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On IR and UV Effects in Hadronic Correlations at Finite Temperature

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I Introduction

aim at **analyzing existence and properties of hadronic excitations at $T > 0$**

$\rightsquigarrow$ Correlator in momentum space defined at Matsubara frequencies $\omega_n = 2\pi T n$

$$\Delta(i\omega_n, \vec{p}) = \oint \frac{dp_0}{2\pi i} \frac{\Delta(p_o, \vec{p})}{p_0 - i\omega_n}$$
$$= \int \frac{dp_0}{2\pi} \frac{1}{p_0 - i\omega_n} \underbrace{\frac{1}{i}\left[\Delta(p_0 + i\epsilon, \vec{p}) - \Delta(p_0 - i\epsilon, \vec{p})\right]}_{\textbf{spectral density } \boldsymbol{\sigma(p_0, \vec{p})}} + \; Subtr.$$

$\Rightarrow$ temporal correlator $\quad G^T(\tau) = T \sum_n e^{-i\omega_n \tau} \Delta(i\omega_n, 0) = \int_0^\infty dp_0 \, \sigma(p_0, 0) \underbrace{\frac{\cosh\left[p_0(\tau - 1/2T)\right]}{\sinh(p_0/2T)}}_{\textbf{kernel } \boldsymbol{K(\tau, p_0; T)}}$

$\Rightarrow$ spatial correlator $\quad G^S(z) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \Delta(0, p) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma(p_0, p)}{p_0}$

as true for all lattice analyses: **control systematic UV and IR effects** at $T=0$ obtain masses and amplitudes, e.g. m_π, f_π from $\sigma(p_0) \sim f_\pi^2\, \delta(p_0^2 - m_\pi^2)$

$$G_\pi^T(\tau) \underset{\tau\to\infty}{\longrightarrow} f_\pi^2\, m_\pi\, \exp(-\, m_\pi\, \tau)$$

Obstacles at $T \neq 0$:

- existence of bound states, resonances etc. usually unknown
  $\rightsquigarrow$ try to establish spectral density
- limited physical extension $1/T$ in temporal direction
  $\rightsquigarrow$ spatial correlations
- limited number of data points N_τ in temporal direction
  $\rightsquigarrow$ anisotropic lattices

- finite size $LT = N_\sigma / N_\tau$
- finite lattice spacing $aT = 1/N_\tau$

Numerical parameters

all results based on

- quenched approximation

  standard plaquette gauge action + non-perturbatively improved Wilson fermion action

  $\Rightarrow$ discretization errors at $\mathcal{O}(a^2)$
- isotropic lattices of various sizes $N_\sigma^3 \times N_\tau$, to study finite volume and finite a effects
- 3 temperatures each below and above T_c: $0.4, 0.6, 0.9 T_c$ and $1.25, 1.5, 3.0 T_c$
- quark masses: up to five at $T < T_c$, $\simeq$ chiral limit above T_c
- renormalization/improvement constants: non-perturbative if available [Lüscher et al.]

  tadpole improved perturbative otherwise
- absolute scale by T_c, relative scales by [Klassen et al.] interpolation of string tension
- statistics between 60 and 120 configurations
- channels analyzed: $PS, S, V_T, V_L, A_T, A_L$ flavor non-singlets

T/T_c	β	$N_\sigma^3 \times N_\tau$	a [fm]	N_{conf}	κ	κ_c
0.44	6.000	$24^3 \times 16$	0.104	60	0.13240 0.13320 0.13420 0.13480	0.13520
0.55	6.136	$24^3 \times 16$	0.083	60	0.13300 0.13400 0.13460 0.13540	0.13571
		$32^3 \times 16$		120	0.13300 0.13400 0.13460 0.13495 0.13540	0.13571
		$48^3 \times 16$		60	0.13300 0.13400 0.13460 0.13495	0.13571
0.93	6.499	$24^3 \times 16$	0.049	120	0.13300 0.13400 0.13460 0.13540	0.13558
		$32^3 \times 16$		120	0.13300 0.13400 0.13460 0.13531 0.13540	0.13558
		$48^3 \times 16$		60	0.13300 0.13400 0.13460 0.13531	0.13558
1.24	6.205	$(64\|32\|24\|16)^3 \times 8$	0.075	(46|61|85|89)	0.13599	0.13580
1.24	6.499	$(48\|36\|24)^3 \times 12$	0.049	(81|81|78)	0.13558	0.13558
1.24	6.721	$(64\|48\|32)^3 \times 16$	0.038	(46|96|81)	0.13507	0.13522
1.49	6.338	$(64\|32\|24\|16)^3 \times 8$	0.062	(46|62|59|60)	0.13581	0.13578
1.49	6.640	$(48\|36\|24)^3 \times 12$	0.041	(61|59|68)	0.13536	0.13535
1.49	6.872	$(64\|48\|32)^3 \times 16$	0.031	(66|62|60)	0.13495	0.13499
2.98	6.872	$(96\|64\|32\|24\|16)^3 \times 8$	0.031	(37|43|65|80|80)	0.13494	0.13499
2.91	7.192	$(48\|36\|24)^3 \times 12$	0.021	(85|80|80)	0.13440	0.13450
2.98	7.457	$(64\|48\|32)^3 \times 16$	0.015	(80|80|60)	0.13390	0.13394

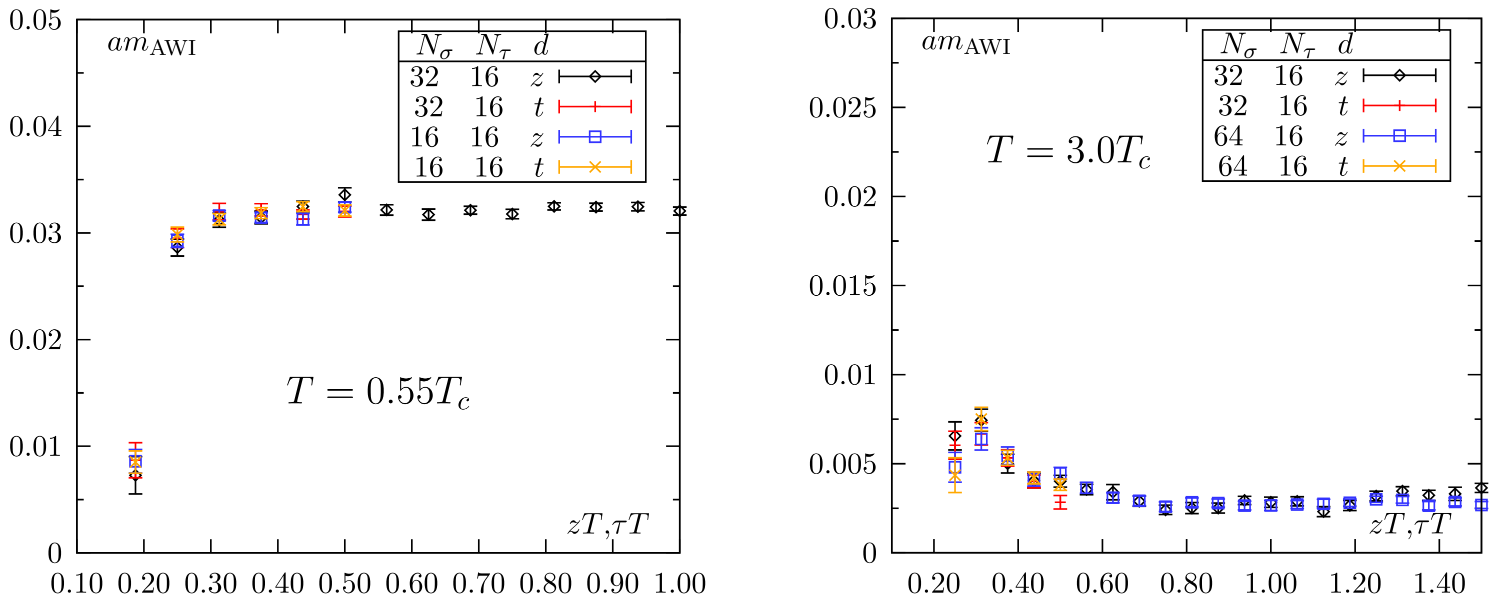
in red: runs with the hypercube truncated fixed point action [Bietenholz, Wiese]

II Quark masses

- for Wilson-type fermions chiral limit, κ_c, usually defined by vanishing Goldstone boson mass m_π
- this definition does not work anymore at $T > T_c$
- use current quark mass defined through axial vector Ward identity: $2\, m_{\mathrm{AWI}}\, P(x) = \partial_\mu A_\mu(x)$
- on the lattice

