Is hot QCD a weakly or strongly coupled medium?

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Disclaimer

Will consider QCD (with $N_{\rm f}$ as a parameter) in full "static" thermal equilibrium at a temperature T.

This is the situation also addressed by all traditional lattice-QCD experiments —

while real experiments need not be fully thermalised, and the system is in any case expanding fast.

Formulation of the problem

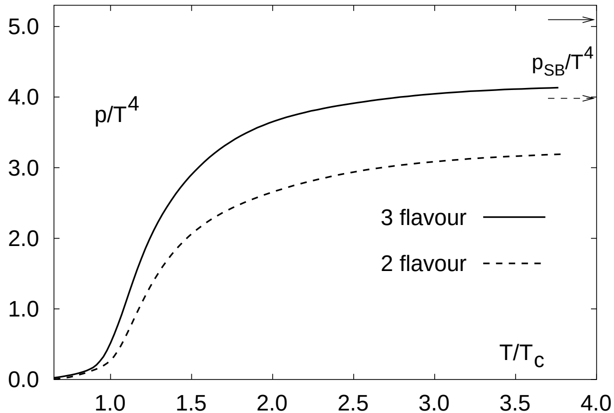
Looking at (lattice) data alone is in general not enough to tell whether the system is weakly or strongly interacting: a single expectation value does not naturally divide into free and interaction parts.

We need to **compare** lattice data with a framework which has the strength of interactions as a parameter.

For QCD perturbation theory, the strength is given by the renormalised gauge coupling $g(\bar{\mu})$.

Comment 1

0^{th} order in g — free partons — does not work "well".



Karsch, Laermann, Peikert [hep-lat/0002003]

But no problem — interactions need not be zero, they just need to be weak enough to be accountable for.

Comment 2

(expansion in g) $\neq$ (expansion in the number of loops)

The perturbative loop expansion breaks down. An expansion in g is at best obtained after an all-orders **resummation** of the colour-electric modes A_0.

Kapusta, NPB 148 (1979) 461

This is often implemented through the Dimensionally Reduced or the Hard Thermal Loop effective theory.

Ginsparg, NPB 170 (1980) 388;
Appelquist, Pisarski, PRD 23 (1981) 2305;
Frenkel, Taylor, NPB 374 (1992) 156;
Braaten, Pisarski, PRD 45 (1992) 1827

Comment 3

(expansion in g) $\neq$ (resummed loop expansion)

The resummed loop expansion still breaks down, at a finite order in g, due to infrared divergences related to the colour-magnetic modes A_i:

$$1 + \#_2 \cdot g^2 + \#_3 \cdot g^3 + ... + \infty \cdot g^n \ .$$

Linde, PLB 96 (1980) 289;
Gross, Pisarski, Yaffe, RMP 53 (1981) 43.

Comment 4

But an expansion in g still exists! (At least for time-independent observables.) It just cannot be determined with resummed perturbation theory.

Kajantie, Laine, Rummukainen, Shaposhnikov, hep-ph/9508379;
Braaten, Nieto, hep-ph/9510408

In other words, coefficients can be non-perturbative, starting from some finite order: $\infty \cdot g^n \longrightarrow \#_n \cdot g^n$.

Implementation (with Dimensional Reduction)

$$\boxed{\text{QCD; } |\mathbf{k}| \sim 2\pi T, gT, g^2T}$$

$\Downarrow$ perturbation theory (1)

$$\boxed{\text{EQCD; } |\mathbf{k}| \sim gT, g^2T}$$

$\Downarrow$ perturbation theory (2)

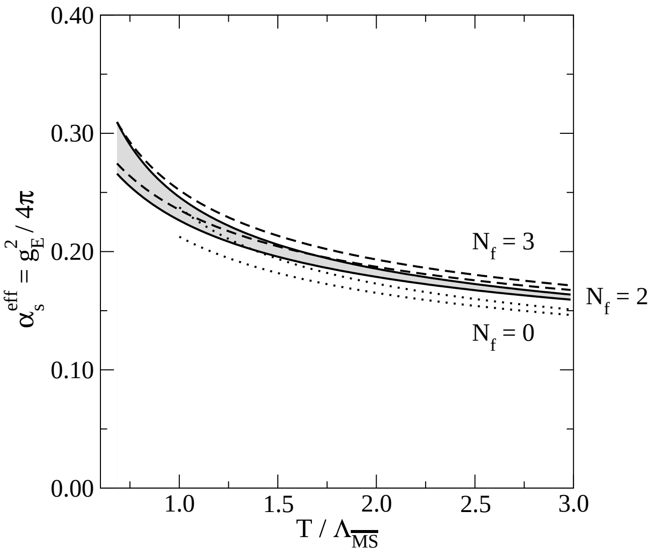
$$\boxed{\text{MQCD; } |\mathbf{k}| \sim g^2T}$$

$\Downarrow$ numerical simulations (3)

$$\boxed{\text{PHYSICS}}$$

Theoretically fine, but how about in practice?

