Supercomputing Relativistic Heavy-Ion Collision Physics

Tata Institute of Fundamental Research

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Aspects of the QCD Phase Transition(s) from the Lattice

- The Finite T Phase Transition : the apeNEXT steps

  K. Jansen, MpL, I. Wetzorke and the APE Collaboration;, in progress

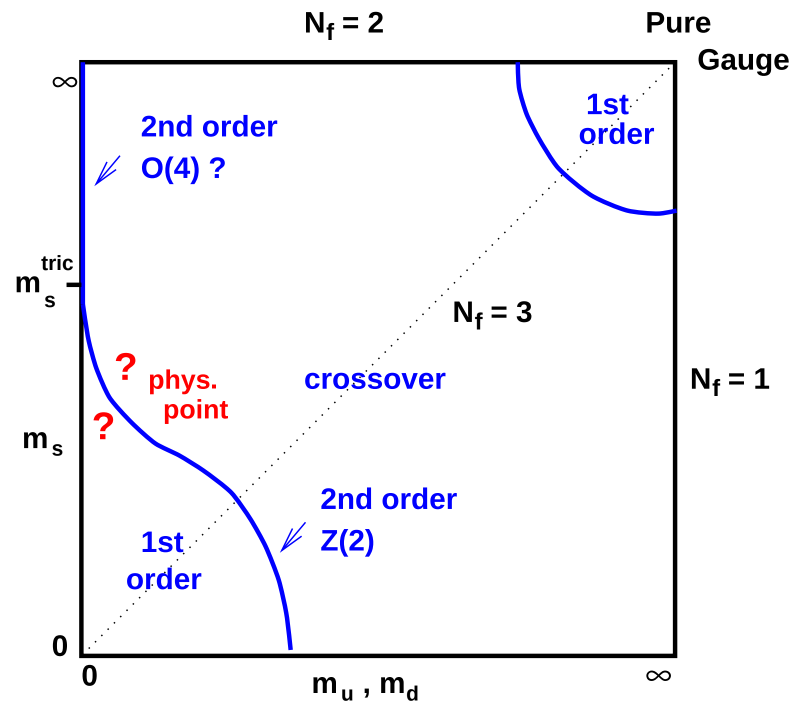
- Chiral Symmetry, Confinement, Topology

  B. Alles, M. D'Elia, MpL, M. Pepe; Phys.Rev.D70:074509,2004, hep-lat/0512xxx

- Beyond $\mu/T \simeq 1$.

  MpL, hep-lat/0509181, M. D'Elia, F. Di Renzo, MpL hep-lat/0511029

The high T, $\mu = 0$ transition



	$U(1)_A$ anomaly	suppressed anomaly at T_c
QCD	$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$	$U(N_f)_L \otimes U(N_f)_R \to U(N_f)_V$
$N_f = 1$	crossover or first order	O(2) or first order
$N_f = 2$	O(4) or first order	$U(2)_L \otimes U(2)_R / U(2)_V$ or first order
$N_f \geq 3$	first order	first order
aQCD	$SU(2N_f) \to SO(2N_f)$	$U(2N_f) \to O(2N_f)$
$N_f = 1$	O(3) or first order	U(2)/O(2) or first order
$N_f = 2$	SU(4)/SO(4) or first order	first order

Symmetry Breaking Patterns for N_f light quarks

Pisarski, Wilczek; original discussion

Basile, Pelissetto, Vicari 2005; RG analysis

Implications on the endpoint in the T,μ plane?

