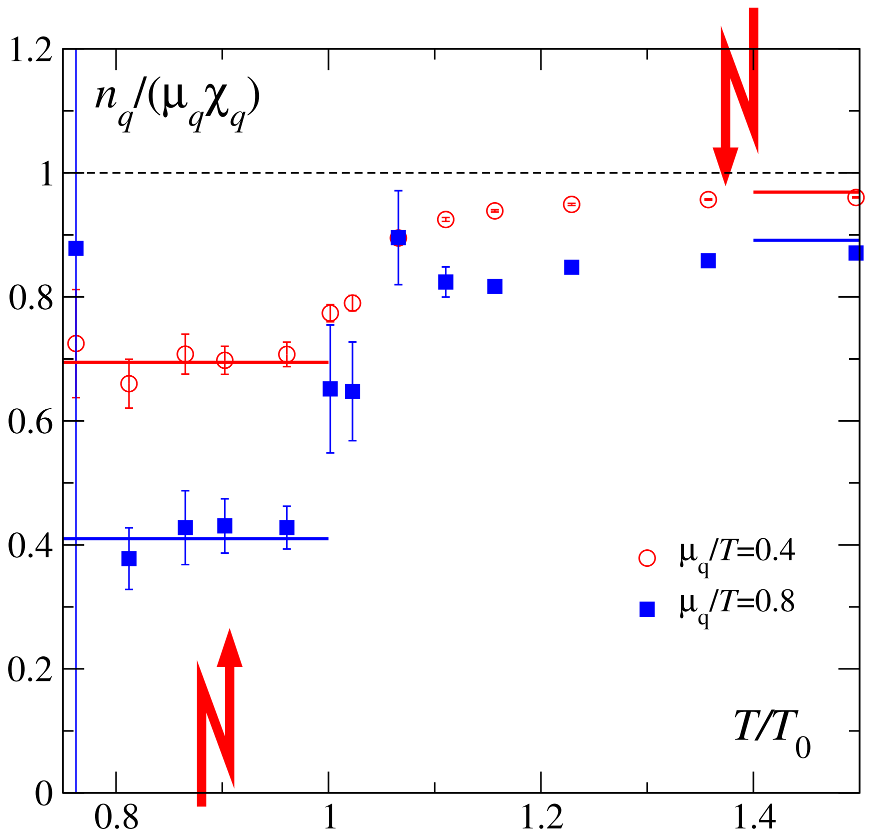

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expect: $\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q} = 0$

at chiral critical point

$m_\pi \simeq 770$ MeV, smaller m_q needed!!

Hadronic fluctuations at $\mu_q = 0$ from Taylor expansion coefficients for $\mu_q > 0$

S. Ejiri, FK, K.Redlich, hep-ph/0509051

- quark number and isospin chemical potentials:

$$\mu_q = \frac{1}{2}(\mu_u + \mu_d), \quad \mu_I = \frac{1}{2}(\mu_u - \mu_d)$$

- expansion coefficients evaluated at $\mu_{q,I} = 0$ are related to hadronic fluctuations at $\mu = 0$:

⇑ baryon number, isospin, charge

event-by-event fluctuations at RHIC and LHC

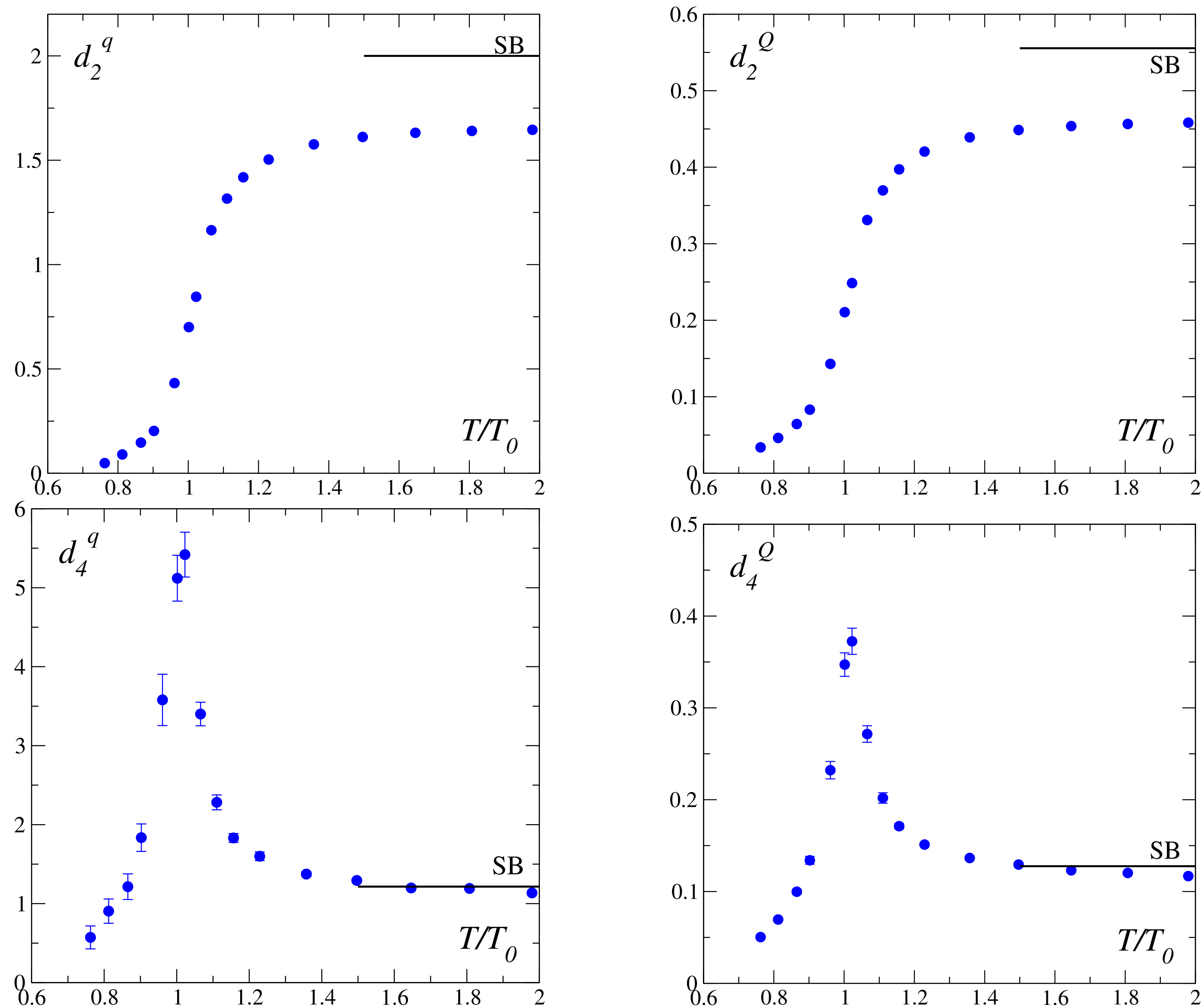
$$d_2^x = \frac{\partial^2 \ln \mathcal{Z}}{\partial(\mu_x/T)^2} = \frac{1}{VT^3}\langle(\delta N_x)^2\rangle_{\mu=0} = \frac{1}{VT^3}\langle N_x^2\rangle_{\mu=0}$$

$$d_4^x = \frac{\partial^4 \ln \mathcal{Z}}{\partial(\mu_x/T)^4} = \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle\right)_{\mu=0} = \frac{1}{VT^3}\left(\langle N_x^4\rangle - 3\langle N_x^2\rangle\right)_{\mu=0}$$

$$\text{with} \quad , \quad x = q, I, Q \quad \text{and} \quad \partial_Q \equiv \frac{2}{3}\frac{\partial}{\partial\mu_u/T} - \frac{1}{3}\frac{\partial}{\partial\mu_d/T}$$

Quark number and charge fluctuations at $\mu_B = 0$; 2-flavor QCD ($m_\pi \simeq 770\ MeV$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508



monotonic increase; close to ideal gas value for $T \gtrsim 1.5 T_c$

develops cusp at T_c

reaches ideal gas value for $T \gtrsim 1.5 T_c$

Charge fluctuations in Boltzmann approximation

- hadronic resonance gas: contributions from isosinglet ($G^{(1)}:\ \eta,\ldots$) and isotriplet ($G^{(3)}:\ \pi,\ldots$) mesons as well as isodoublet ($F^{(2)}:\ p,n,\ldots$) and isoquartet ($F^{(4)}:\ \Delta,\ldots$) baryons

$$\begin{aligned}\frac{p(T,\mu_q,\mu_I)}{T^4} \simeq\ & G^{(1)}(T) + G^{(3)}(T)\frac{1}{3}\left(2\cosh\left(\frac{2\mu_I}{T}\right)+1\right) \\ & + F^{(2)}(T)\cosh\left(\frac{3\mu_q}{T}\right)\cosh\left(\frac{\mu_I}{T}\right) \\ & + F^{(4)}(T)\frac{1}{2}\cosh\left(\frac{3\mu_q}{T}\right)\left[\cosh\left(\frac{\mu_I}{T}\right)+\cosh\left(\frac{3\mu_I}{T}\right)\right]\end{aligned}$$

- charge fluctuations at $\mu_q = \mu_I = 0$;
  isospin quartet $F^{(4)}$ contains baryons carrying charge 2

$$R^Q_{4,2} \equiv \frac{d^Q_4}{d^Q_2} = \frac{4G^{(3)} + 3F^{(2)} + 27F^{(4)}}{4G^{(3)} + 3F^{(2)} + 9F^{(4)}} \rightarrow 1 \text{ for } T \rightarrow 0$$

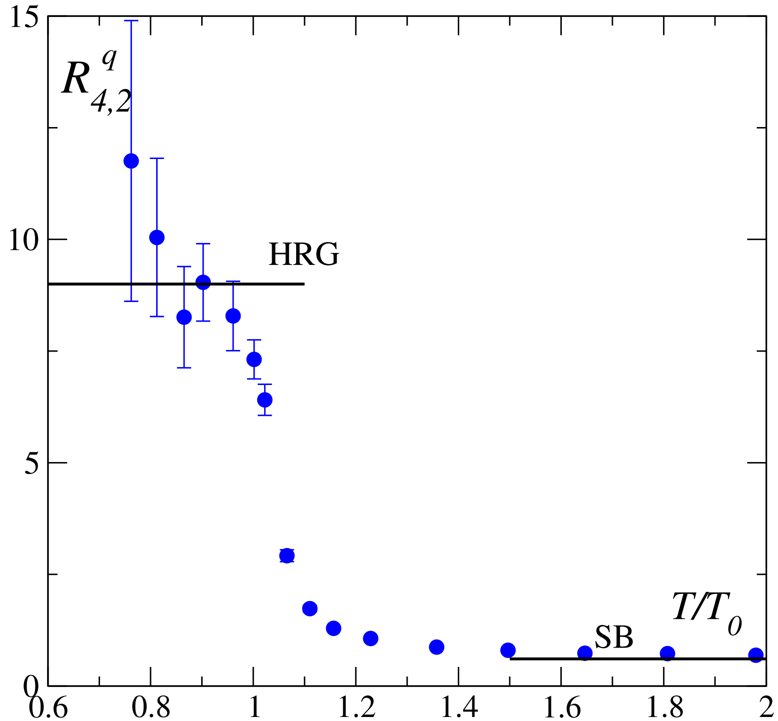
  contribution of doubly charged baryons increases quartic relative to quadratic fluctuations

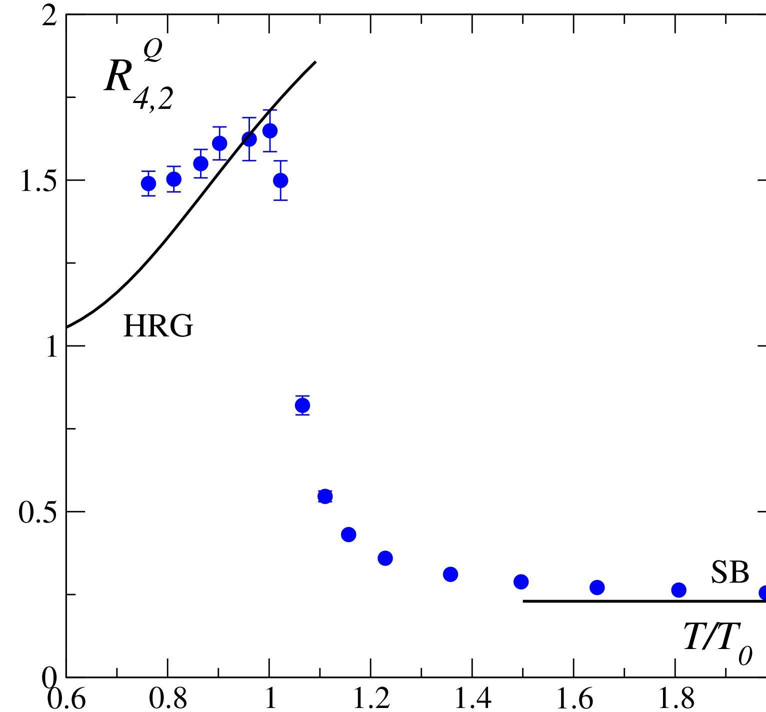
Cumulant ratios

- ratios of cumulants reflect carriers of baryon number and charge

$$R^x_{4,2} = d^x_4 / d^x_2 \quad , \quad x = q,\ Q$$

$$R^q_{4,2} = \begin{cases} 9 & ,\mathrm{HRG} \\ \frac{6}{\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases} \qquad R^Q_{4,2} = \begin{cases} 1 & ,\mathrm{HRG}, T \to 0 \\ \frac{34}{15\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases}$$