Expansion parameter related to step (1):



Laine, Schröder, hep-lat/0509104

Step (2) may be more critical (theory more IR), and should perhaps be carried out numerically.

To be quite sure, let us benchmark a bit!

With the given framework, we can start comparing 4d lattice data with weak-coupling predictions.

We need to take observables for which (infinite volume) and continuum limits can be taken in 4d lattice QCD — which presently confines us mostly to $N_{\rm f} = 0$.

Observables can be divided into various categories:

Category 1: UV-dominated

Pressure:

$$p(T,\boldsymbol{\mu}) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \left\{ \mathrm{Tr} \left[\exp \left(-\frac{\mathcal{H}_{\mathsf{QCD}} - \mu_i \mathcal{Q}_i}{T} \right) \right] \right\}$$

Parametrically:

$$\begin{aligned} \frac{p}{T^4} \sim 1 \; + g^2_{(1)} \quad & \quad + g^4_{(1)} \ln \quad & \quad + g^6_{(1)} (\ln + [\mathsf{pert}]) + \ldots \\ + g^3_{(2)} \; & + g^4_{(2)} \ln \; + g^5_{(2)} \; & + g^6_{(2)} (\ln + [\mathsf{pert}]) + \ldots \\ & & + g^6_{(3)} (\ln + [\mathsf{non\text{-}pert}]) \; . \end{aligned}$$

Susceptibilities: $\frac{\partial^2}{\partial\mu_i \partial\mu_j} p$.

Here [non-pert] drops out but may re-emerge later.

Category 2: UV – IR -mixed

Mesonic correlation lengths:
$\langle \pi(\mathbf{x})\pi(\mathbf{0})\rangle \sim P(|\mathbf{x}|)e^{-M(T)|\mathbf{x}|}$.

$$\frac{M}{T} \sim 1 \; + g^2_{(1)} + \ldots$$
$$+ g^2_{(2)} + \ldots$$
$$+ \ldots \; .$$

Debye screening in the static potential:
$\langle \mathrm{Pol}(\mathbf{x})\mathrm{Pol}(\mathbf{0})\rangle \sim Q(|\mathbf{x}|)e^{-M(T)|\mathbf{x}|}$.

$$\frac{M}{T} \sim g_{(2)} \; + g^2_{(2)}(\ln + [\text{pert}]) + \ldots$$
$$+ g^2_{(3)}(\ln + [\text{non-pert}]) \; .$$

Category 3: IR-dominated

Spatial string tension:

$$\sigma_s = - \lim_{L_1 \to \infty} \lim_{L_2 \to \infty} \frac{1}{L_1 L_2} \ln \langle W_s(L_1, L_2) \rangle \; .$$

This time the behaviour is

$$\frac{\sqrt{\sigma_s}}{T} \sim g^2_{(3)} [\text{non-pert}] \; .$$

"Magnetic" screening masses have the same structure, as has the "strong sphaleron rate".

Catch

Surely UV-dominated observables are the easiest?

No, the opposite is the case in general!

Need to account for all scales ($2\pi T$, gT, g^2T) $\Rightarrow$ for UV-dominated need to work up to the highest order!

In other words, it is impossible to guess the order of magnitude of non-perturbative coefficients ($\infty \to \#_n$), unlike of perturbative coefficients, and it can be large.