The Universality Class of Nf=2 QCD from the Lattice:

Staggered Fermions:

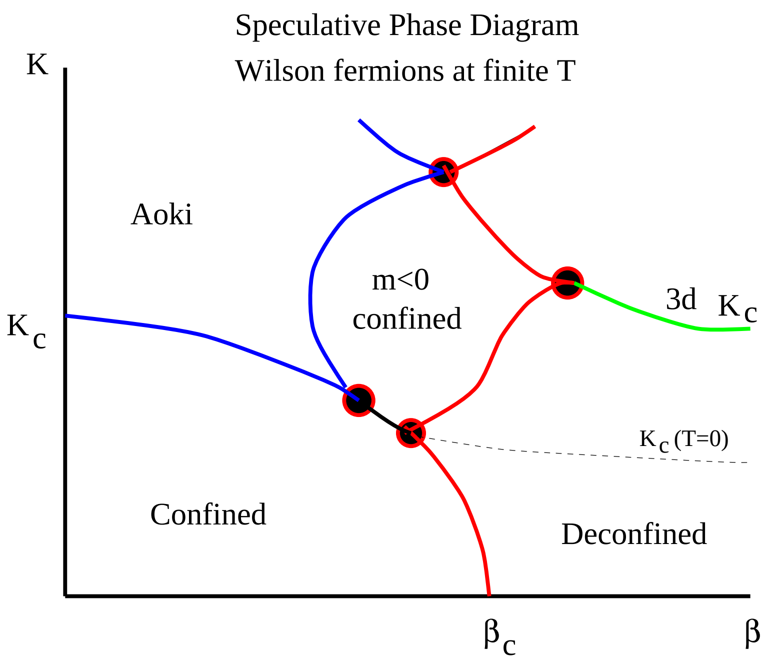
- *O(2) or O(4) with Nt=8 but scaling window very narrow - other behavious cannot be ruled out (Kogut Sinclair 2004)*

- *O(2) at strong couplint very high precision low masses Chandrasekharan Strouthos*
- *First order (O(2) / O(4) ruled out) Nt=4 Pisa Group*

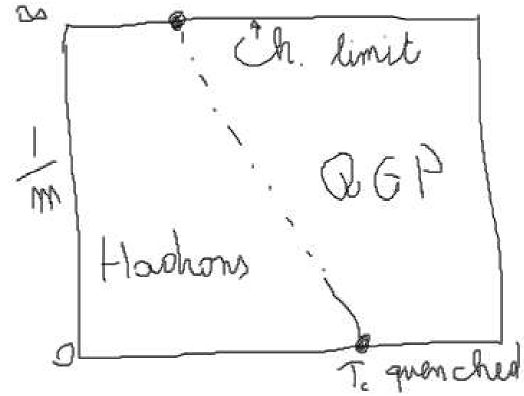
Wilson fermions:

- *Apparently compatible with O(4) scaling Nt=4- CP-Pacs 2001*

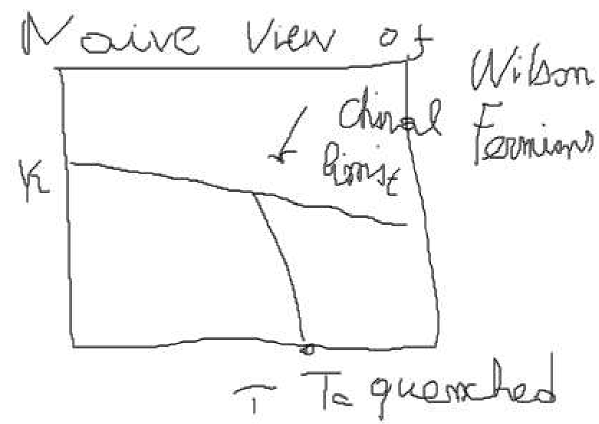
Wilson fermions might hold the key to a better understanding of QCD thermodynamics, however:



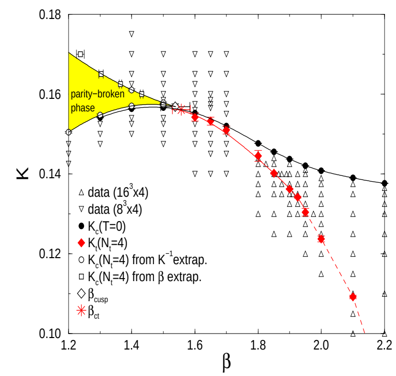
"...This diagram is wonderfully complex, probably incomplete, and may takeome time to map out" **Michael Creutz, 1996**