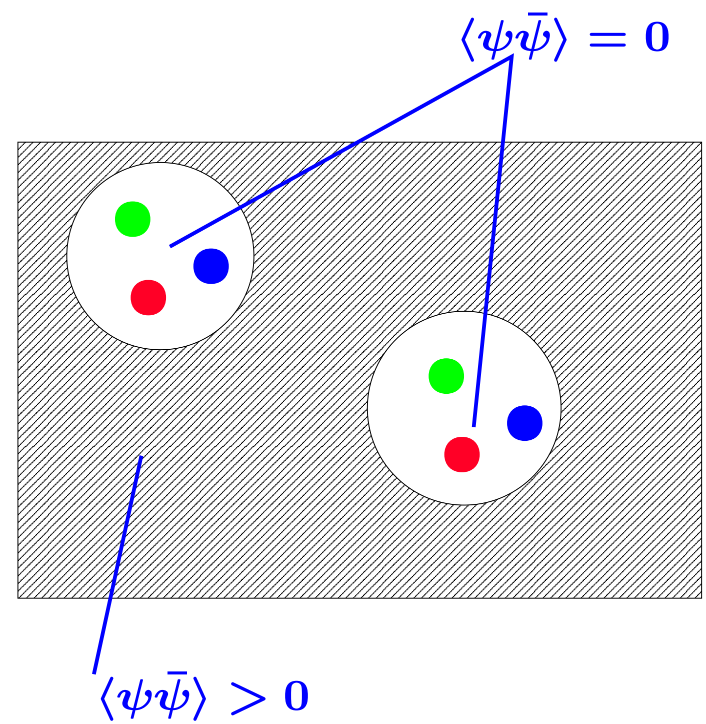




In-medium properties of hadrons

- properties of light quark hadrons reflect chiral symmetry breaking: Goldstone pion, non-degenerate parity partners

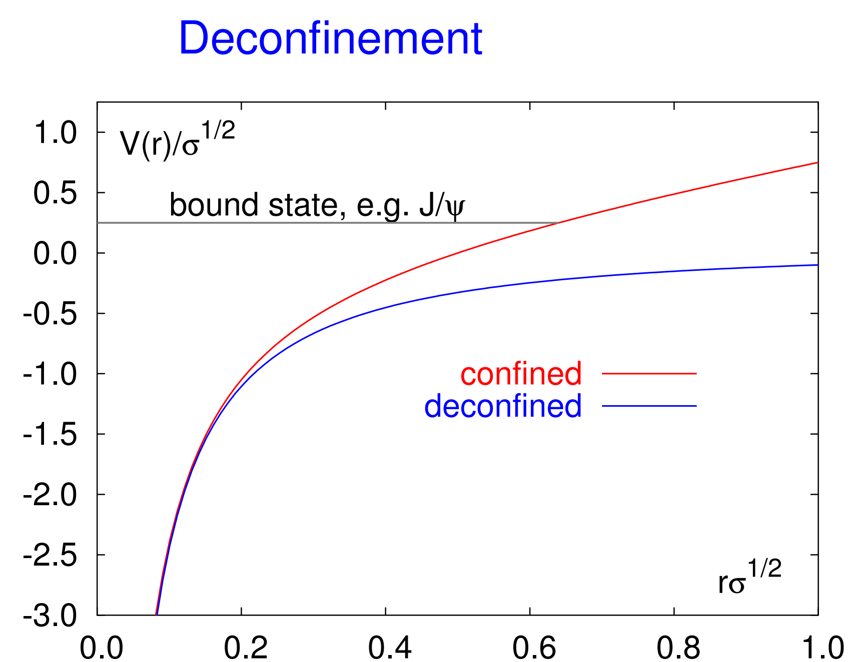
chiral symmetry restoration



In-medium properties of hadrons

- properties of light quark hadrons reflect chiral symmetry breaking: Goldstone pion, non-degenerate parity partners

- properties of heavy quark hadrons reflect structure of the heavy quark potential: quarkonium spectra



In-medium properties of hadrons

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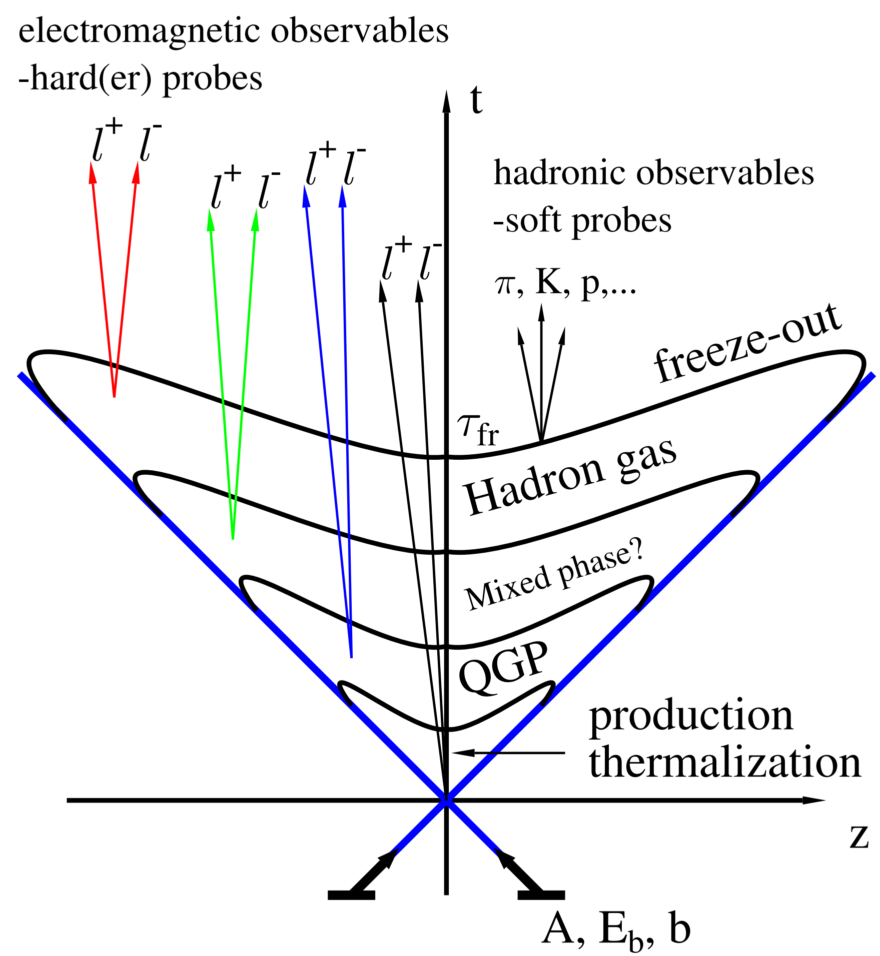
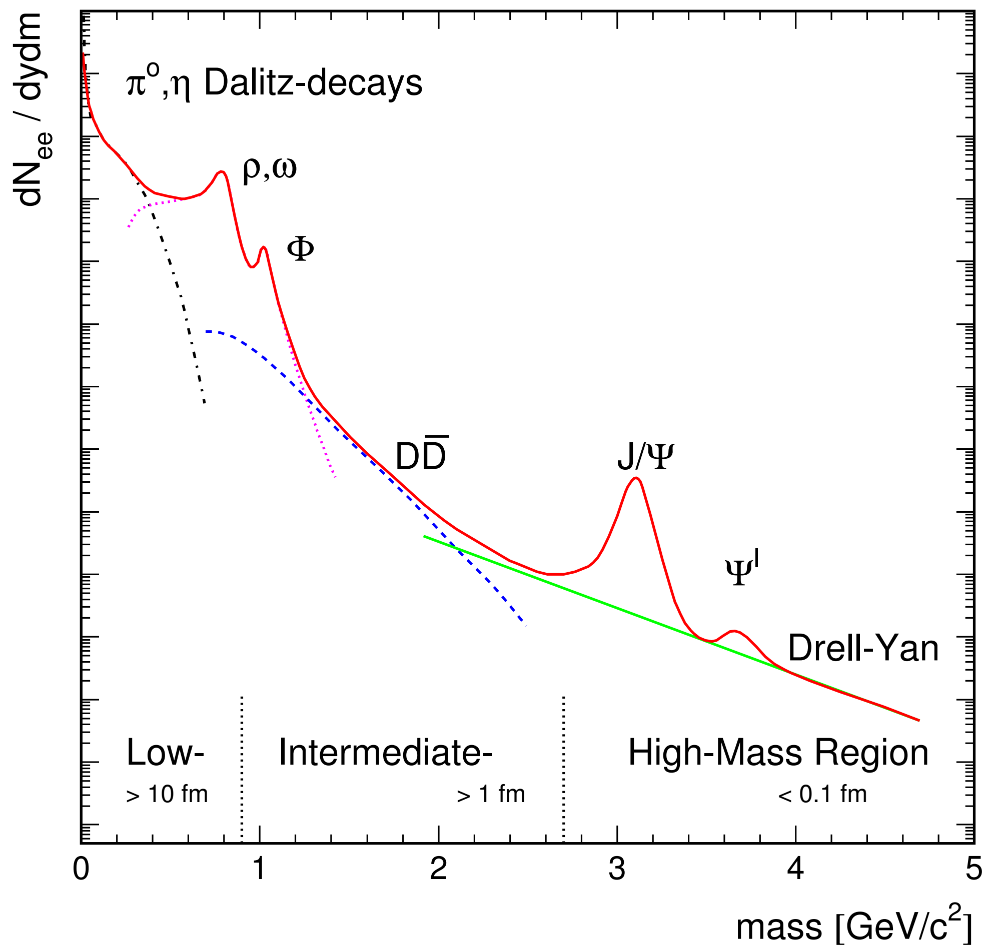
thermal modifications of chiral condensate and heavy quark potential will influence the hadron spectrum



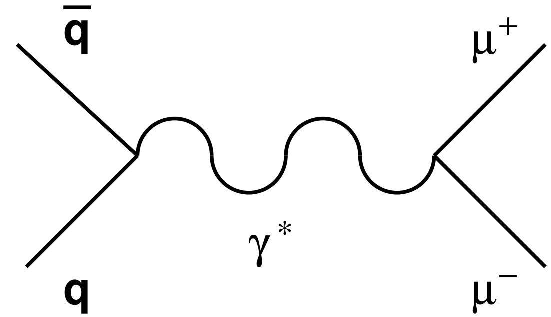
observable consequences in dilepton spectra

Thermal vector meson properties from dilepton rates in heavy ion collisions

dilepton rate vs. invariant mass of l^+l^- pair

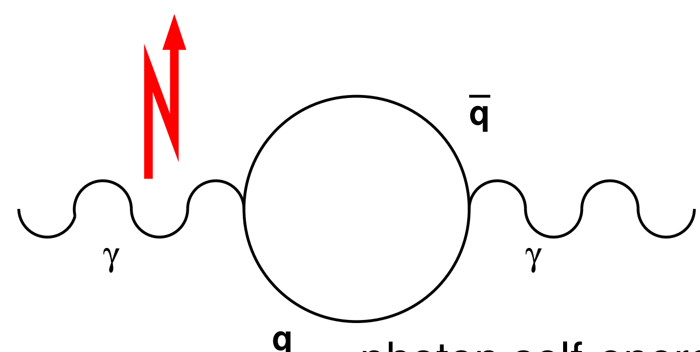


differential cross-section for $\mu^+\mu^-$ pair production



dilepton pair (e^+e^-, $\mu^+\mu^-$) production through annihilation of "thermal" $\bar{q}q$-pairs in hot and dense matter