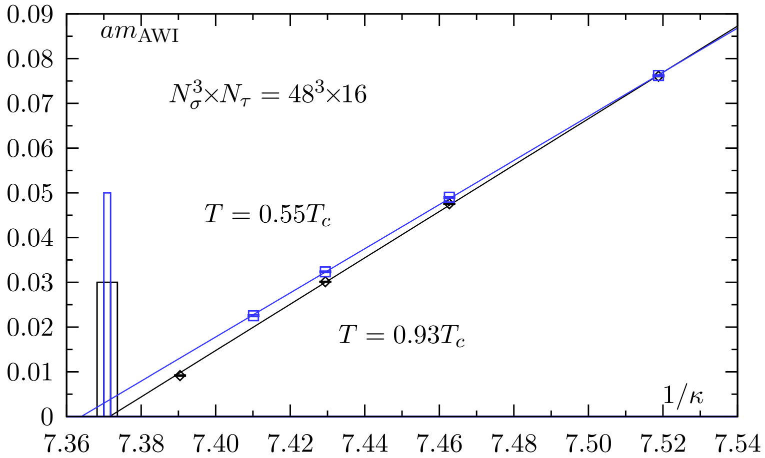
$$2\, m_{\mathrm{AWI}} = Z_A \frac{\langle\, \Delta_\mu A_\mu(x) P(0)\, \rangle}{\langle\, P(x) P(0)\, \rangle}$$

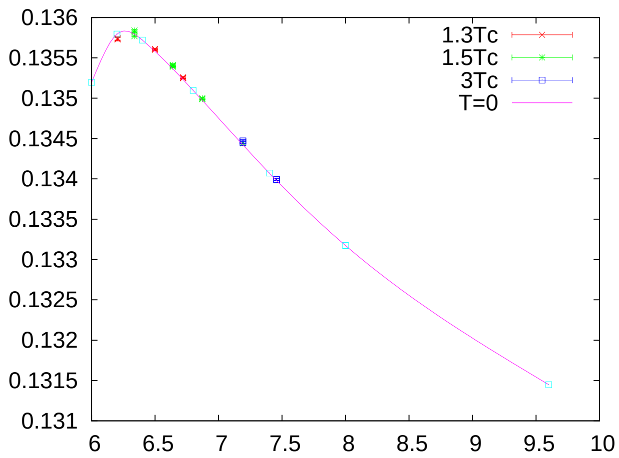
- comparison temporal - spatial correlators:



QUARK MASSES

- below T_c: $\kappa_c^{\rm AWI} = \kappa_c^{\pi} \simeq \kappa_c^{\pi}(T=0)$





- above T_c: simulation at (interpolated) $\kappa_c(T=0) \longrightarrow$ compute κ_c via

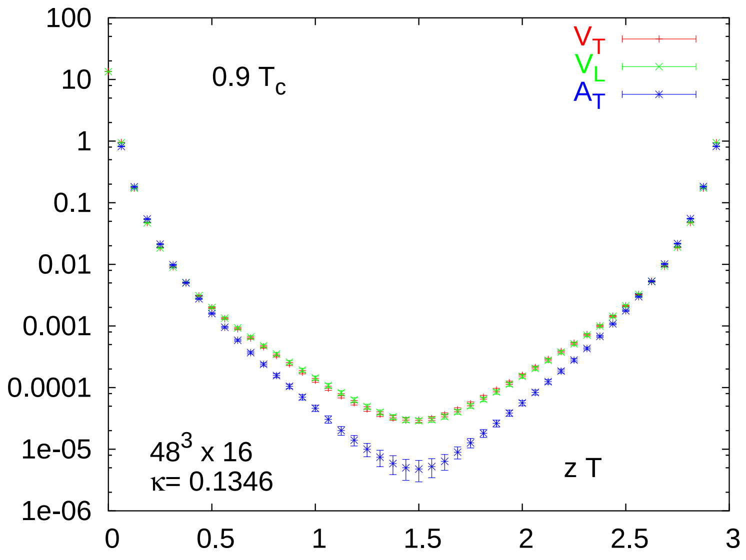
$$a\,m_{\rm AWI} = \frac{Z_m Z_P}{Z_A}\left\{1 + \left[b_m + (b_P - b_A)\right] am_{\rm bare}\right\} am_{\rm bare} \qquad \text{where} \qquad a\,m_{\rm bare} = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

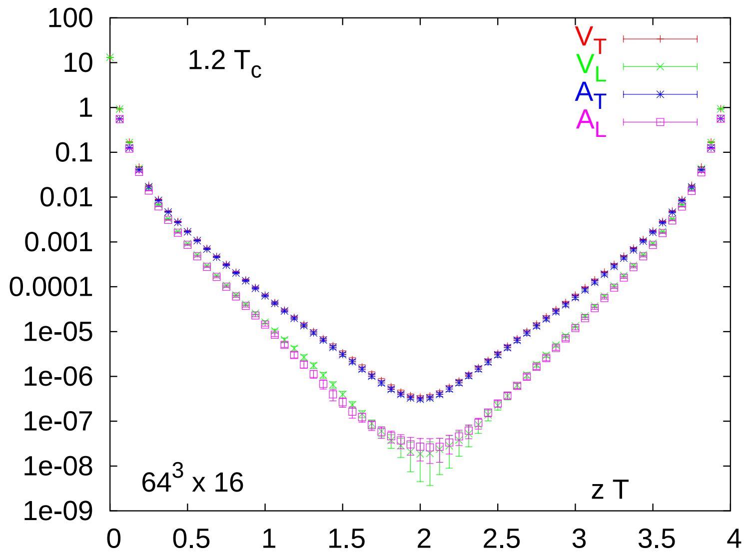
$\Rightarrow$ no temperature effects on κ_c visible

III Symmetry restorations

- at $T \neq 0$, for spatial correlations: rotational $SO(3) \rightarrow SO(2) \times Z(2)$ [S.Gupta]

$$\Rightarrow \quad V_T \neq V_L, \quad A_T \neq A_L \quad \text{possible}$$



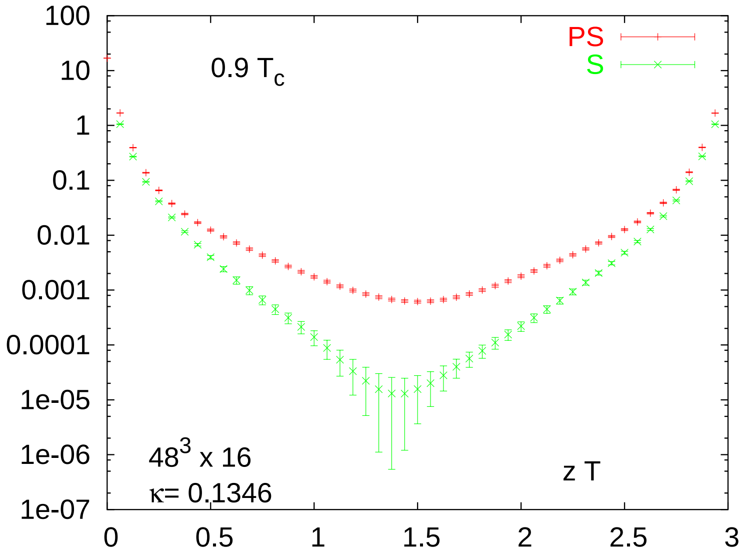


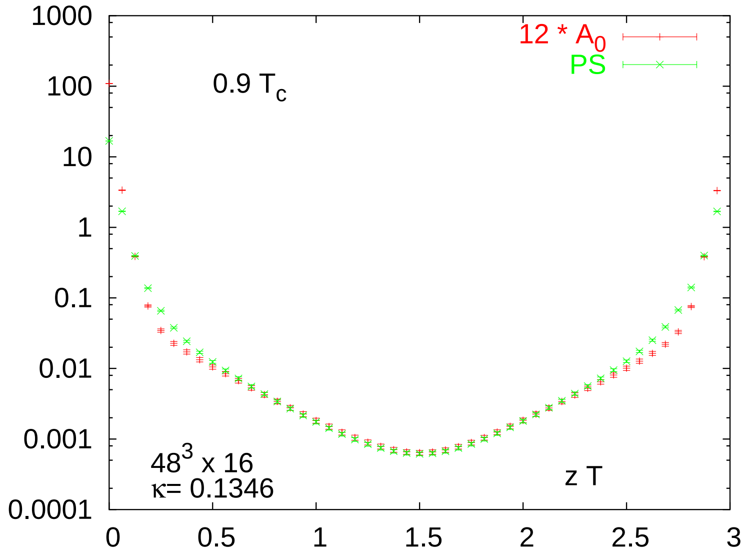
$T < T_c$: $V_T = V_L \neq A_T = A_L$ $\qquad$ $T > T_c$: $V_T = A_T \neq V_L = A_L$

- at $T > T_c$, chiral symmetry restoration $SU_V(N_F) \rightarrow SU_L(N_F) \times SU_R(N_F)$