IR-dominated example: spatial string tension

Effective theory determination:

$$\frac{\sqrt{\sigma_s}}{T} = 0.553(1) g^2_{(3)}$$ Lucini, Teper, hep-lat/0206027

$$g^2_{(3)} = g^2_{(2)}[1 + g_{(2)} + g^2_{(2)} + \ldots]$$ Giovannangeli, hep-ph/0312307

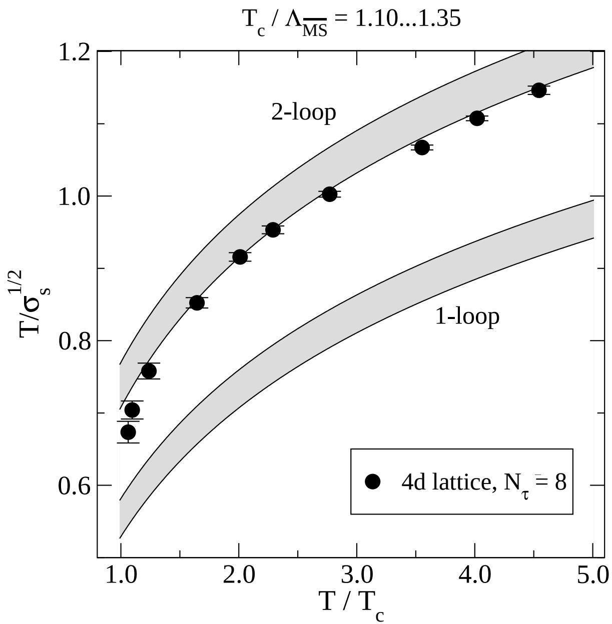
(Funnily, this series converges very fast indeed.)

$$g^2_{(2)} = g^2_{(1)}[1 + g^2_{(1)} + g^4_{(1)} + \ldots]$$ Laine, Schröder, hep-ph/0503061

(This series is less rapid, due to large logarithms.)

$g_{(1)} \equiv g(\bar{\mu})$.

Comparison with 4d lattice:



⇒ Result purely non-perturbative, yet "understandable" !

UV-IR-mixed example: Debye screening

After step (1),

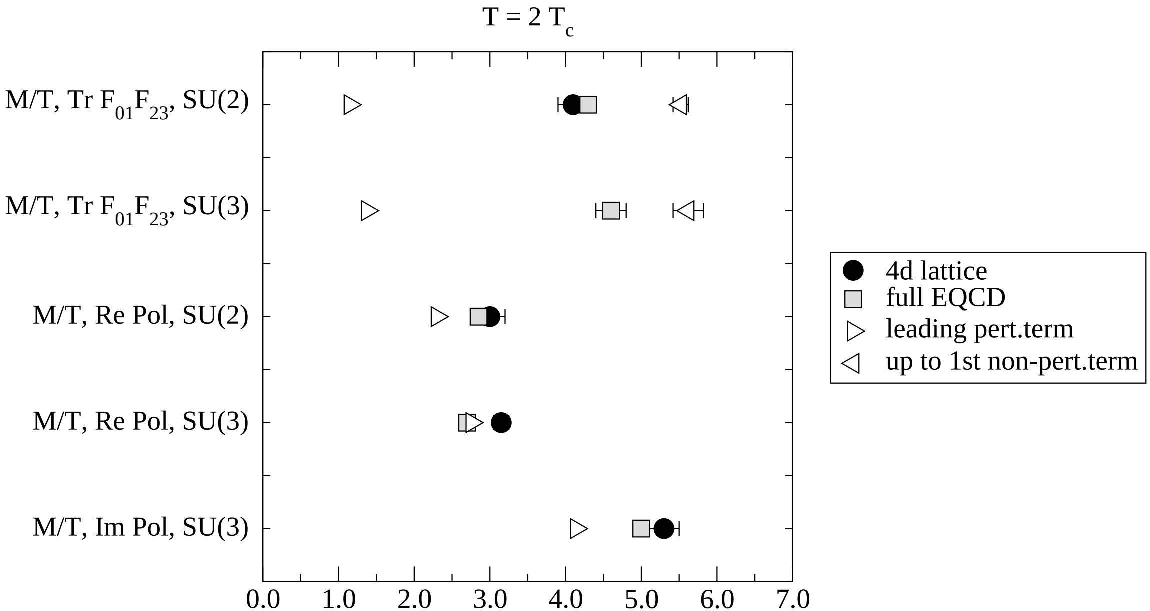
$$\mathrm{Pol}(\mathbf{x}) \approx 1 - \frac{g_{(1)}^2}{2T^2}\,\mathrm{Tr}[A_0^2(\mathbf{x})]\ .$$

The correlation length is thus determined by

$$\big\langle \mathrm{Tr}[A_0^2(\mathbf{x})]\,\mathrm{Tr}[A_0^2(\mathbf{0})]\big\rangle \sim \mathcal{Q}(|\mathbf{x}|)e^{-M|\mathbf{x}|}\ ,$$

which can directly be measured with EQCD.