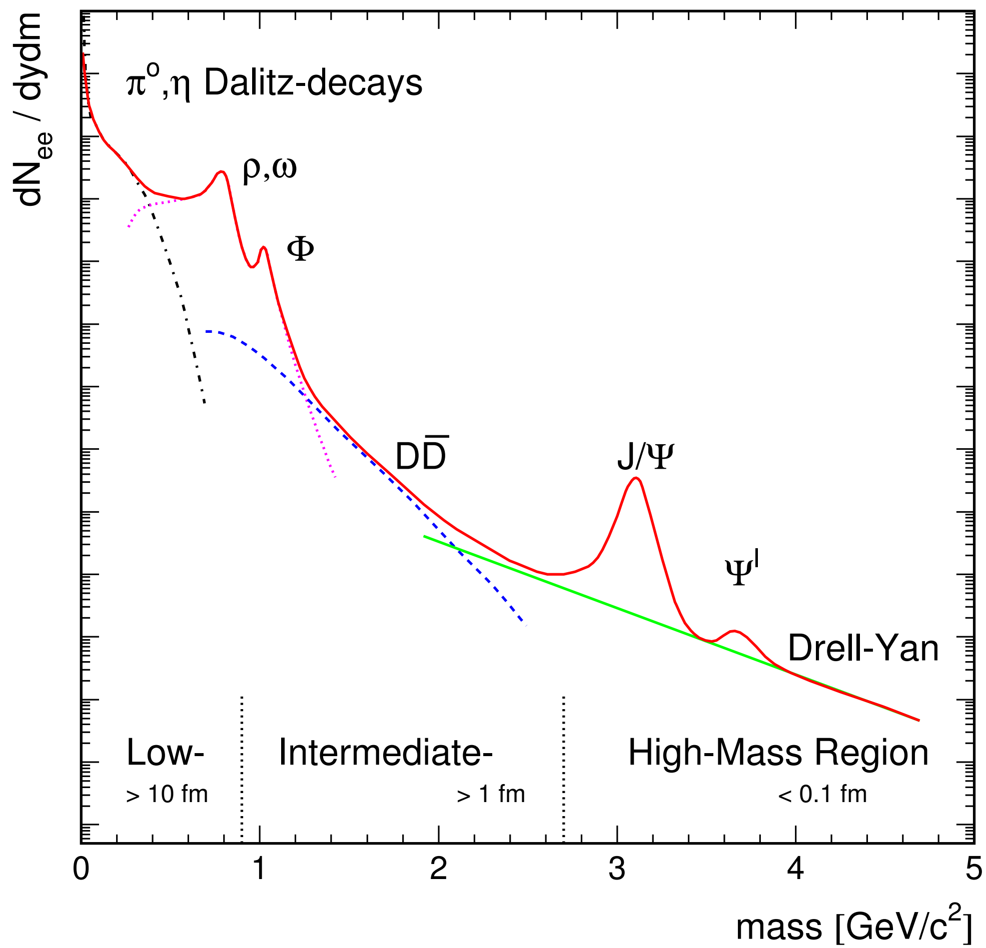
rate $\sim |q\bar{q} \rightarrow \gamma^*|^2 \cdot |l^+l^- \rightarrow \gamma^*|^2$

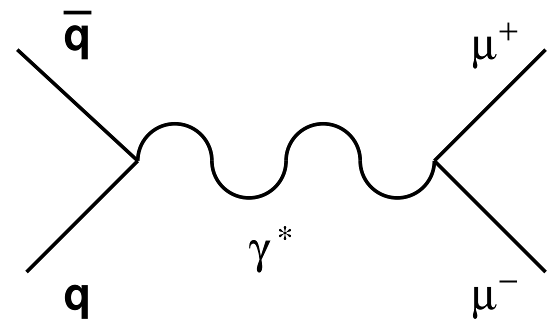


photon self-energy

Thermal vector meson properties from dilepton rates in heavy ion collisions

dilepton rate vs. invariant mass of l^+l^- pair





dilepton pair (e^+e^-, $\mu^+\mu^-$) production through annihilation of "thermal" $\bar{q}q$-pairs in hot and dense matter

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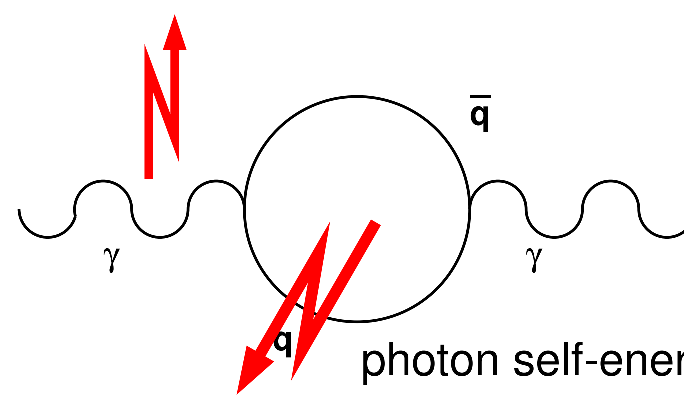


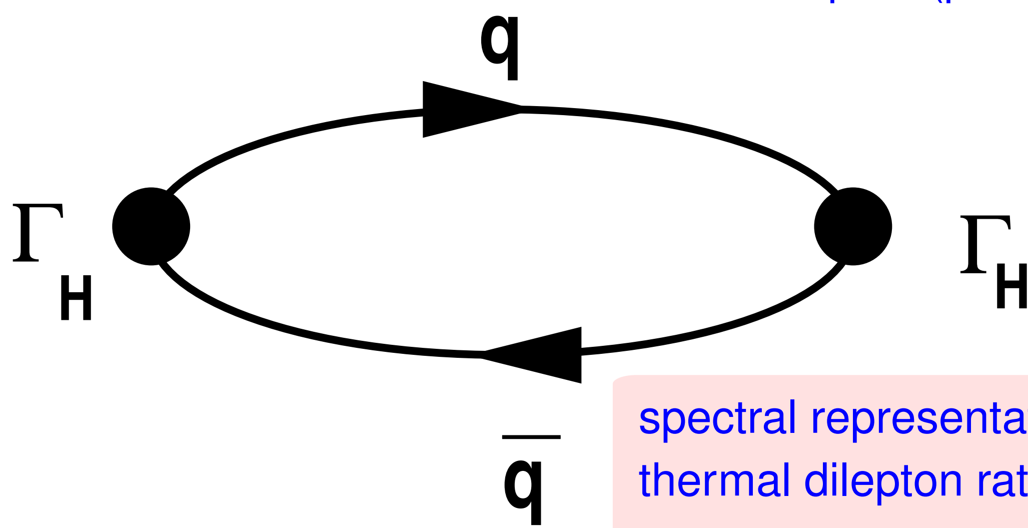
photon self-energy

differential cross-section for $\mu^+\mu^-$ pair production $\Rightarrow$ thermal meson correlation function

Thermal meson correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$-pair

spectral representation of correlator $\Rightarrow$ in-medium properties of hadrons; thermal dilepton (photon) rates



spectral representation of thermal dilepton rate

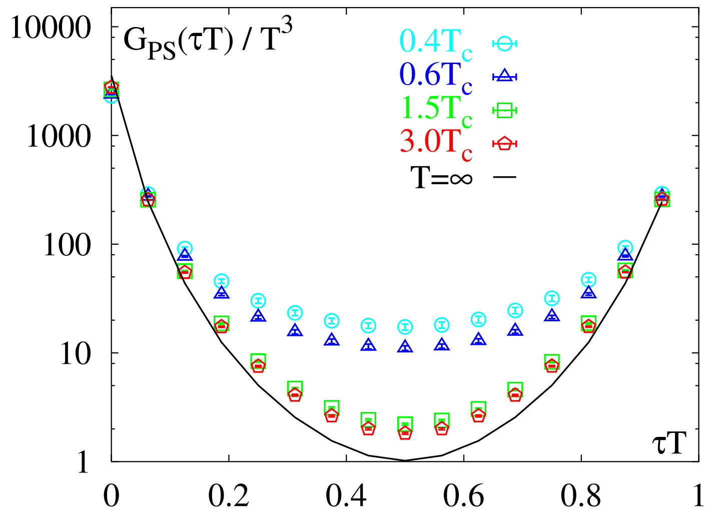
$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

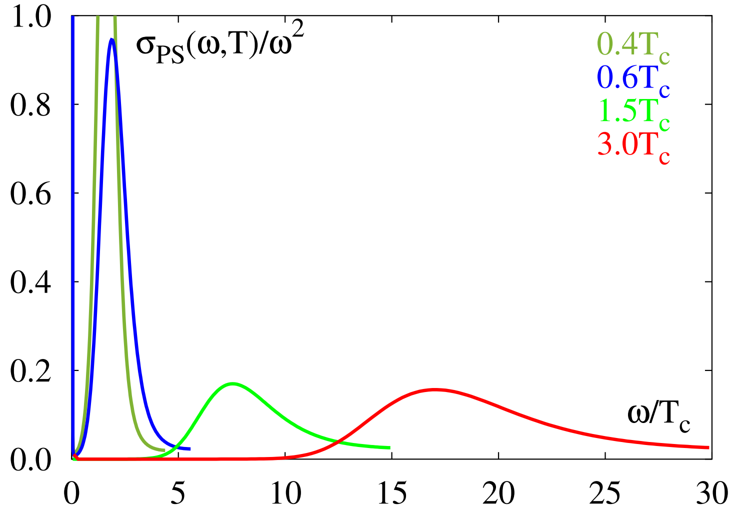
spectral representation of Euclidean correlation functions

$$G_H^\beta(\tau,\vec{r}) = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\, \sigma_H(\omega,\vec{p},T)\, \mathrm{e}^{i\vec{p}\vec{r}}\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