Pseudoscalar - Scalar sector

- below T_c



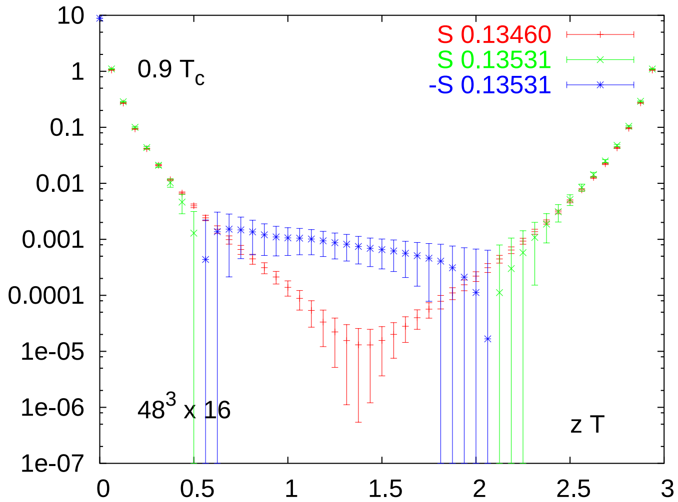


- pseudoscalar π much lighter than scalar a_0/δ

- π couples to axialvector 0-component A_0 with relative strength $1/12$

Symmetry restorations

- pseudo/scalar correlations at small quark mass are plagued by socalled exceptional configurations



- related to zero modes see e.g. [Gavai, Gupta, Lacaze]

- zero mode contribution to the (integrated) correlation function is

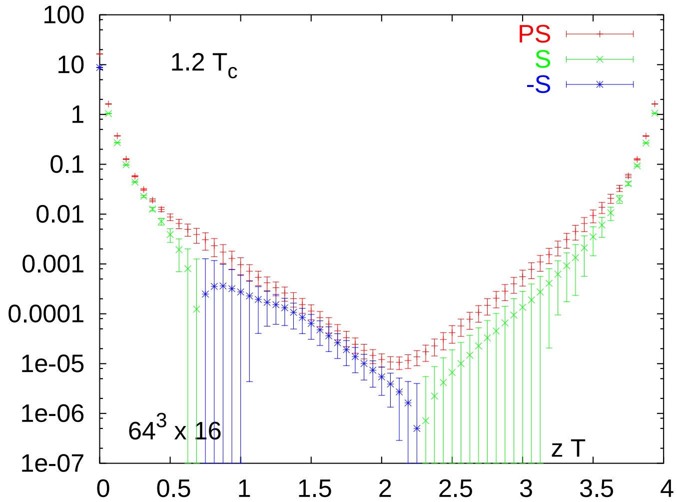
$$\int d^4x\ G_{PS,S}(x) = \pm \frac{|Q|}{m^2 V} + \ldots$$

Symmetry restorations

- at/above T_c

- effective $U_A(1)$ restoration would predict $\pi - a_0/\delta$ degeneracy

- not observed at $T = 1.25 T_c$:



- equivalent to saying that topologically non-trivial configurations survive up to (at least) $1.25 T_c$

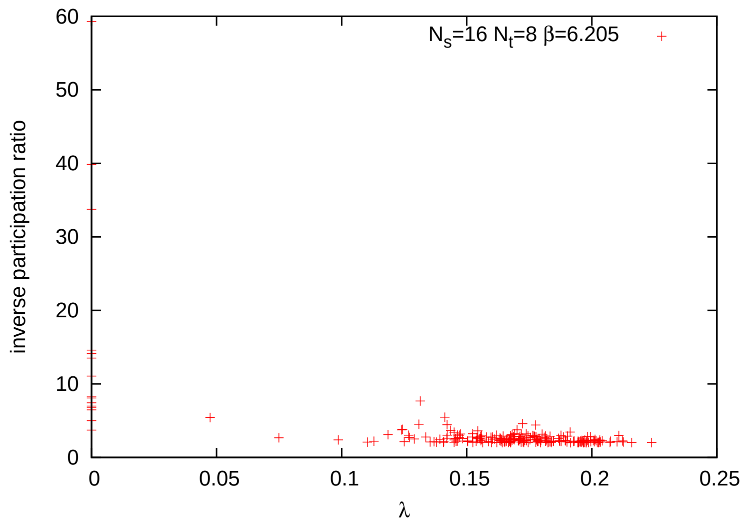
- study of zero modes with GW Dirac operator:

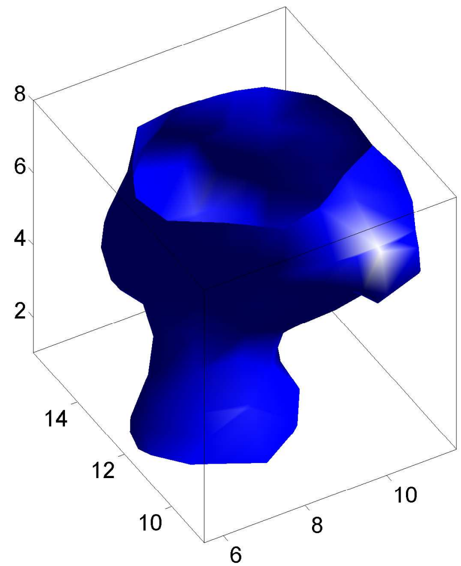
- there is an exact index theorem relating topological charges Q with zero modes $Q = n_+ - n_-$
  where $n_\pm$ is the number of zero modes with chirality $\gamma_5\psi_0 = \pm\psi_0$

- at $1.25T_c$ we found plenty of zero modes with $Q = 1$

- they are localized, measured by inverse participation ratio p

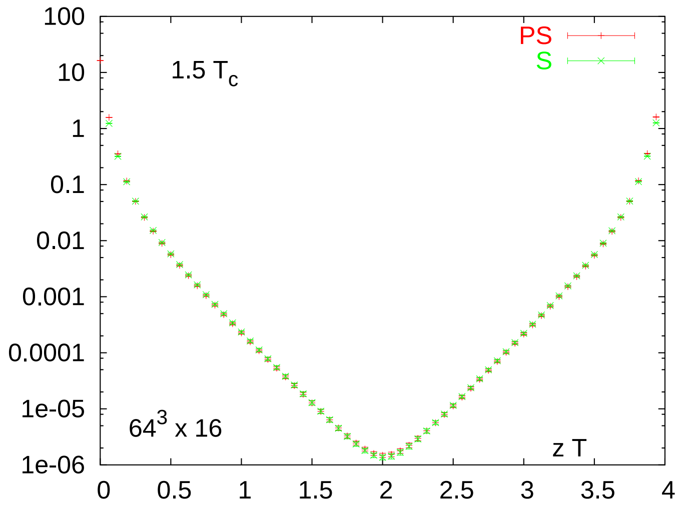
$$p = V\sum_x \psi^\dagger(x)\psi(x) = \begin{cases} \text{V} & \text{localized on a single site} \\ 1 & \text{completely delocalized} \end{cases}$$

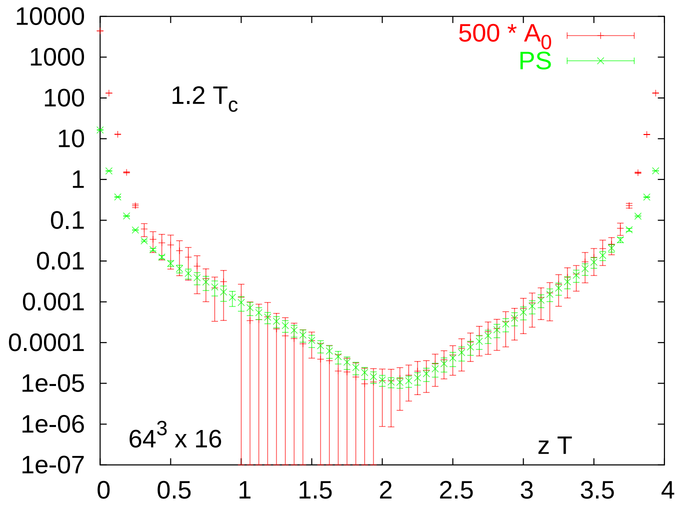




Symmetry restorations

- at $1.5T_c$: $m_{\mathrm{PS}} = m_{\mathrm{S}}$





- aside: A_0 couples to PS with much less strength $\simeq 1/500$

IV Spatial correlations

- $G^S(z)$ depends on the same spectral density, but relation more involved
- expect

$$G^S(z) \underset{z \to \infty}{\longrightarrow} \text{Ampl.} \times \exp(-\, m_{\text{screen}}\, z)$$