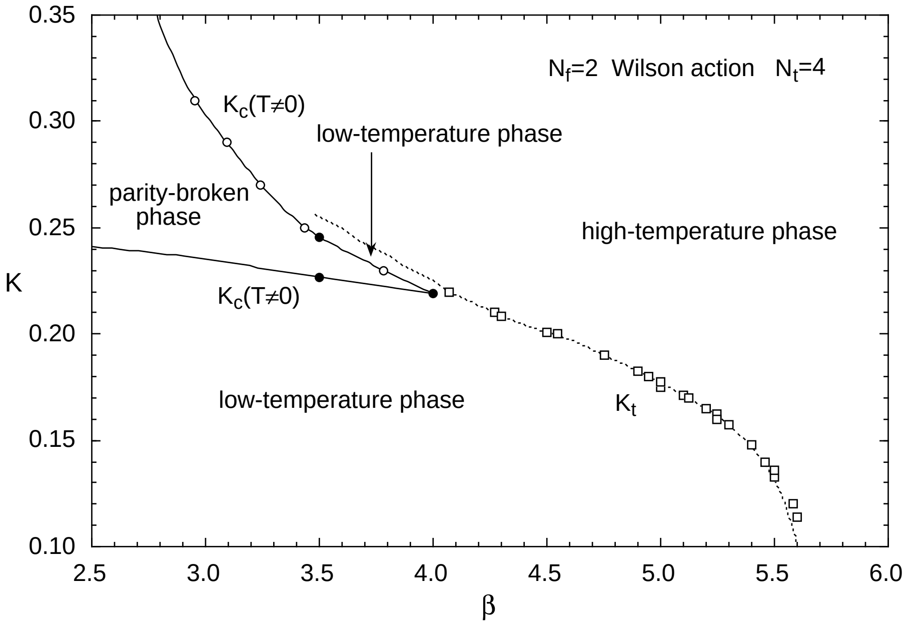
Phases of QCD in the T, mass plane

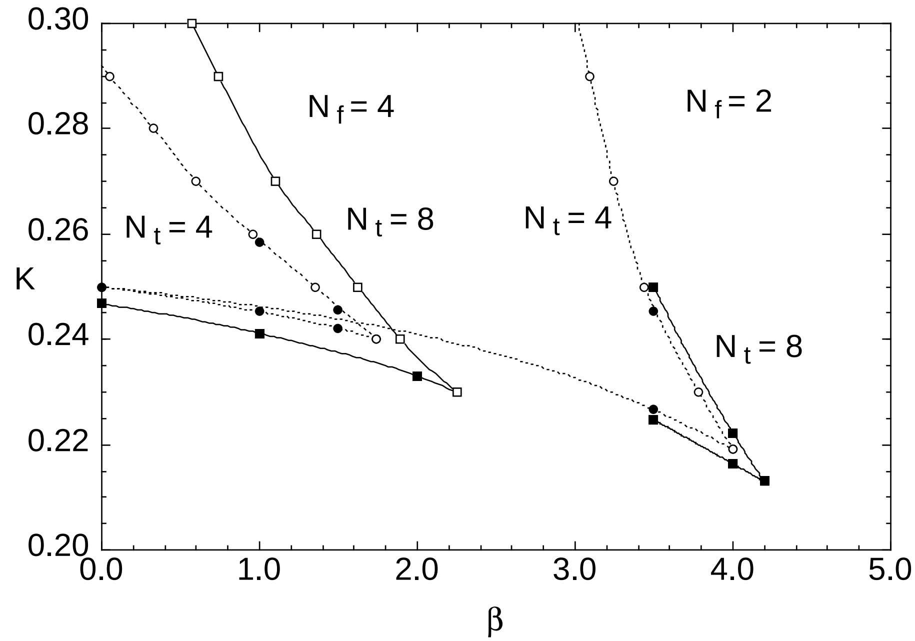


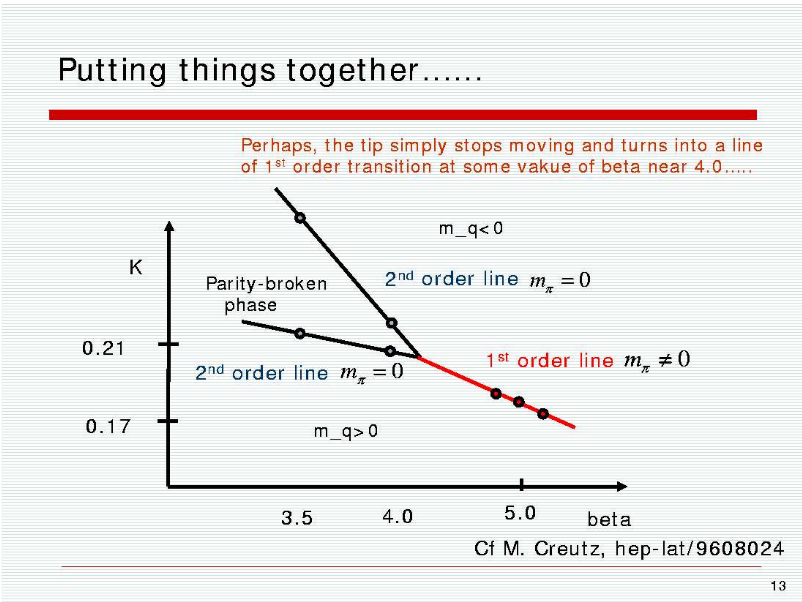
Phases of Wilson Fermions: The naive view



Phases of Wilson Fermions : Lattice results







Open questions (Akira Ukawa 2004)

Phases of Wilson Fermions :