Light quark correlation functions and spectral functions

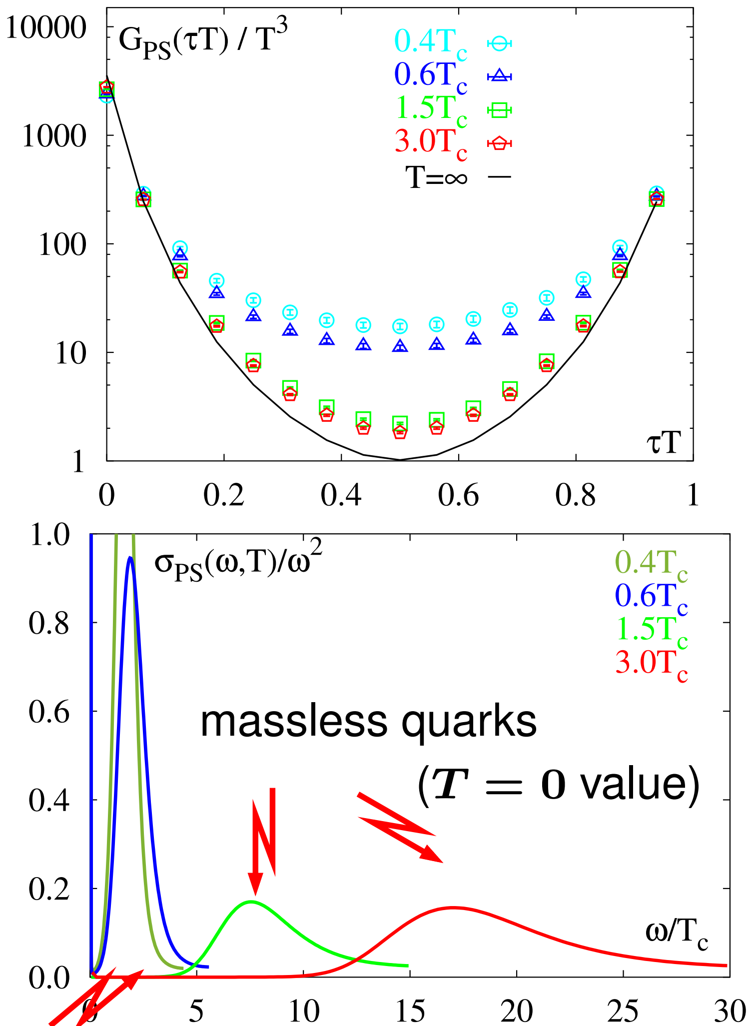
pseudo-scalar spectral functions





Light quark correlation functions and spectral functions

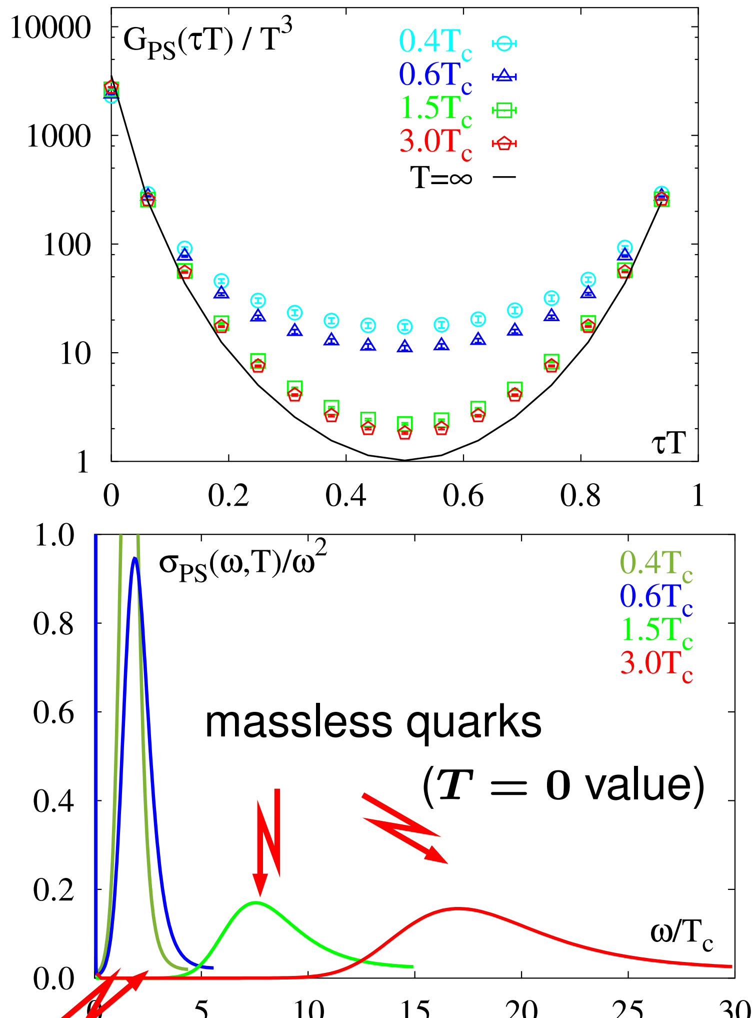
pseudo-scalar spectral functions



quark masses adjusted to give identical m_{PS}

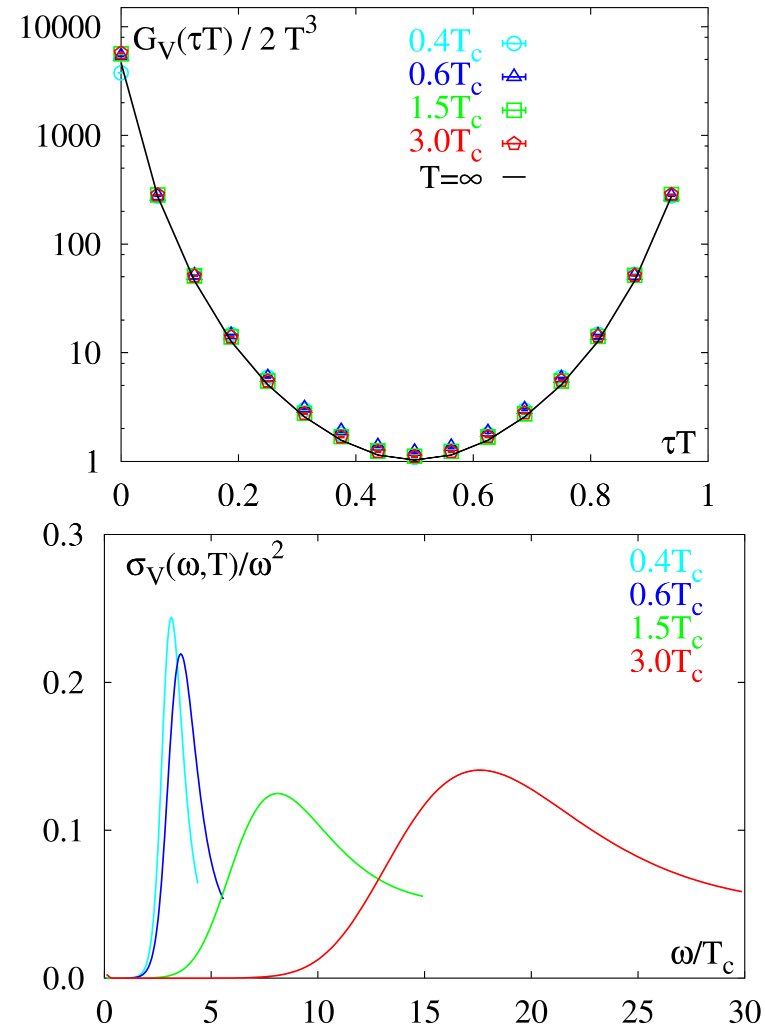
Light quark correlation functions and spectral functions

pseudo-scalar spectral functions



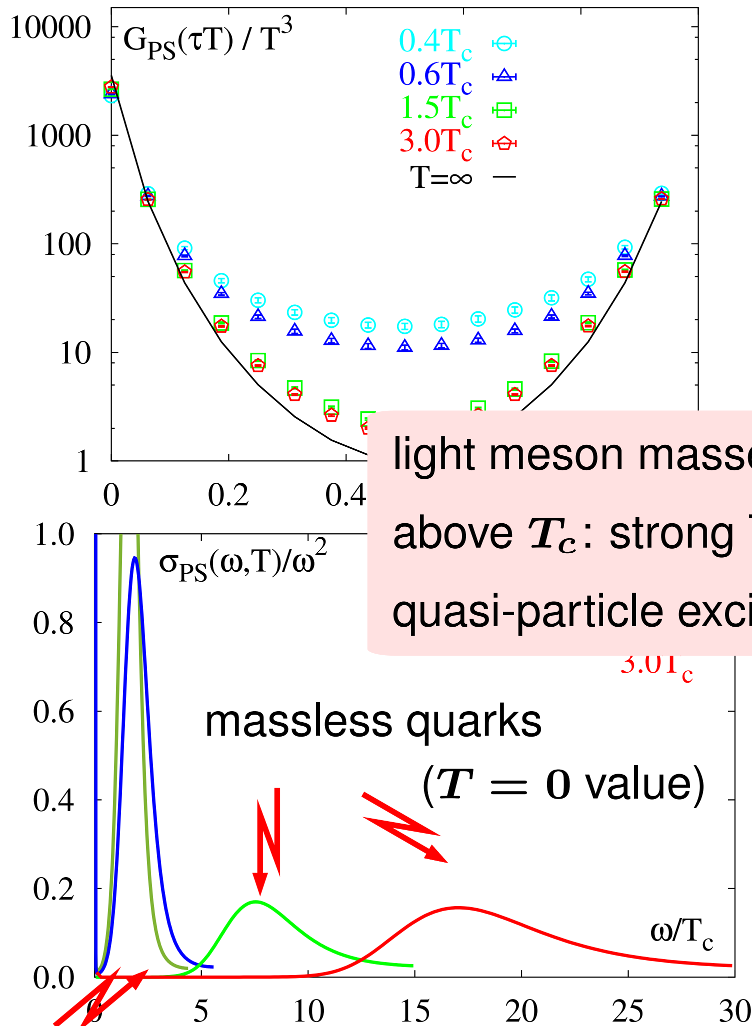
quark masses adjusted to
give identical m_{PS}

vector spectral functions



Light quark correlation functions and spectral functions

pseudo-scalar spectral functions



quark masses adjusted to give identical m_{PS}

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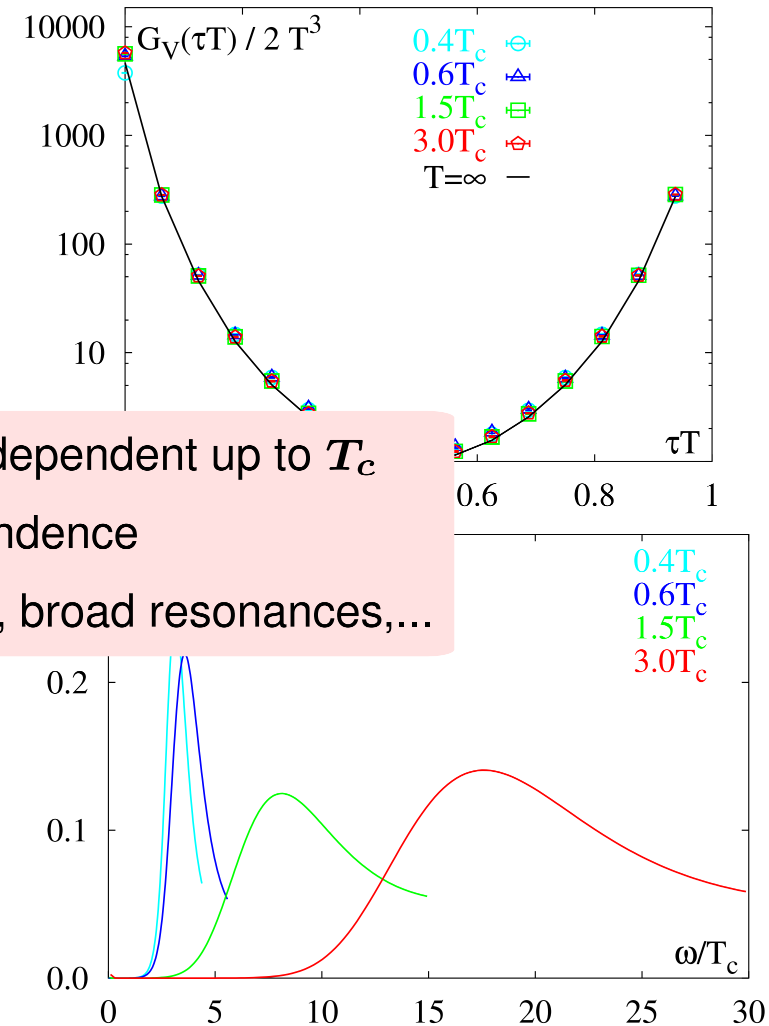
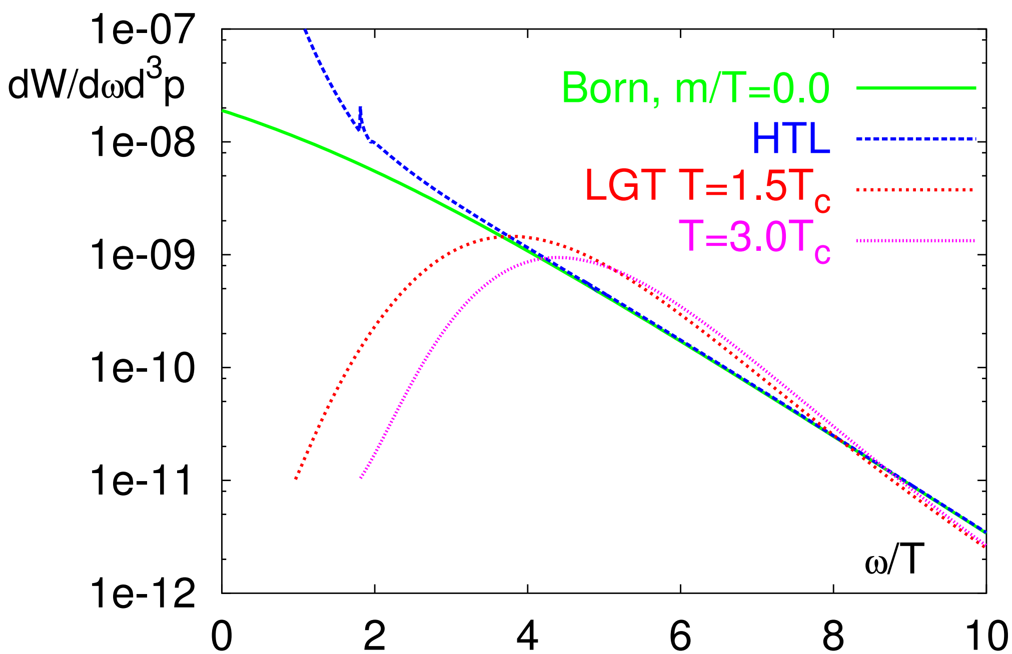


light meson masses T-independent up to T_c

above T_c: strong T-dependence

quasi-particle excitations, broad resonances,...

Dilepton rate: HTL and lattice calculations



thermal dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(e^{\omega/T}-1)}$$

HTL and lattice disagree for

$\omega/T \lesssim (3-4)$

- infra-red sensitivity of HTL-calculations ⇔ "massless gluon" cut in HTL-propagator
- infra-red sensitivity of lattice calculations ⇔ thermodynamic limit, $V \to \infty$
- $VT^3 = (N_\sigma/N_\tau)^3 < \infty$ ⇒ momentum cut-off: $p/T > 2\pi N_\tau/N_\sigma$

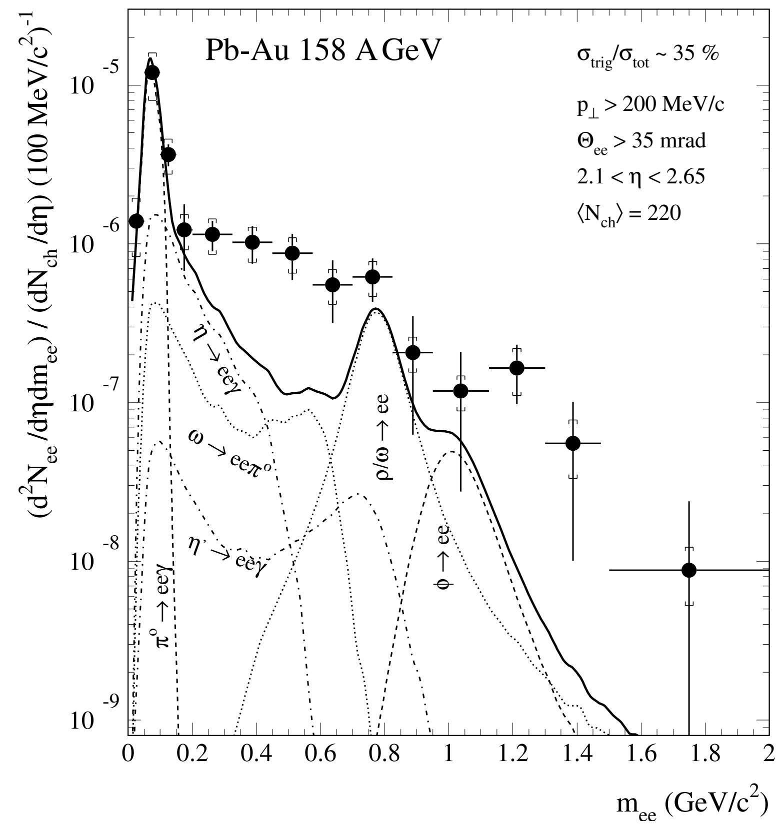
⇛ need large lattices to analyze infra-red regime