- however, $m_{\text{screen}}(T) \neq m(T)$ in general :

  look for zeros of $G^{-1}(p) = p_0^2 + \vec{p}^{\,2} + m_0^2 + \Pi(p_0, \vec{p}, T)$

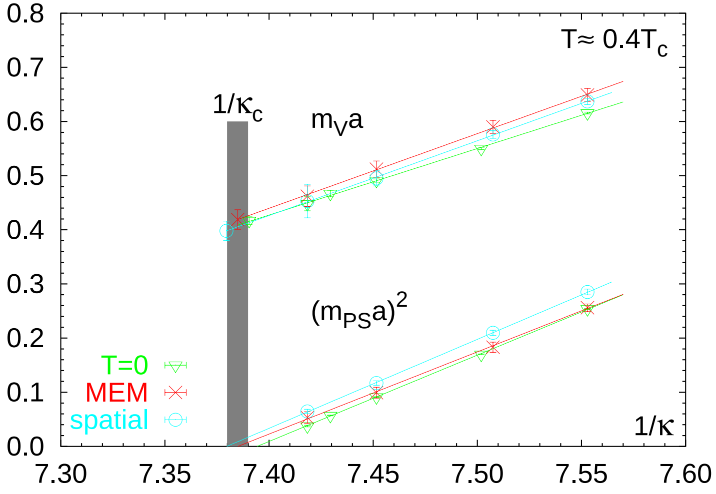
$$\vec{p} = 0 : \qquad -\, p_0^2 \;=\; m_0^2 \;+\; \Pi(p_0, \vec{0}, T) \;=\; m^2$$

$$p_0 = 0 : \qquad -\, \vec{p}^{\,2} \;=\; m_0^2 \;+\; \Pi(0, \vec{p}, T) \;=\; m_{\text{screen}}^2$$

- still, contains physical information

screening masses below T_c

- at $0.4T_c$:



comparison with $T = 0$: [Göckeler et al.]

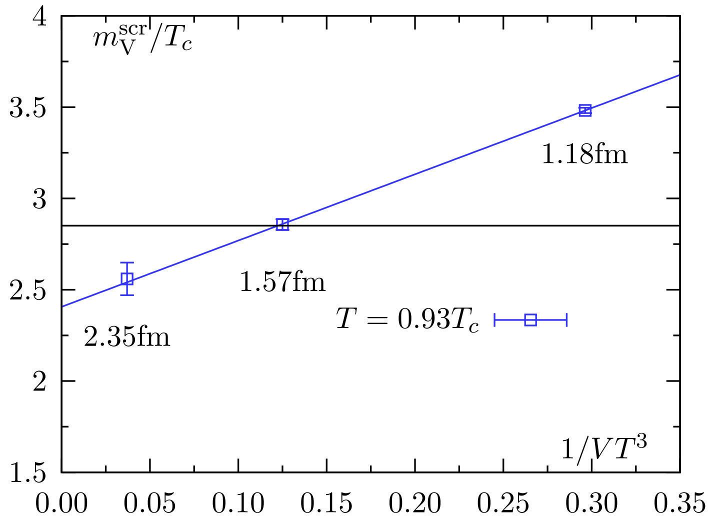
- no difference

- masses from MEM : likewise

L = 2.5 fm

- at $0.55T_c$: between L = 2.7 fm and L = 4.0 fm: no finite size effect seen

- at $0.93T_c$:



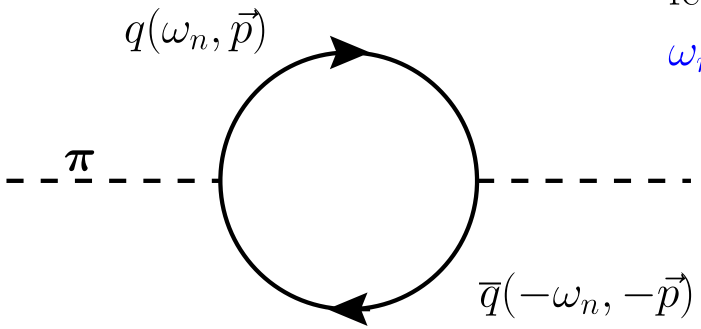
in the accessible volumes

finite volume effects well described by

$$m(L) = m(\infty) + c\,\frac{1}{L^3}$$ [Fukugita et al.]

Spatial correlators

At T large: expect free quarks



fermion Matsubara
$\omega_n = \pi T(2n+1)$

- Continuum: [Friman, Florkowski]

$$G_\pi^S(z) = \frac{N_c T}{2\pi z^2 \sinh(2\pi T z)} \left[1 + 2\pi T z \coth(2\pi T z)\right]$$

define effective (z-dependent) screening mass $m_{\text{screen}}^{\text{eff}}(z) = -\dfrac{1}{G(z)} \dfrac{\partial G(z)}{\partial z} \simeq 2\pi T \left\{1 + \dfrac{1}{2\pi T z} + \ldots\right\}$

- Lattice (Wilson fermions)

$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1,k_2,\omega_n} \frac{1}{(1+M)^2} \frac{1}{\sinh^2(E_1 N_\sigma/2)} \left\{b_M \cosh\left[2E_1\left(\frac{N_\sigma}{2} - z\right)\right] + d_M\right\}$$

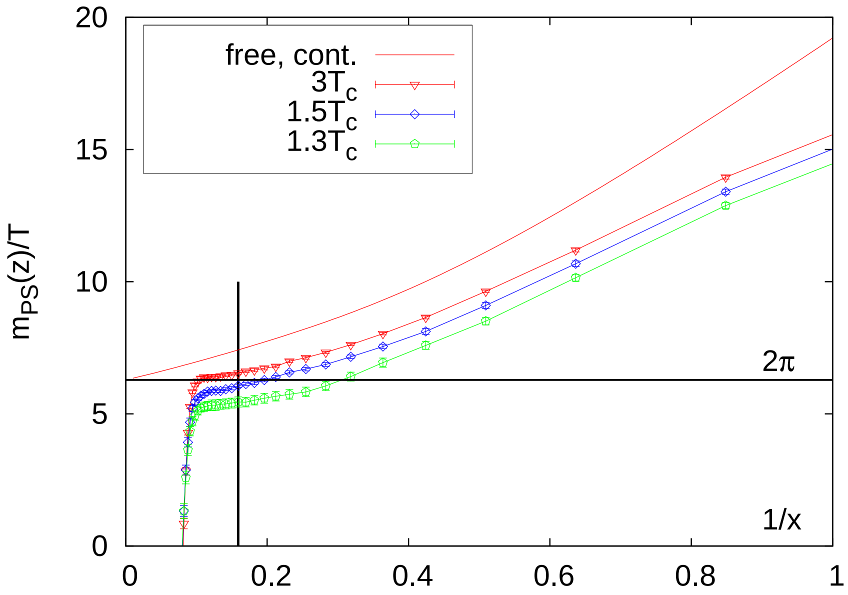
where $M = \Sigma_{1,2,4}(1 - \cos(k_i))$

$$\cosh(E_1) = 1 + \frac{\Sigma_{1,2,4} \sin^2(k_i) + M^2}{2(1+M)}$$

	b_M	d_M
π	1	0
$\frac{1}{2}(\rho_1+\rho_2)$	$1 - \frac{1}{2}\frac{\sin^2(k_1)+\sin^2(k_2)}{\sinh^2(E_1)}$	$-\frac{1}{2}\frac{\sin^2(k_1)+\sin^2(k_2)}{\sinh^2(E_1)}$
ρ_3	0	1
ρ_4	$1 - \frac{1}{2}\frac{\sin^2(k_4)}{\sinh^2(E_1)}$	$-\frac{1}{2}\frac{\sin^2(k_4)}{\sinh^2(E_1)}$

screening masses above T_c:

- comparison continuum free with interacting lattice



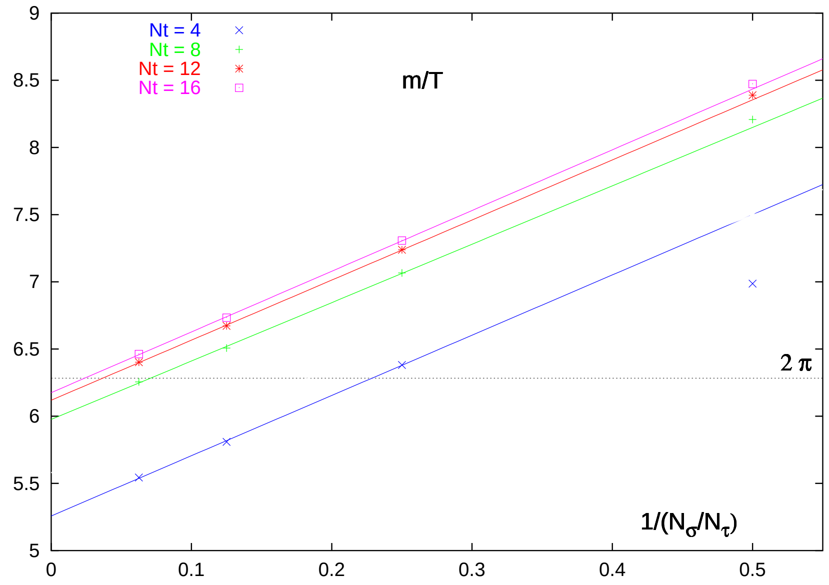
$x = 2\pi T z$

- in the continuum: $2\pi T$ reached only at $z \to \infty$

- similarly, lattice data has no clear plateau

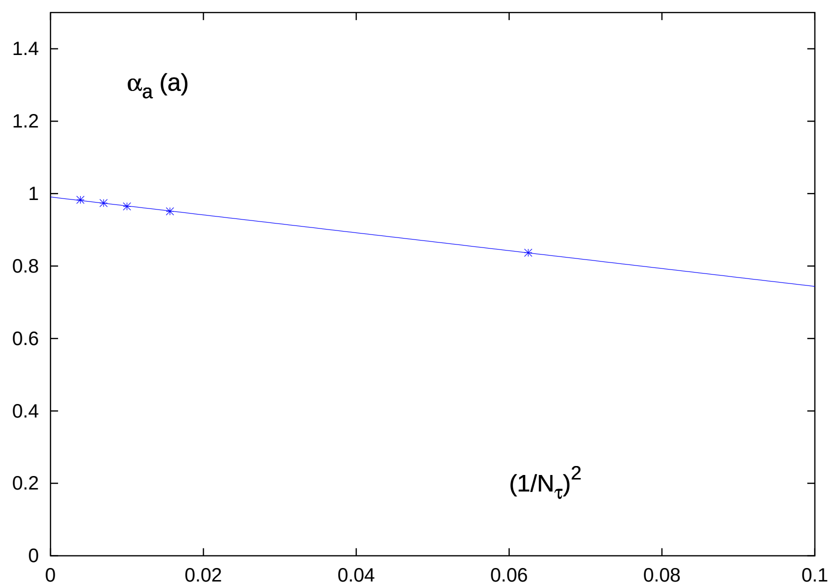
- for definiteness, quote $m_{\text{screen}} = m^{\text{eff}}_{\text{screen}}(z = N_\sigma/4)$ (indicated by vertical line)

$\Rightarrow$ motivates analysis of lattice artefacts in the free lattice case



finite volume effect

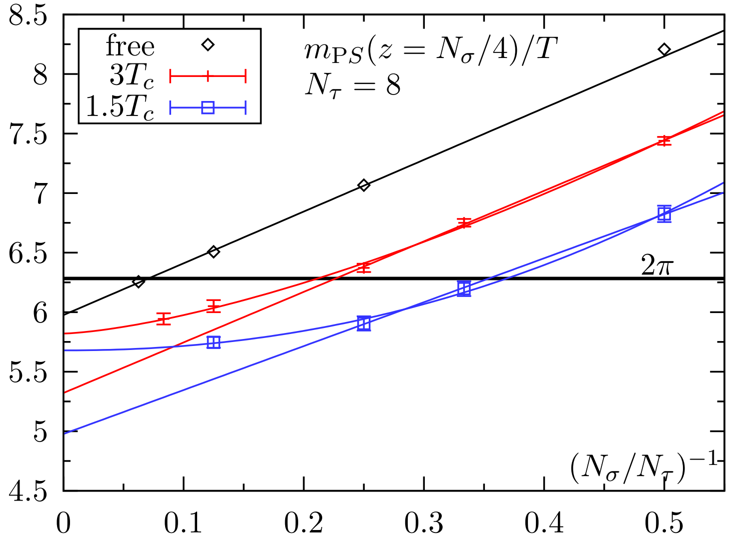
$$m_{\mathrm{screen}}(L,a) = 2\pi T\,\alpha(a)$$
$$\star\ (1+\gamma_V \frac{2}{\pi}\frac{N_\tau}{N_\sigma})$$

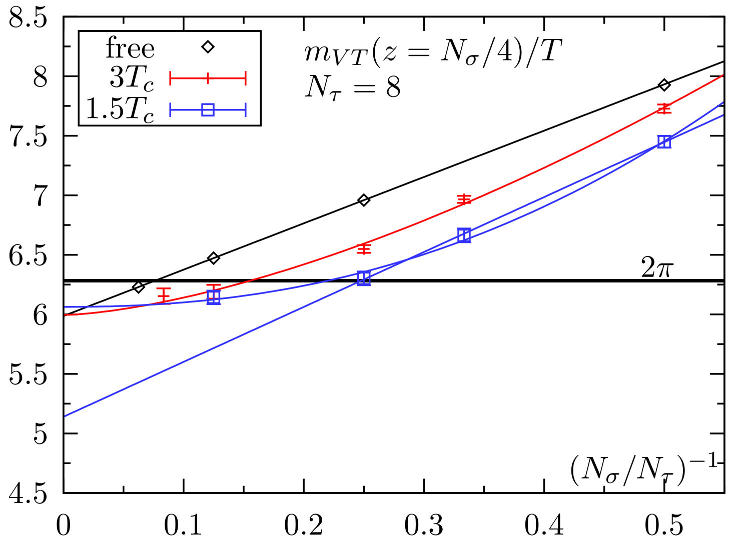


finite lattice spacing effect

$$\alpha(a) = 1 - \gamma_a \frac{\pi^2}{3}\left(\frac{1}{N_\tau}\right)^2$$

- interactive lattice data: finite volume effects: $L \to \infty$





$$T \le T_c: \quad m_{\text{screen}}(L,a) = m_{\text{screen}}(a)\left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma}\right)^3\right]$$

$$T > T_c: \quad m_{\text{screen}}(L,a) = m_{\text{screen}}(a)\left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma}\right)^p\right]$$

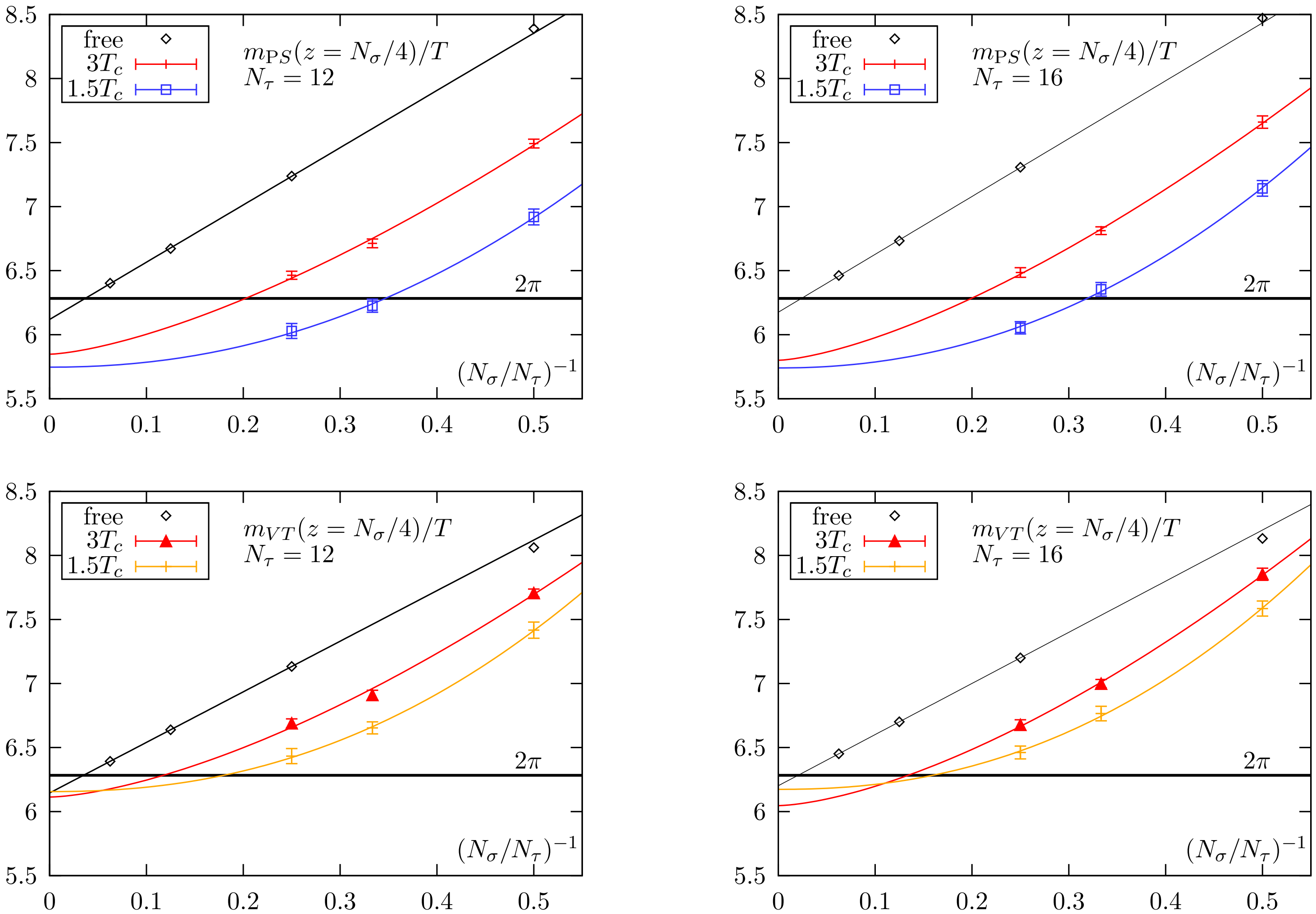
$$T = \infty: \quad m_{\text{screen}}(L,a) = m_{\text{screen}}(a)\left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma}\right)^1\right]$$

fit results :

p	PS	V
$1.5T_c$	2.1(4)	2.2(5)
$3.0T_c$	1.5(2)	1.5(2)