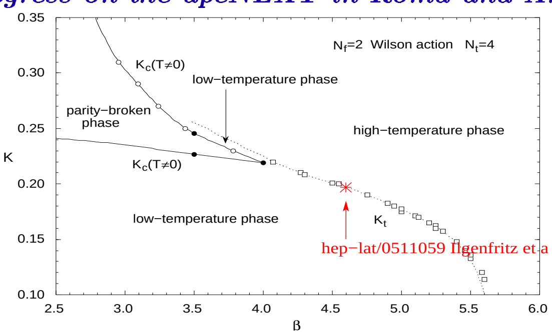
The (ape)NEXT steps

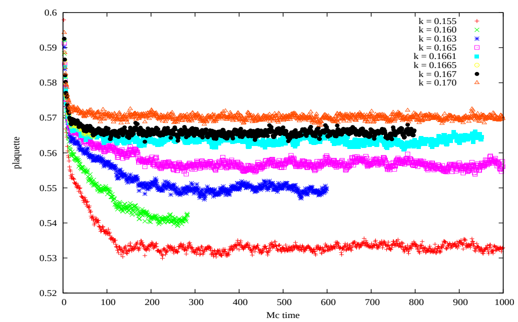
QCD Thermodynamics with Wilson Fermions

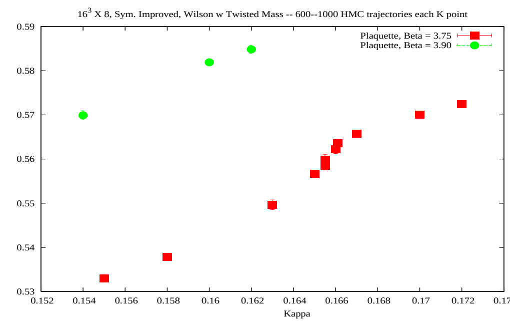
Twisted Mass, Symanzik Improved, Nt=8

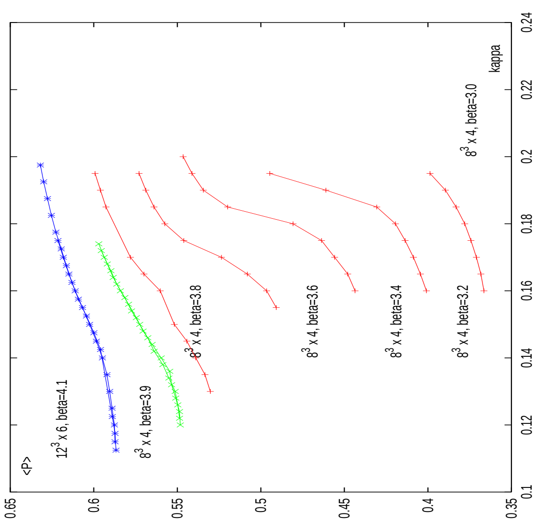
K. Jansen, M.P. Lombardo, I. Wetzorke and the APE Collaboration