⇛ in future also thermal photon rates

Thermal dilepton enhancement and the disappearance of the ρ-meson

low mass dilepton rate

differential cross-section for e^+e^- pair production



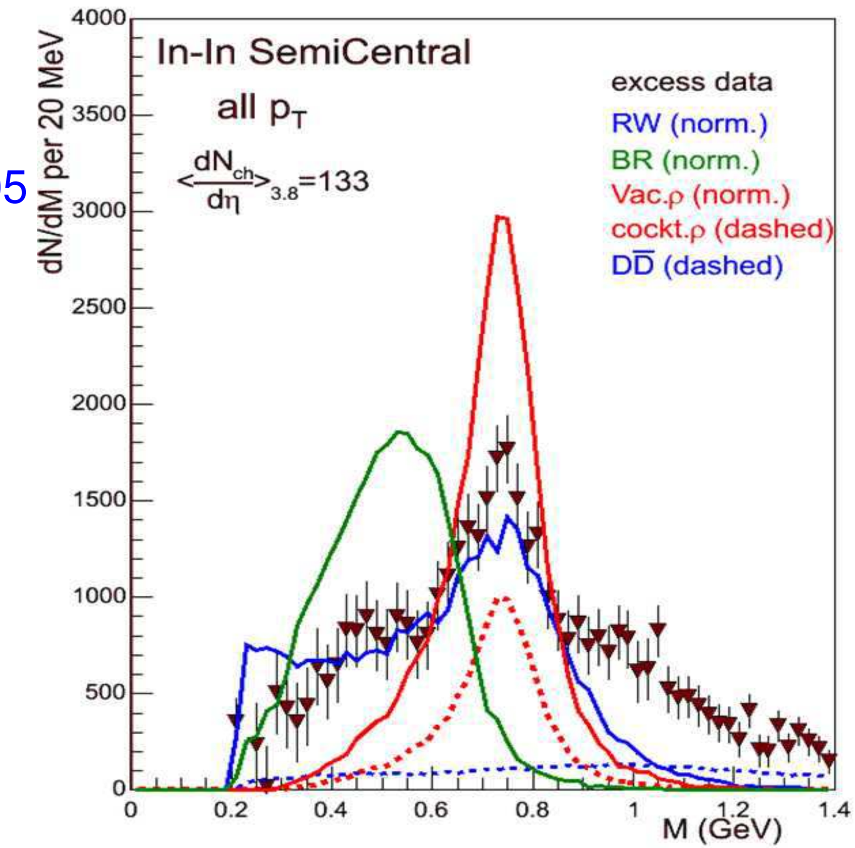
invariant mass of the pair

G. Agakichiev (CERES Collaboration), Nucl.Phys. A661 (1999) 23

Thermal dilepton enhancement and the disappearance of the ρ-meson

low mass dilepton rate

rho spectral function: CERES, QM 2005

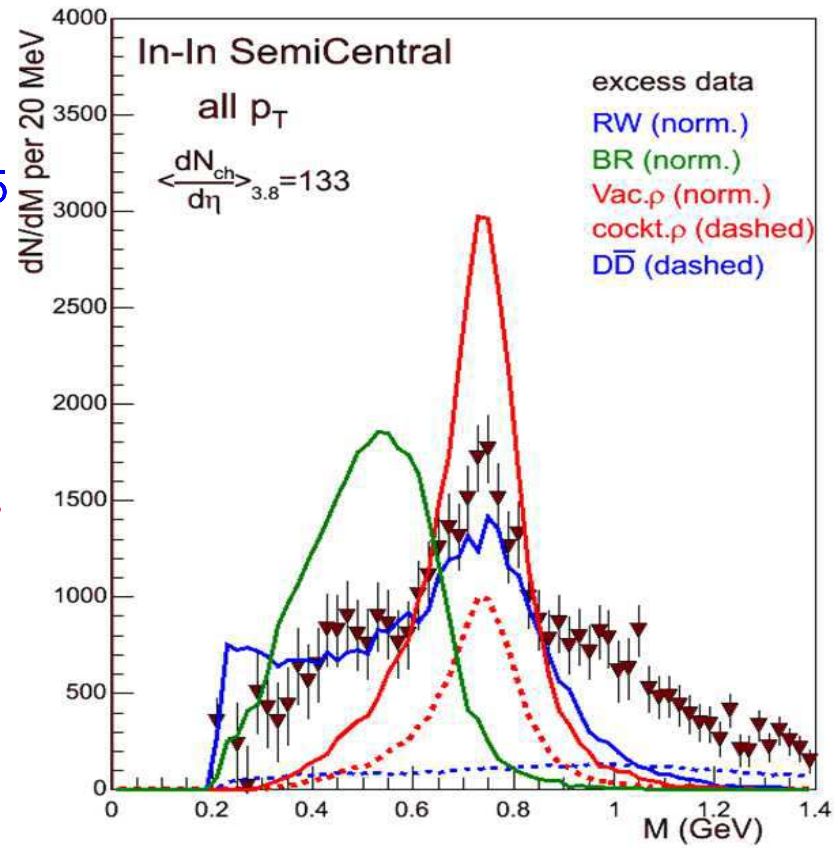


invariant mass of the pair

Thermal dilepton enhancement and the disappearance of the ρ-meson

low mass dilepton rate

rho spectral function: CERES, QM 2005



invariant mass of the pair

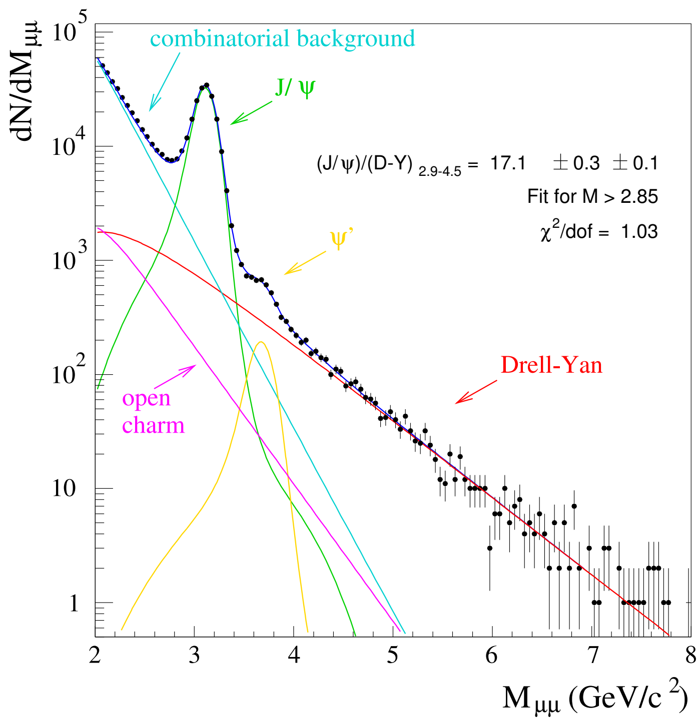
Future:

need analysis of thermal correlators with light dynamical quarks to understand resonance broadening

Charmonium suppression in heavy ion collisions (SPS, RHIC)

Charmonium production: dilepton rate

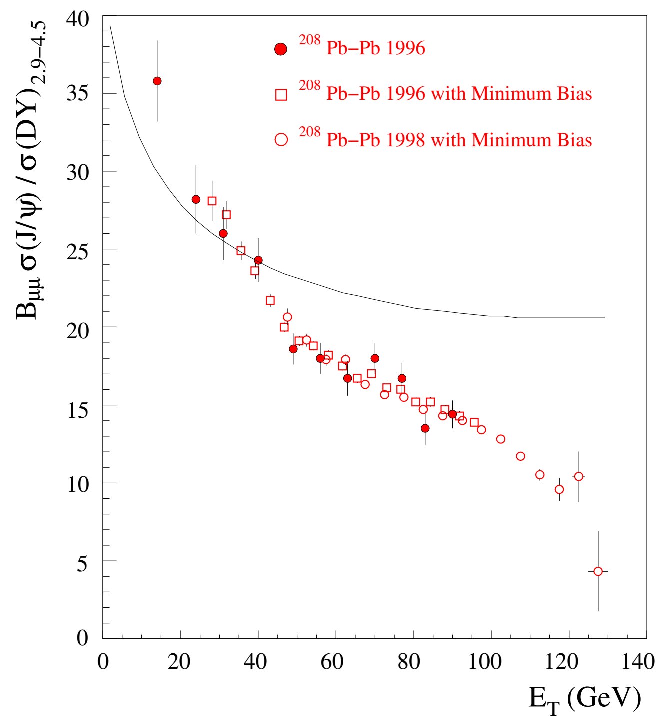
Suppression Pattern: RHIC



invariant mass of the pair

differential cross-section for $\mu^+\mu^-$ pair production

Suppression Pattern: SPS



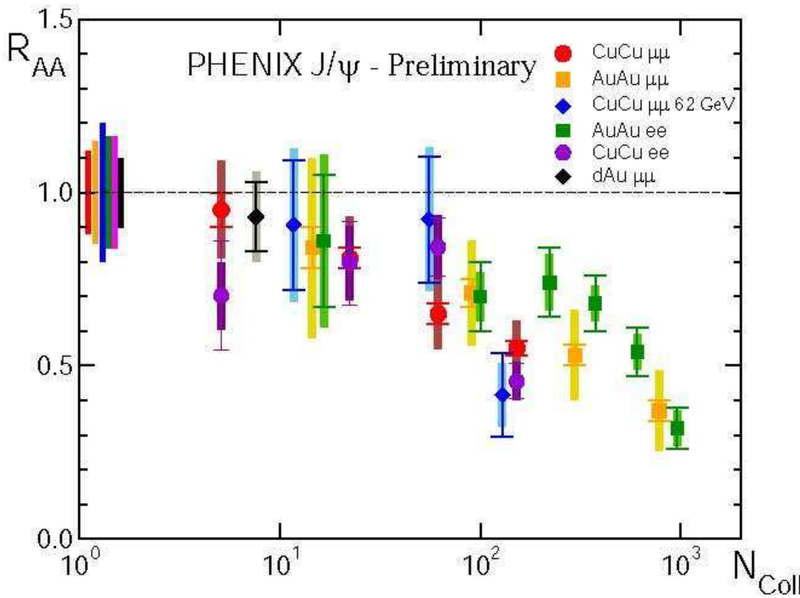
$\sim$ energy density

total cross-section for $\mu^+\mu^-$ pair production

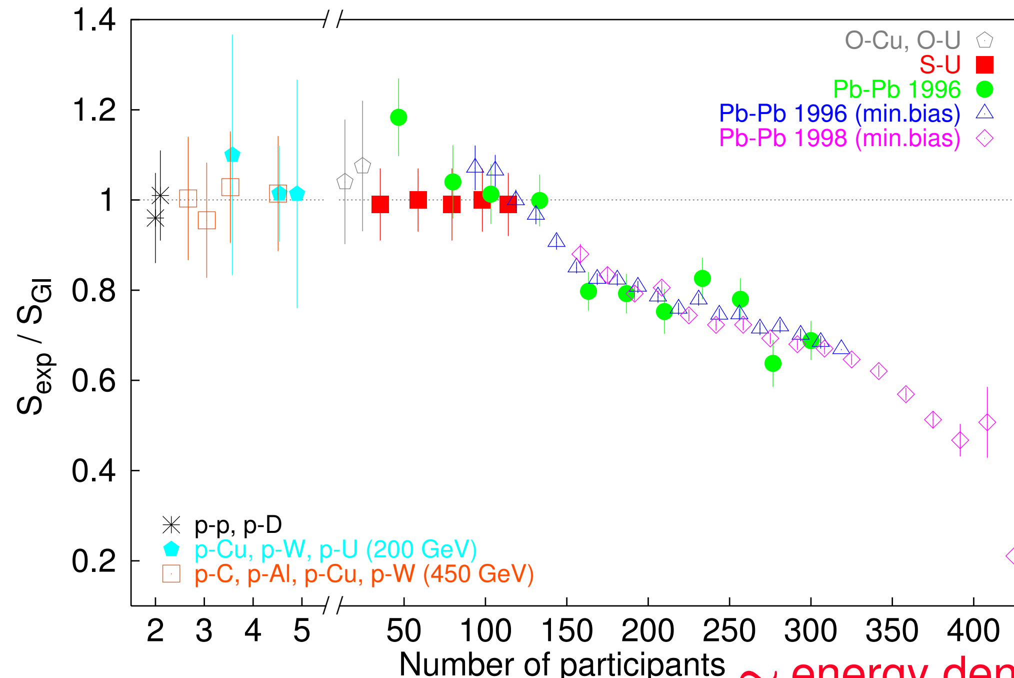
M.C.Abreu (NA50), Phys.Lett. B477 (2000) 28

Charmonium suppression in heavy ion collisions (SPS, RHIC)

Suppression Pattern: RHIC



Suppression Pattern: SPS



∼ energy density

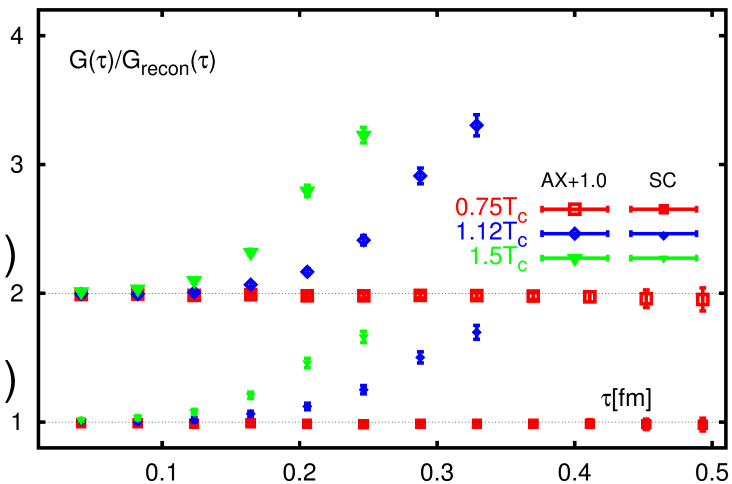
measured A-A rate normalized to
rate expected from known p-A collisions