Comparison with 4d lattice:



Laine, hep-ph/0301011

$\Rightarrow$ Results partly non-perturbative, yet "understandable" !

UV-dominated example: pressure

There are again two possibilities: the conservative one is to do numerics already after step (1),

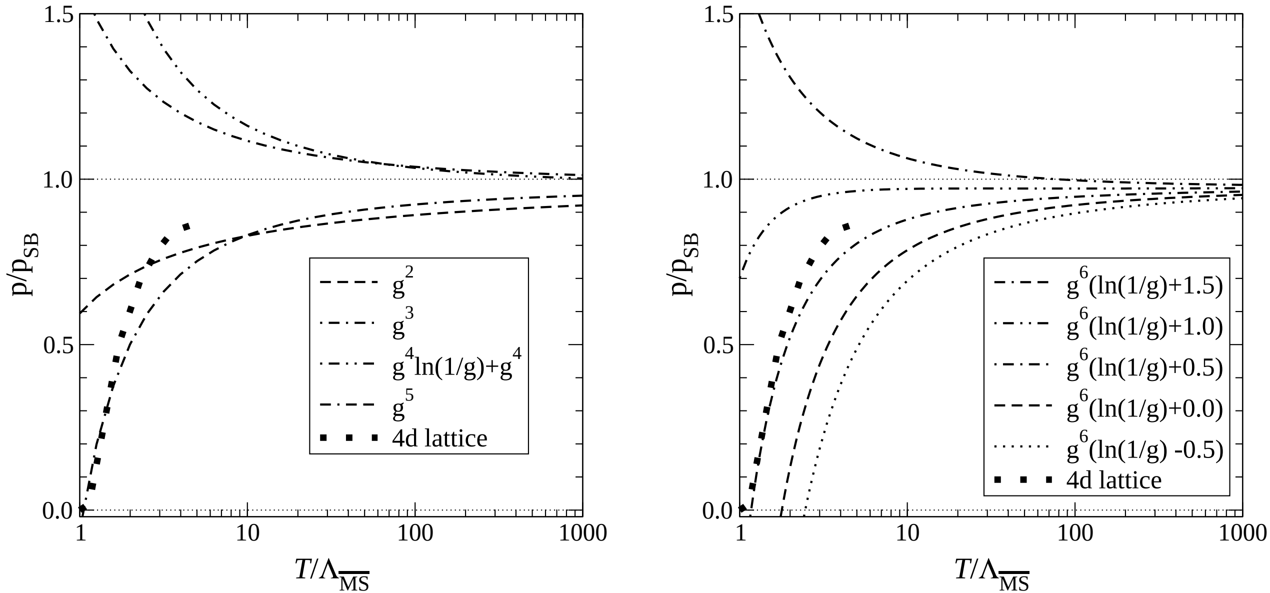
$$\frac{p}{T^4} = 1+g^2_{(1)}+g^4_{(1)}(\ln+C_4)+g^6_{(1)}(\ln+C_6)+\frac{p_{\mathsf{EQCD}}}{T^4}\ ,$$

$$p_{\mathsf{EQCD}} = \lim_{V\to\infty}\frac{T}{V}\ln\Big[\int\mathcal{D}A_i\mathcal{D}A_0\exp\Big(-S_{\mathsf{EQCD}}\Big)\Big]\ .$$

The full reduction requires also step (2), up to the first non-perturbative coefficient.

Neither setup is complete yet! (Since C_6 is missing.)

Comparison with 4d lattice ($N_\mathrm{f} = 0$; after both steps):



Kajantie, Laine, Rummukainen, Schröder 2002

$\Rightarrow$ Non-perturbative coefficient again numerically very significant, but afterwards everything OK?

Future (?)

Given that the approach works at least qualitatively (within 15% at $2T_\mathrm{c}$), it should be extended to situations where its strengths become even more apparent.

For instance, dependence on the properties of quarks.

Moreover, real-time observables like spectral functions (& viscosities?) can hopefully also be addressed, without recourse to additional ingredients like MEM.

Is hot QCD a weakly or strongly coupled medium?

In a way it is both! Weak-coupling expansion does give the correct picture, but there are remnant confining effects in the form of non-perturbative coefficients.

In any case it appears to be an **understandable** medium, once the non-perturbative terms are fixed.

Hopefully we can use this understanding to crosscheck those lattice-QCD results which insert additional model assuptions into their analysis.