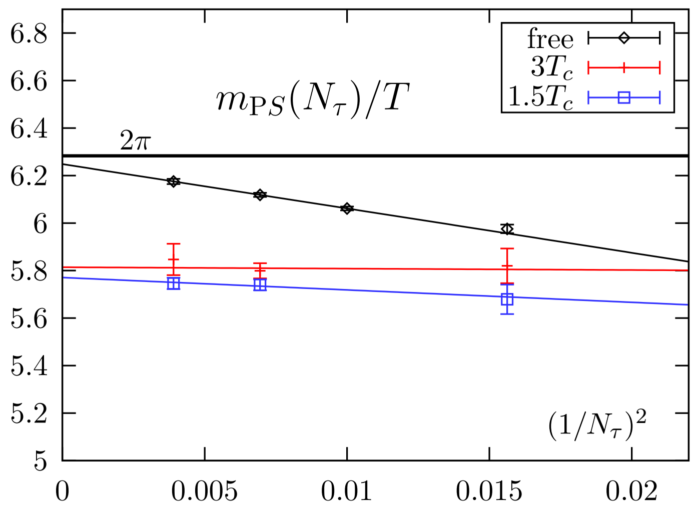
aspect ratios $N_\sigma/N_\tau = 8, 12$ necessary to disentangle linear from power $p \neq 1$

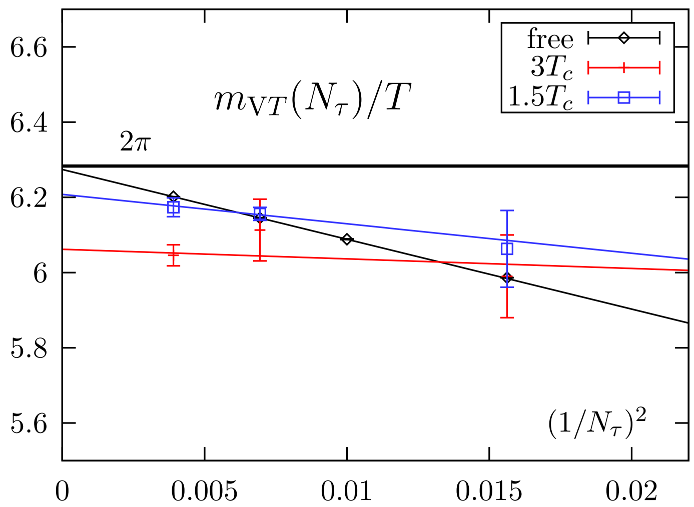
$N_\tau = 12$ $\quad\quad$ $N_\tau = 16$



- at any fixed volume: data below free case
- finite volume effects smaller than in free case

- finally: finite lattice spacing effects: $a \longrightarrow 0$

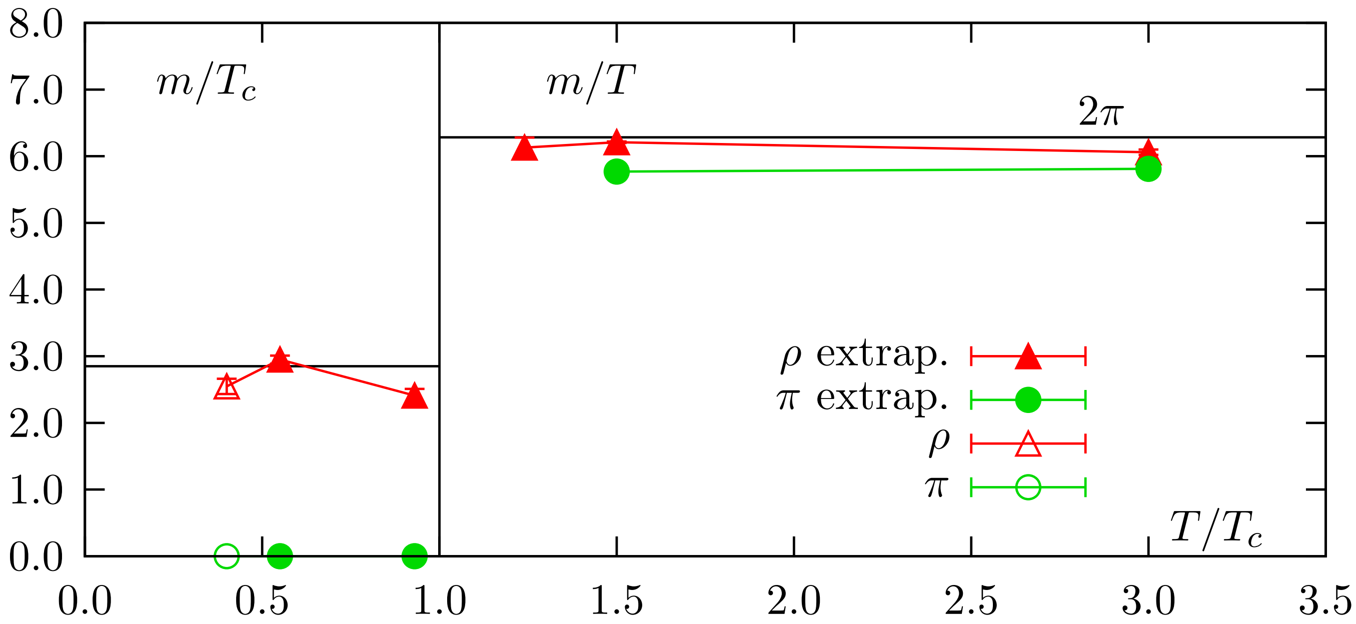




- non-perturbatively $\mathrm{O}(a)$ improved action $\Rightarrow$ discretization errors at $\mathrm{O}(a^2)$ also in interacting case

- considerably smaller discretization effects than in free case summary:

- $L \to \infty$ extrapolated
- $a \to 0$ extrapolated at $T > T_c$



$T < T_c$:

- weak T dependence

$T > T_c$:

- data some % below $2\pi T$
- at variance with [Laine, Vepsäläinen]

V Temporal correlations

- at $T \neq 0$: hampered by the limited extent $1/T$ of the system
- fits require ansätze, i.e. knowledge about existence of hadronic excitations
- information about existence of excitations, masses and widths of bound states/resonances encoded in spectral density $\sigma(\omega)$

$\Rightarrow$ try to extract spectral density from correlation functions

- important guidance comes from free case relevant at large T and large ω:

$$G_M^T(\tau) = \frac{1}{N_\sigma^3} \sum_{\vec{k}} \frac{1}{(1+M)^2} \frac{1}{\cosh^2(E_1 N_\tau/2)} \left\{ b_M \cosh\left[2E_1\left(\frac{N_\tau}{2} - \tau\right)\right] + d_M \right\}$$

$$M = \Sigma_i (1 - \cos(k_i))$$

$$\cosh(E_1) = 1 + \frac{\Sigma_i \sin^2(k_i) + M^2}{2(1+M)}$$

	b_M	d_M
π	1	0
ρ	$3 - \Sigma_i \frac{\sin^2(k_i)}{\sinh^2(E_1)}$	$\Sigma_i \frac{\sin^2(k_i)}{\sinh^2(E_1)} - 1$

MEM in a nutshell   for details see M. Asakawa at al., Prog.Part.Nucl.Phys. 46 (2001) 459

$$D(\tau,\vec{p}) \quad = \quad \int d\omega \; \sigma(\omega,\vec{p}) \times K(\omega,\tau)$$