Heavy quark spectral functions and correlation functions

data for $G_H(\tau, T)$ over reconstructed correlation functions at T from data below T_c

(χ_{c1})

(χ_{c0})



scalar and axial-vector correlation functions:

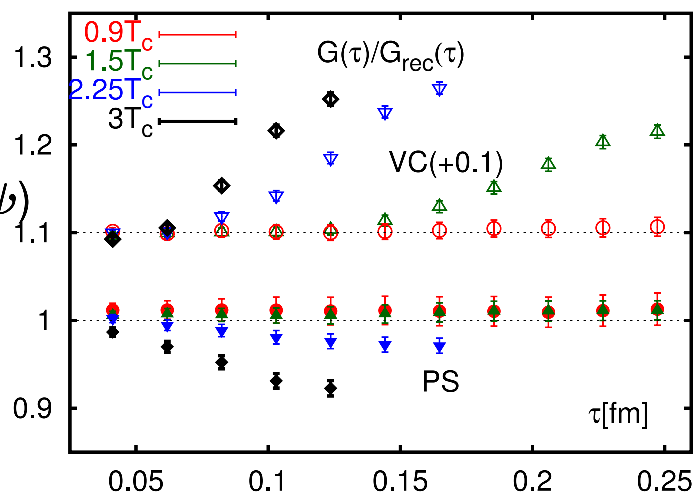
strong temperature dependence just above T_c for χ_c states

(normalized at $T < T_c$)

$(48^3 \times N_\tau,\ N_\tau = 12,\ 16,\ 24,\ a = 0.04$ fm)

(J/ψ)

(η_c)



vector and pseudoscalar correlation functions:

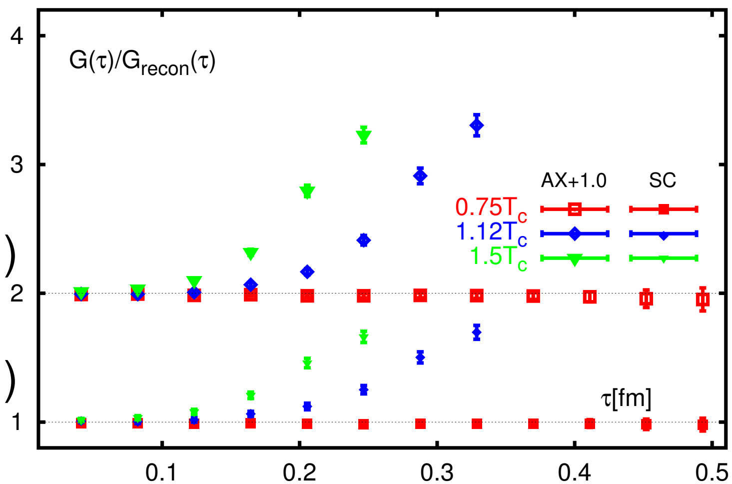
no temperature dependence for η_c up to $1.5\ T_c$;
only mild but systematic temperature dependence of J/ψ

(normalized at $T < T_c$)

$(N_\sigma = 40,\ 48,\ 64,$
$N_\tau = 12,\ 16,\ 24,\ 40,\ a = 0.02$ fm)

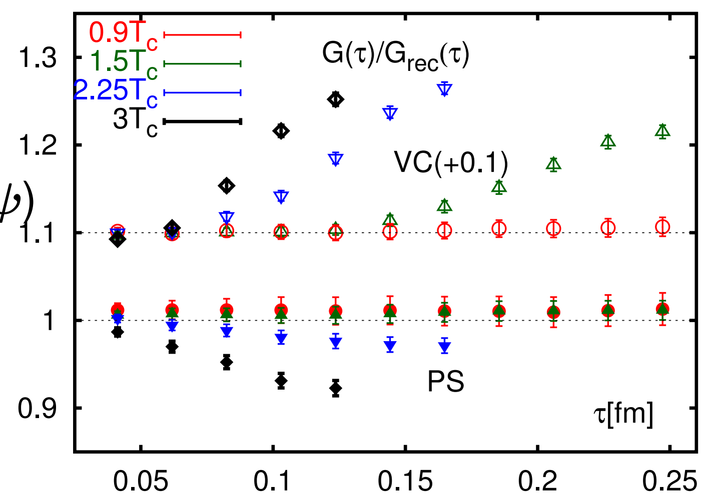
S. Datta et al., Phys. Rev. D69 (2004) 094507

Heavy quark spectral functions and correlation functions



(χ_{c1})

(χ_{c0})



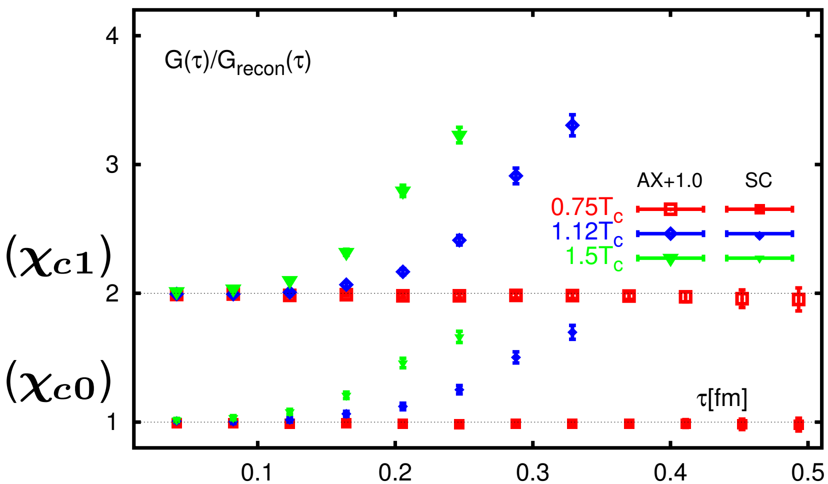
(J/ψ)

(η_c)

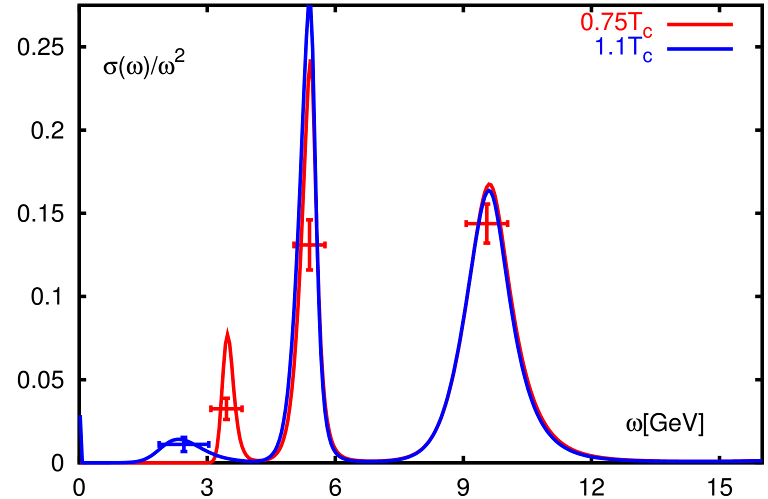
pattern seen in correlation functions also visible in spectral functions

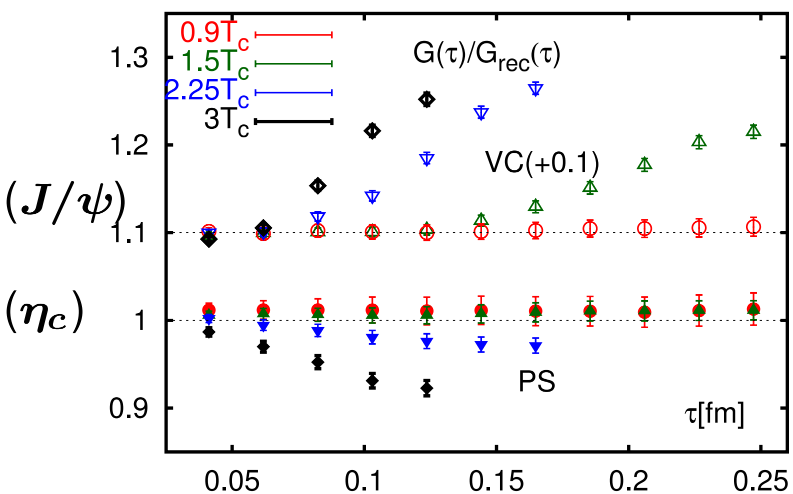
S. Datta et al., Phys. Rev. D69 (2004) 094507

Heavy quark spectral functions and correlation functions

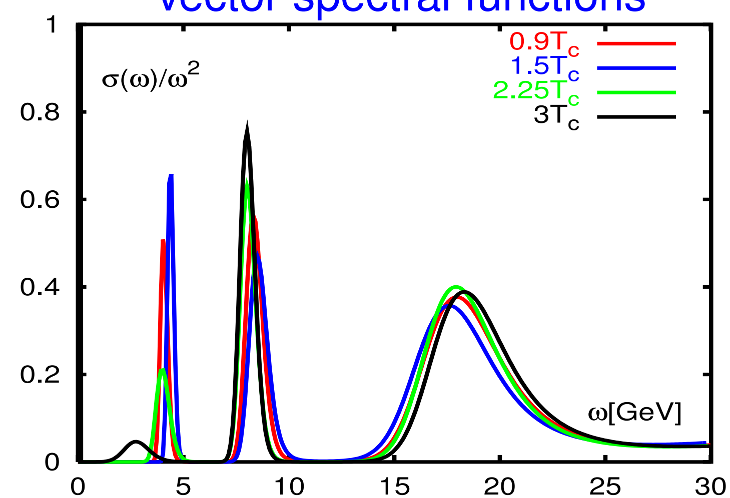


scalar spectral functions



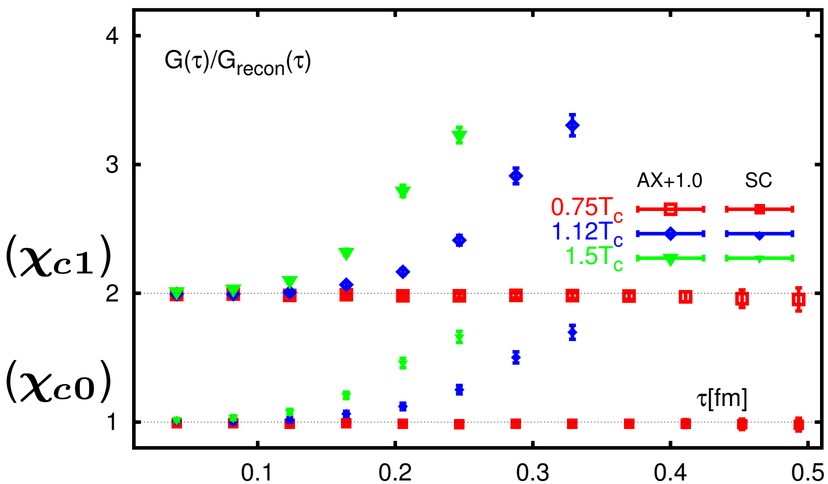


vector spectral functions

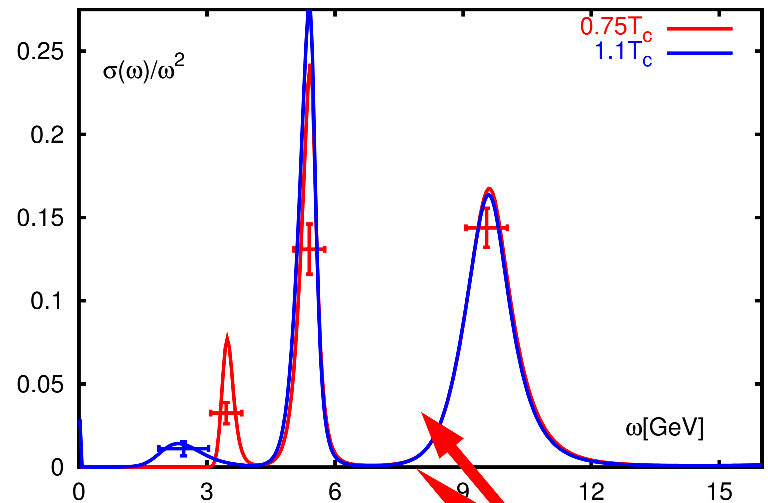


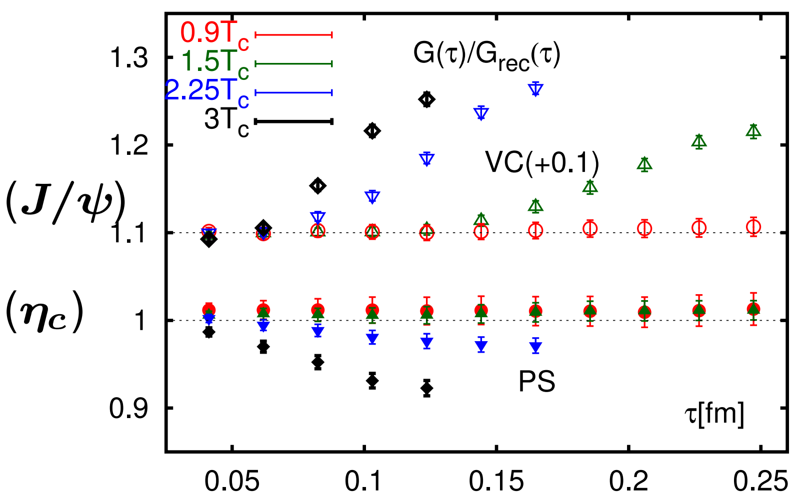
S. Datta et al., Phys. Rev. D69 (2004) 094507

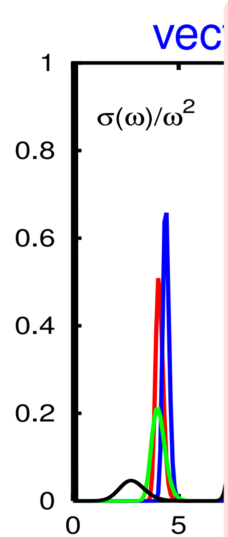
Heavy quark spectral functions and correlation functions



scalar spectral functions







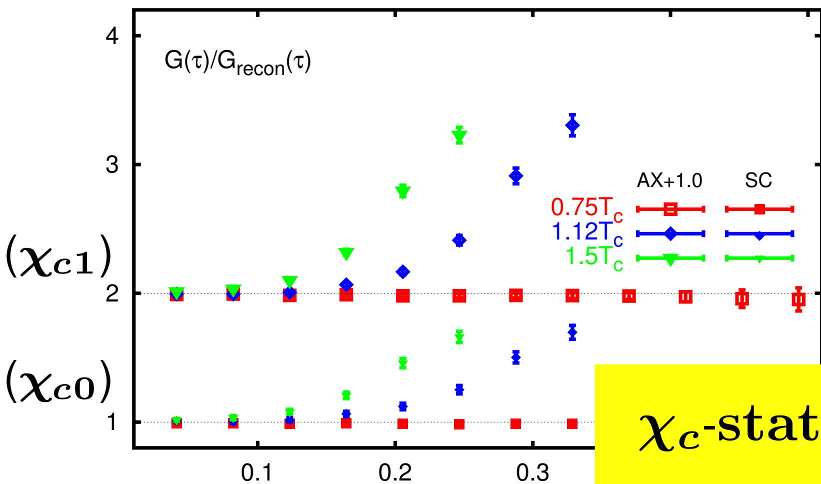
ultra-violet cut-off effects:
Wilson-doubler;
finite Brillouin zone;