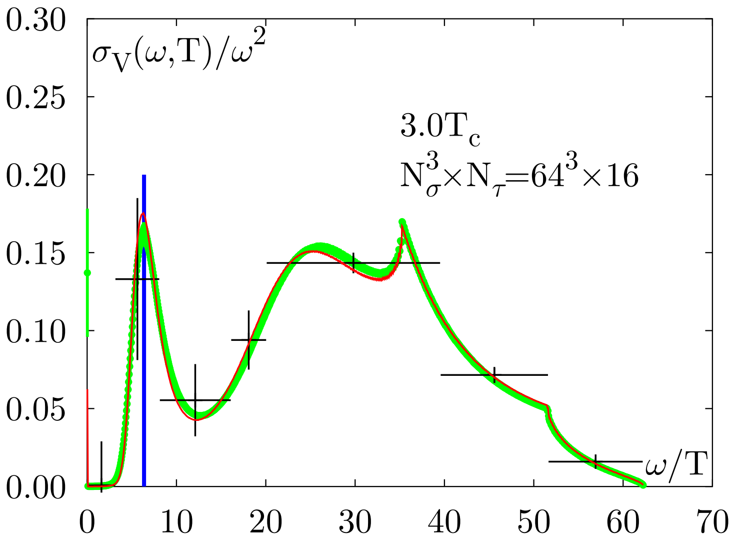
data at discrete τ continuous fct. of ω $\Rightarrow$ ill-posed problem

$\rightarrow$ Maximum Entropy Method: look for most probable $\tilde{\sigma}$ given

- data D
- prior knowledge H: $\sigma(\omega) > 0$, $\sigma(\omega \rightarrow \infty) =$ pert. th.

- $P[\tilde{\sigma}|DH] \sim P[D|\tilde{\sigma}H]P[\tilde{\sigma}|H]$ Bayes
- $P[D|\tilde{\sigma}H] \sim e^{-L}$ with $L = \frac{1}{2}\sum_{i,j}(D-D_{\tilde{\sigma}})(\tau_i)\; \overbrace{C^{-1}_{i,j}}^{\text{covariance matrix}}\; (D-D_{\tilde{\sigma}})(\tau_j)$
- $P[\tilde{\sigma}|H] \rightarrow P[\tilde{\sigma}|H\alpha] \sim e^{\alpha S}$ with $S = \int d\omega\, [\tilde{\sigma}(\omega) - \overbrace{m(\omega)}^{\text{default model}} - \tilde{\sigma}(\omega) \log \frac{\tilde{\sigma}(\omega)}{m(\omega)}]$
- look for maximum of $\exp\{\alpha S - L\} \quad \rightarrow \quad \tilde{\sigma}_\alpha(\omega)$
- $\sigma_{out}(\omega) \;\simeq\; \int d\alpha\; \tilde{\sigma}_\alpha(\omega)\; P[\alpha|DH]$

... MEM errors



fluctuation of mean σ by jackknife

systematic uncertainty via averaging over ω interval I:

$$\langle\sigma\rangle_I \;\sim\; \int_I d\omega\,\sigma(\omega) \;/\; \int_I d\omega$$

$$\langle(\delta\sigma)^2\rangle_I \;\sim\; \int_{I\times I} d\omega\, d\omega'\,\delta\sigma(\omega)\,\delta\sigma(\omega') \;/\; \int_{I\times I} d\omega\, d\omega'$$

recall
$$G^T(\tau) = \int_0^\infty dp_0\, \sigma(p_0,0)\, \frac{\cosh\left[p_0(\tau - 1/2T)\right]}{\sinh(p_0/2T)}$$

- control lattice UV cut-off effects ($a \neq 0$)
- at $T = 0$ and $T < T_c$: poles or resonances $\Rightarrow$ much less severe an issue
- at $T > T_c$: expect dominant contribution from two-quark cut
  from which any non-perturbative contribution has to be disentangled

$\rightsquigarrow$ study lattice artefacts from free lattice spectral density

$$\frac{G_M^{\mathrm{free,L}}(\tau)}{T^3} = \left(\frac{N_\tau}{N_\sigma}\right)^3 \int \frac{d^3k}{(2\pi)^3} \frac{1}{(1+M(\vec{k}))^2} \frac{b_M(\vec{k})}{\cosh^2(E(\vec{k})N_\tau/2)} \cosh\left[2E(\vec{k})\left(\frac{N_\tau}{2} - \tau\right)\right] + \text{const}$$

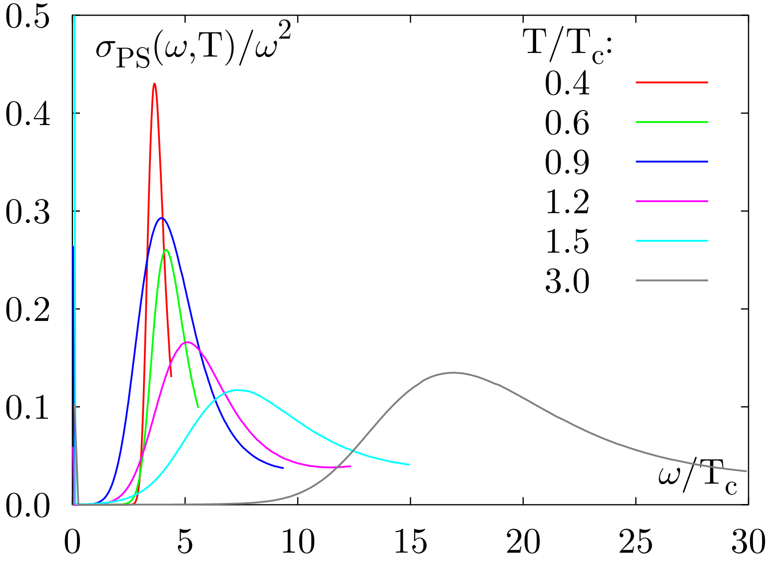
$$= \int_0^{\omega_{\max}} d\omega \qquad \sigma_M^L(\omega a) \qquad K(\omega/T, \tau T)$$

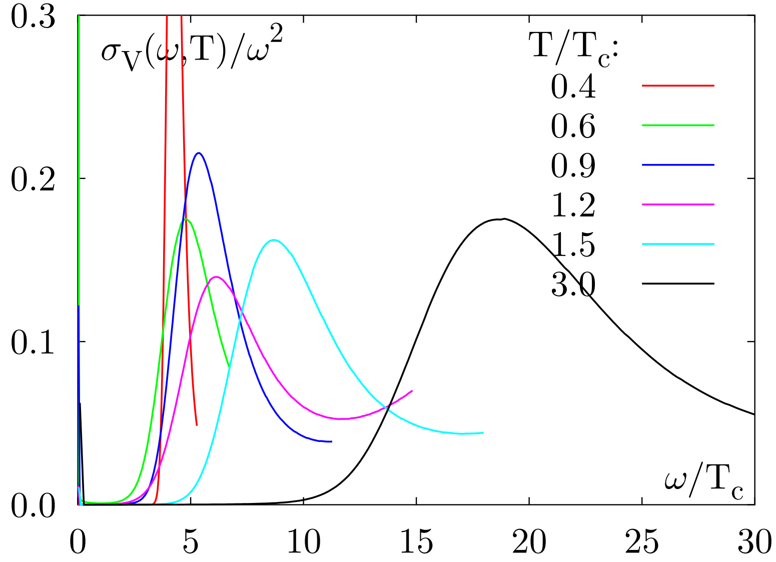
cut-off effects in σ — continuum kernel

either by variable transformation $2E(\vec{k}) \rightarrow \omega$
or by binning into ω intervals

high frequency part of σ^L
used as default model m

MEM results: overview all T





gross features:

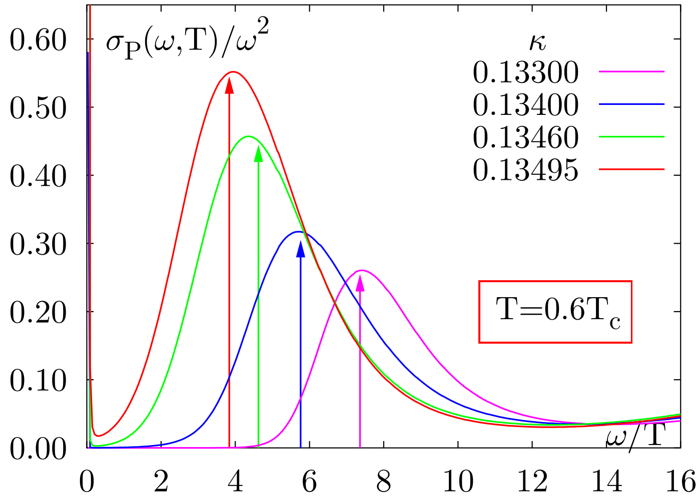
- quark masses shown: at $T < T_c$: roughly the same in physical units

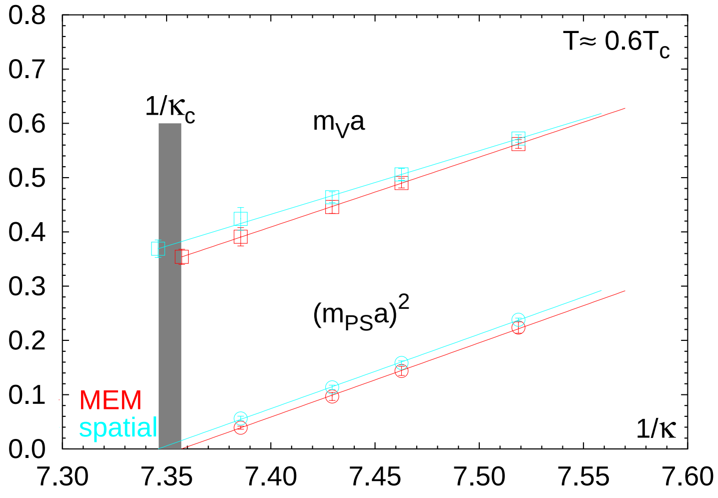
  at $T > T_c$: about chiral limit
- below T_c: peaks become broader and heights decrease with T
- slight move of ρ peak below T_c
- above T_c: considerable broadening and movement with T

caveat:

widths and heights subject to statistics

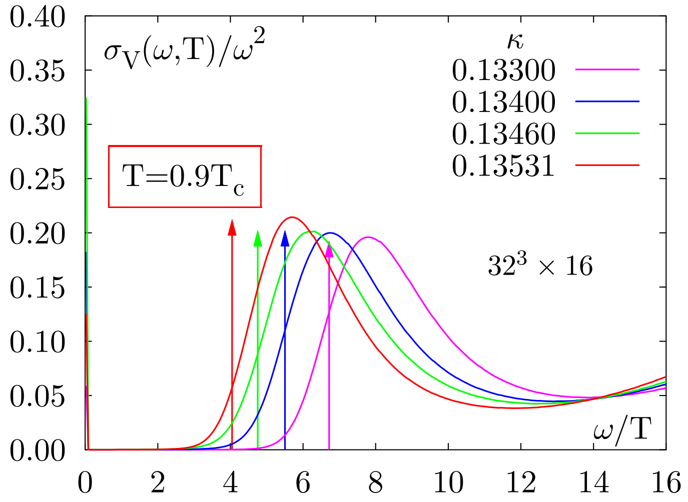
MEM results below T_c: compare with screening masses

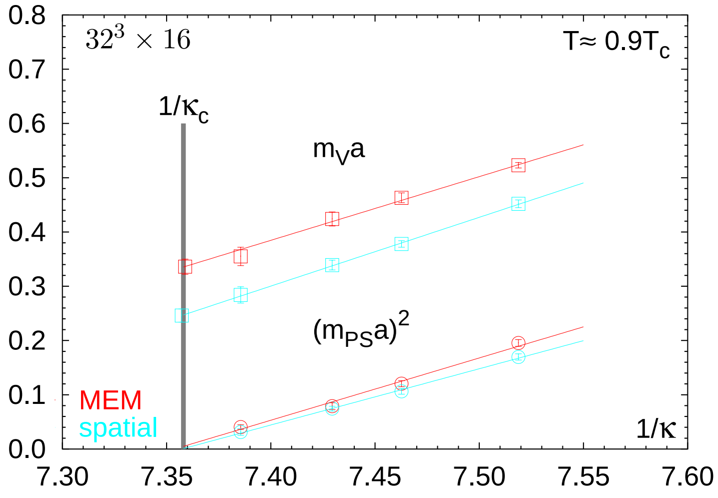




$T = 0.6T_c$:

no significant differences





$T = 0.9T_c$:

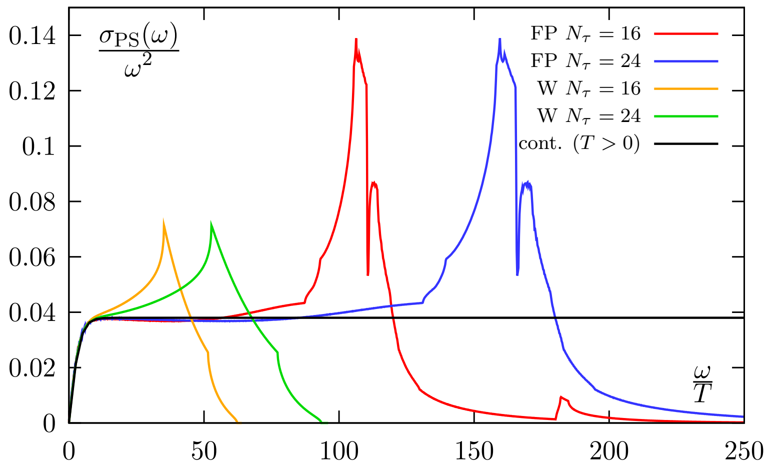
ρ enhanced over screening mass

arrows denote screening mass position

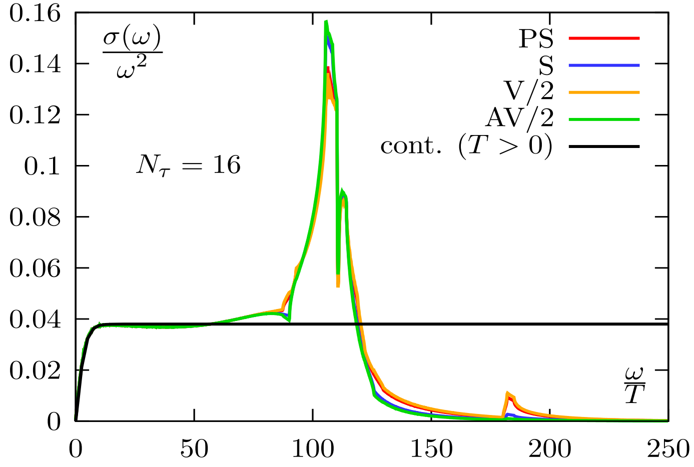
above T_c : exploring the hypercube truncated fixed point action

- motivation - arising from free quark lattice study

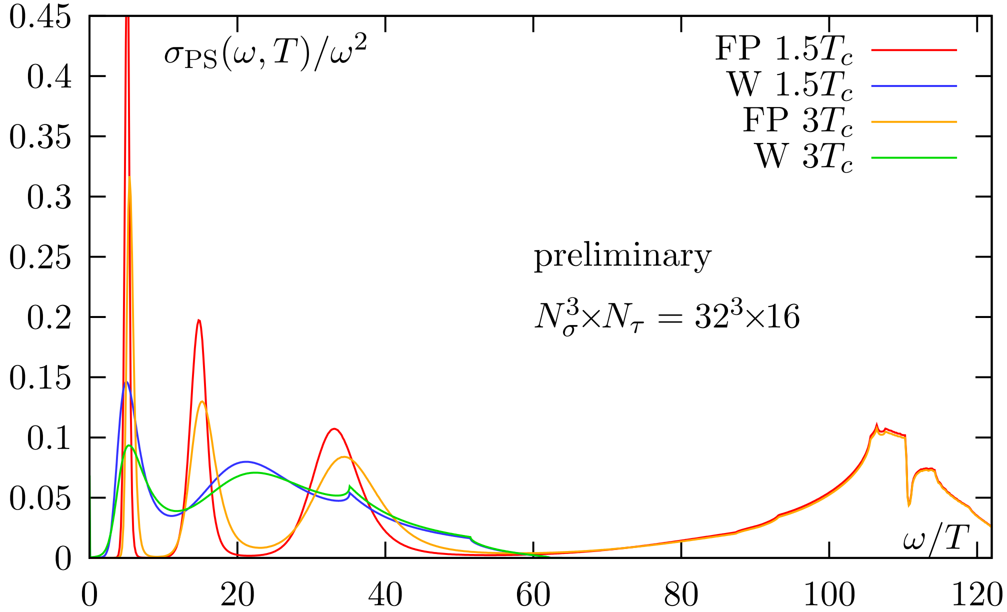
cut-off effects shifted much deeper into the UV



better chiral properties



MEM results so far



IV Outlook

in order to fully control

- spatial correlations
  - $T < T_c$: $a \rightarrow 0$ limit missing, although no big effects are expected anymore
  - $T \simeq T_c$: chiral limit needs to be improved
  - $T > T_c$: aspect ratios $N_\sigma/N_\tau = 8, 12$ helpful for the $V \rightarrow \infty$ limit
- temporal correlations
  - $T > T_c$: further studies of the systematic effects due to the lattice cut-off
  - $T > T_c$: increase sensitivity for the very infrared $\omega \rightarrow 0$