need to get better control over lattice cut-off effects

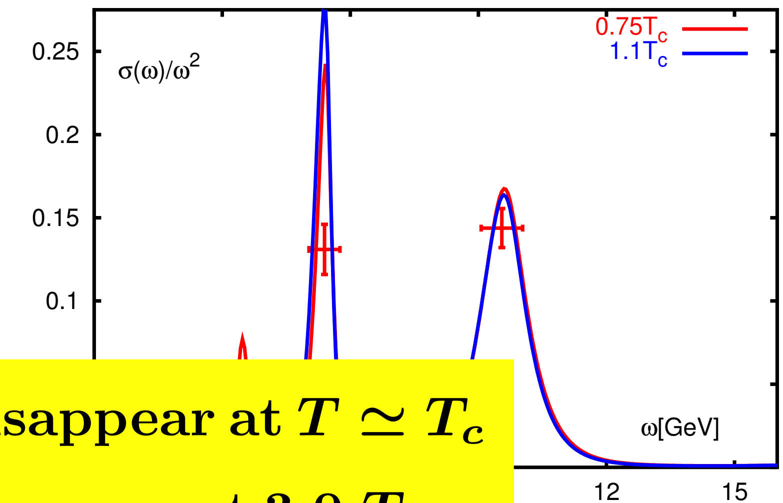
resolution statistics limitted

S. Datta et al., Phys. Rev. D69 (2004) 094507

Heavy quark spectral functions and correlation functions

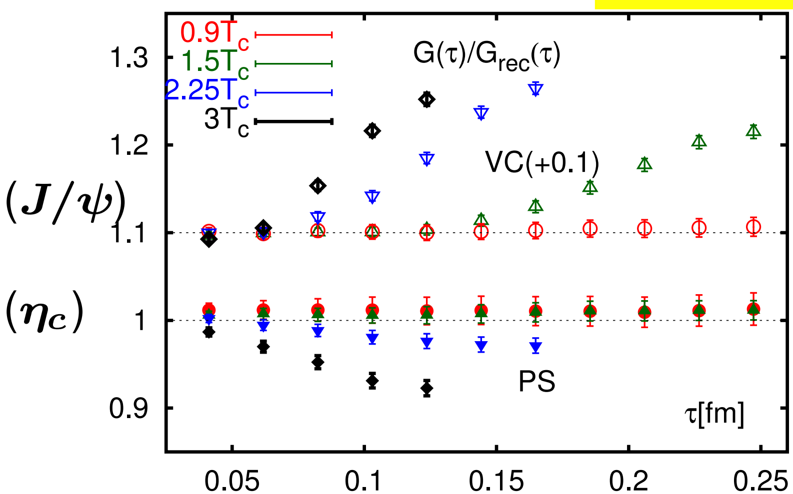


scalar spectral functions

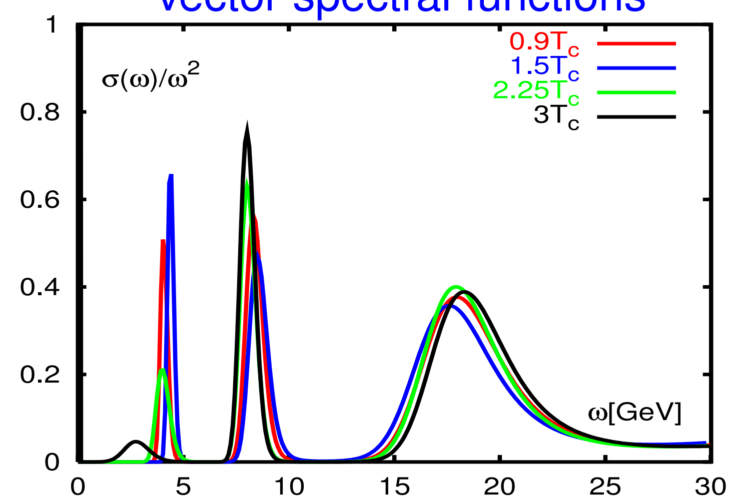


χ_c-states disappear at $T \simeq T_c$

J/ψ and η_c gone at 3.0 T_c

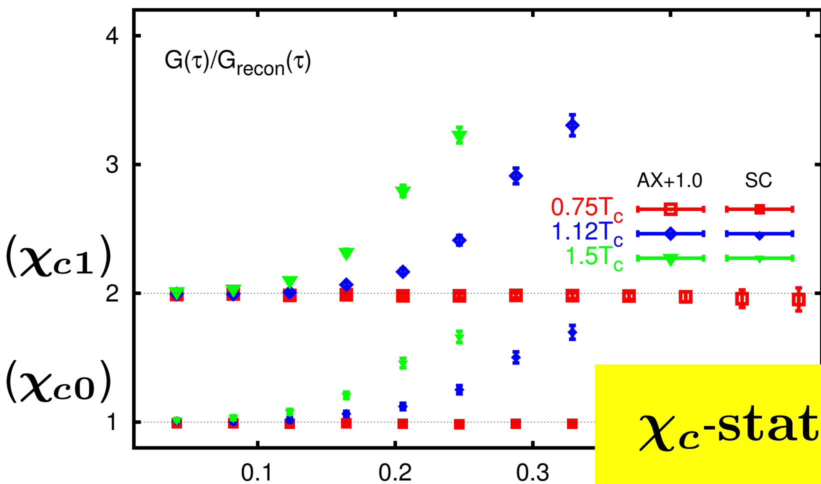


vector spectral functions

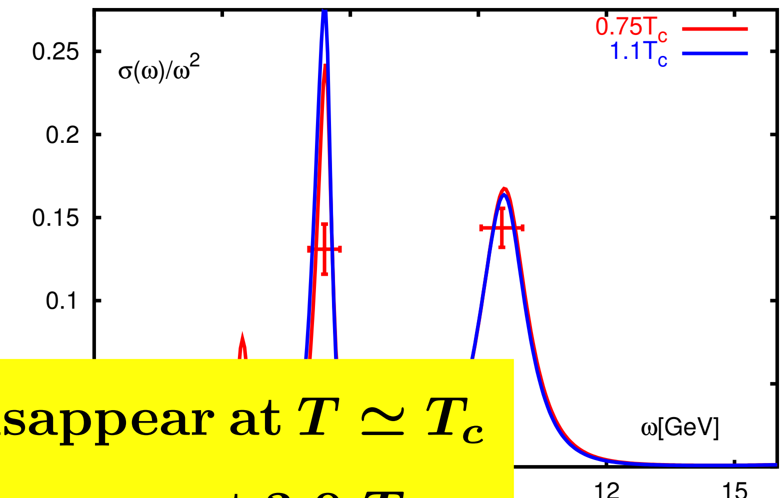


S. Datta et al., Phys. Rev. D69 (2004) 094507

Heavy quark spectral functions and correlation functions



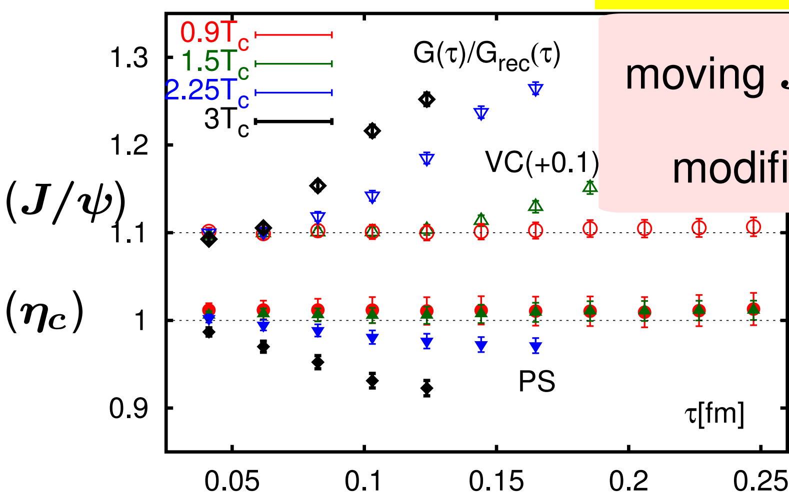
scalar spectral functions



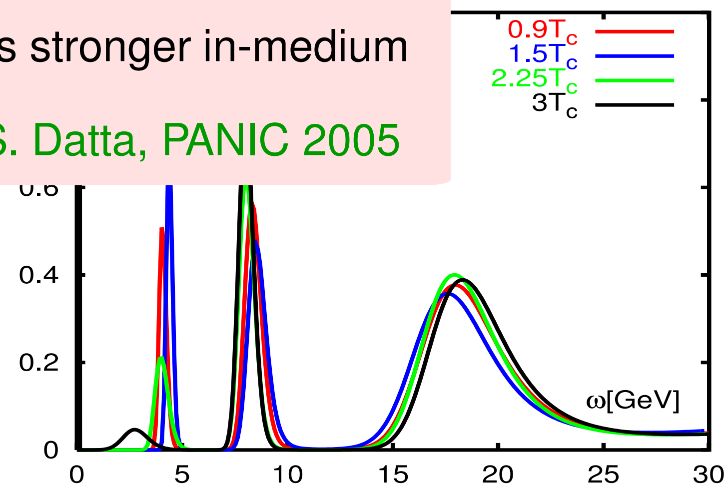
χ_c-states disappear at $T \simeq T_c$

J/ψ and η_c gone at 3.0 T_c

moving J/ψ shows stronger in-medium modification: S. Datta, PANIC 2005



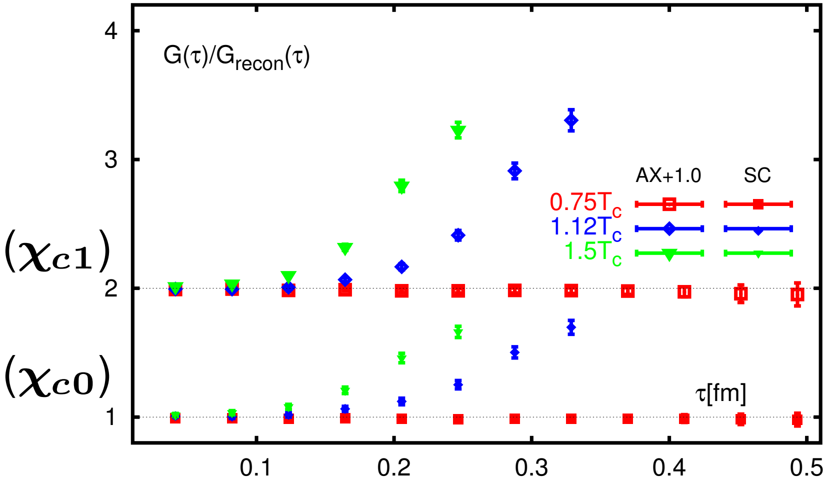
functions

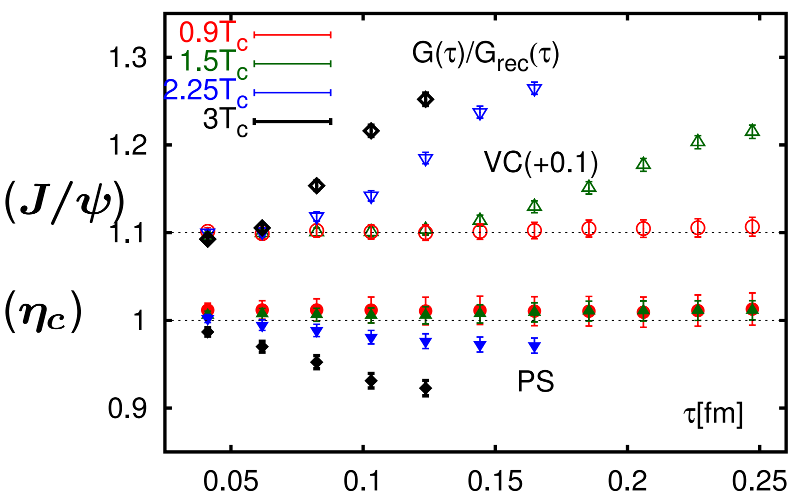


S. Datta et al., Phys. Rev. D69 (2004) 094507

Heavy quark spectral functions and correlation functions

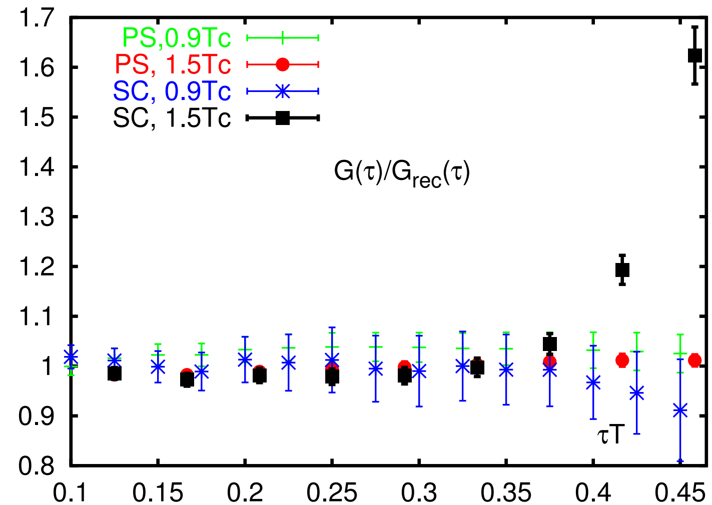
charm





S. Datta et al., Phys. Rev. D69 (2004) 094507

bottom – is coming up



first results on bottomonium at high T

(more difficult, finer lattices needed)

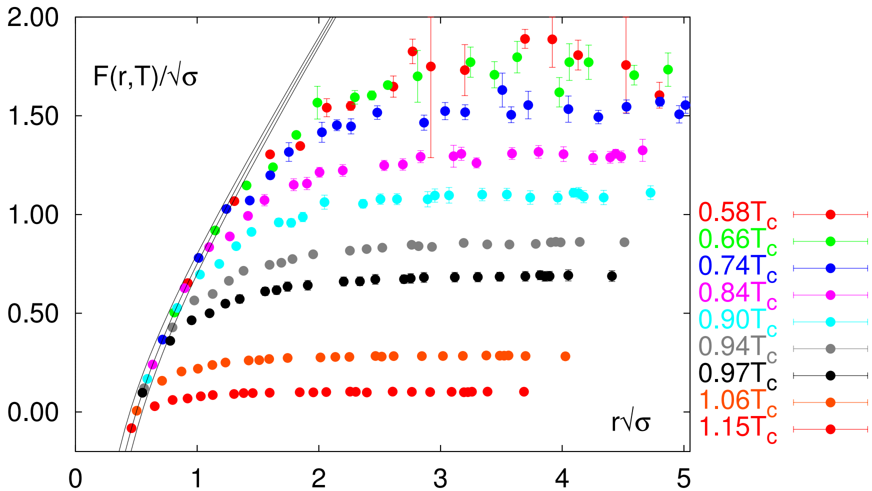
K. Petrov et al., hep-lat/0509138

S. Datta, PANIC 2005

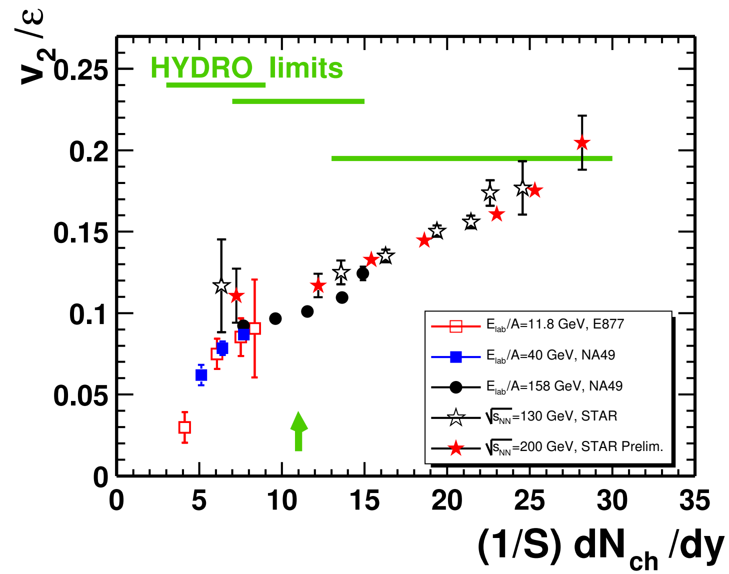
χ_b modified at $1.5\ T_c$

η_b unmodified at $1.5\ T_c$

Strongly coupled QCD



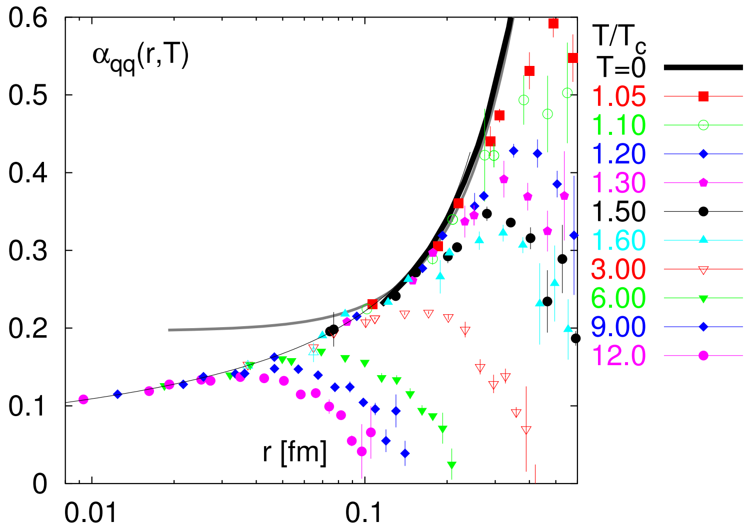
$\bar{q}q$-potential at finite-T (LGT)



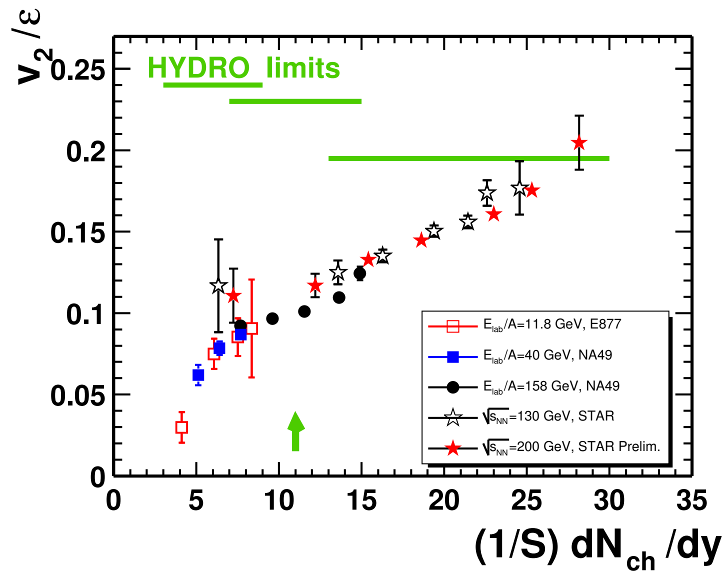
elliptic flow (RHIC)

- T-dependence of $\bar{q}q$ interaction at short and medium distance reflects asymptotic freedom as well as remnants of confinement
- large elliptic flow suggest the creation of an almost perfect fluid in a heavy ion collision at RHIC

Strongly coupled QCD



running coupling at finite-T (LGT)

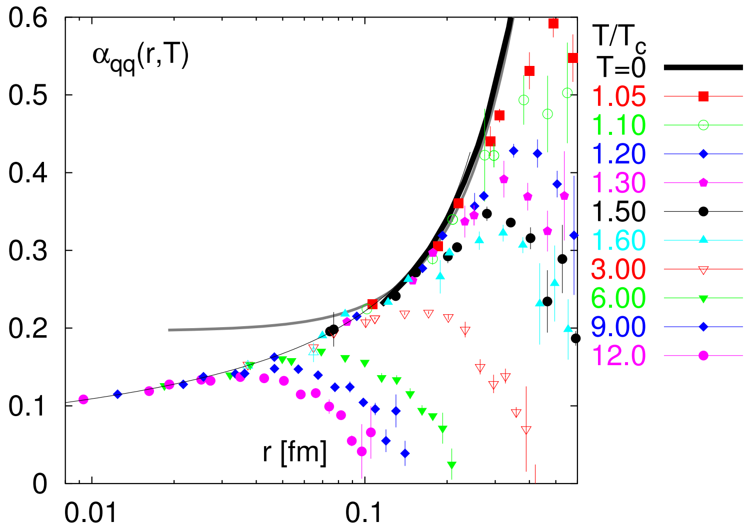


elliptic flow (RHIC)

- T-dependence of $\bar{q}q$ interaction at short and medium distance reflects asymptotic freedom as well as remnants of confinement
- large elliptic flow suggest the creation of an almost perfect fluid in a heavy ion collision at RHIC

Is there evidence for colored bound states or density correlations that could give support to the sQCD scenario suggesting a fluid phase generated at RHIC ?

Strongly coupled QCD



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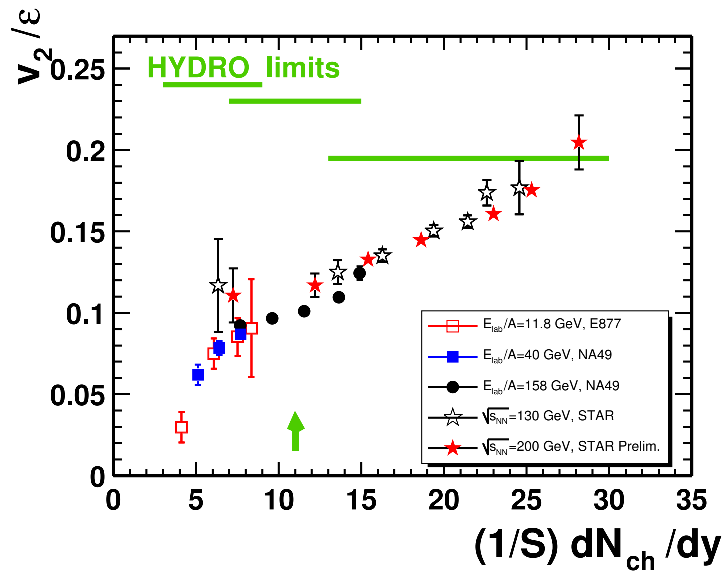

elliptic flow (RHIC)

- T-dependence of $\bar{q}q$ interaction at short and medium distance reflects asymptotic freedom as well as remnants of confinement
- large elliptic flow suggest the creation of an almost perfect fluid in a heavy ion collision at RHIC

- spectral analysis of thermal correlation functions opens the possibility for a direct analysis of colored bound states in the QGP (in analogy to quarkonium spectroscopy)
- density-density correlation functions should reflect characteristics of a fluid (well defined interparticle separation)

Conclusions

- Bulk thermodynamics is currently under intensive study:
  uncertainites on $T_c \simeq 175$ MeV are still about 10%;
  the EoS shows little "m_q" and "a" dependence for $T \geq T_c$.
- The last word on the QCD phase diagram is not yet spoken:
  universal properties of the transition in 2-flavor QCD still have to be established;
  the location of the chiral critical point still is uncertain
- finite density calculations are making steady progress; bulk thermodynamics and susceptibilities in the regime of interest from AGS(FAIR) to LHC are within reach of current methods
- Heavy quark bound states exist well above T_c:
  charmonium studies get refined, excited states dissolve close to T_c;
  first results for bottomonium are coming up.