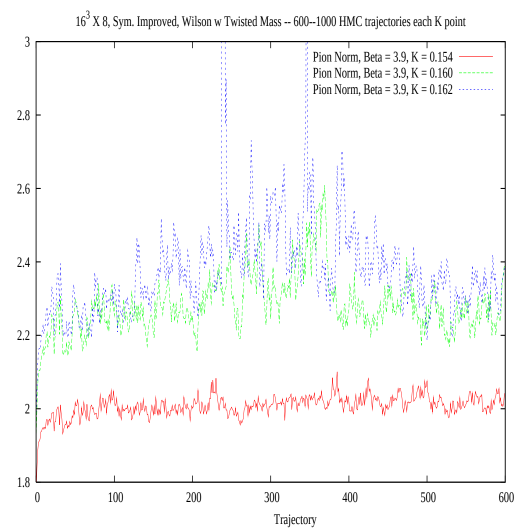
in progress on the apeNEXT in Roma and Amaro.

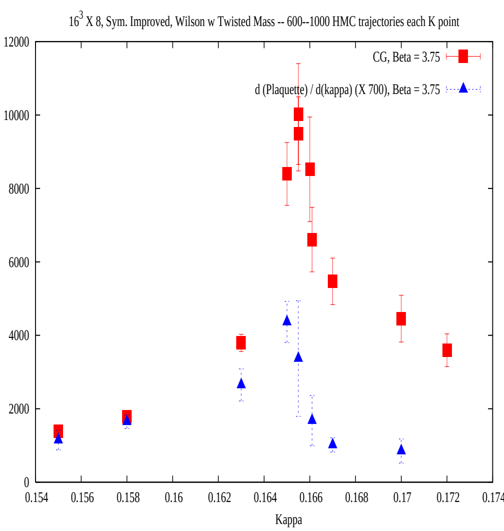


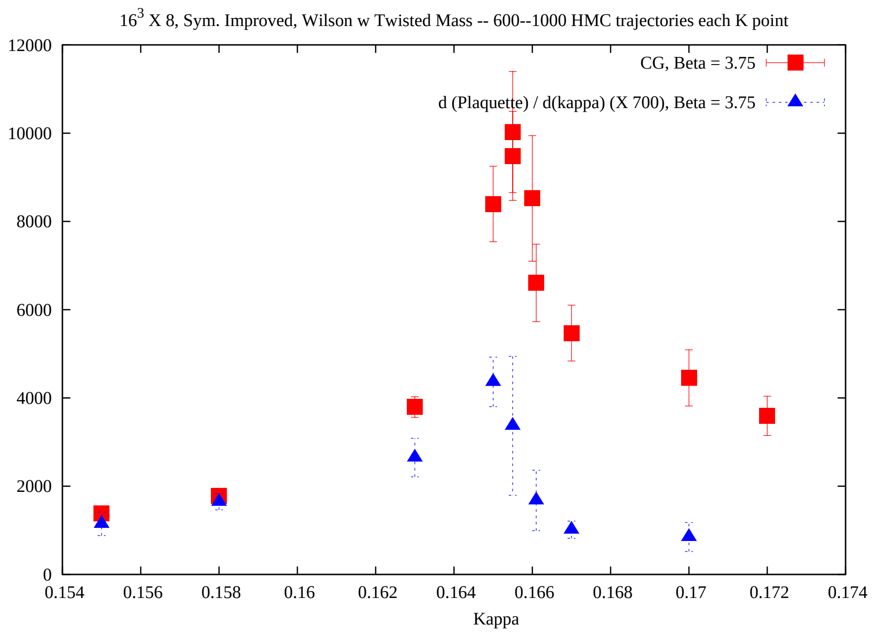


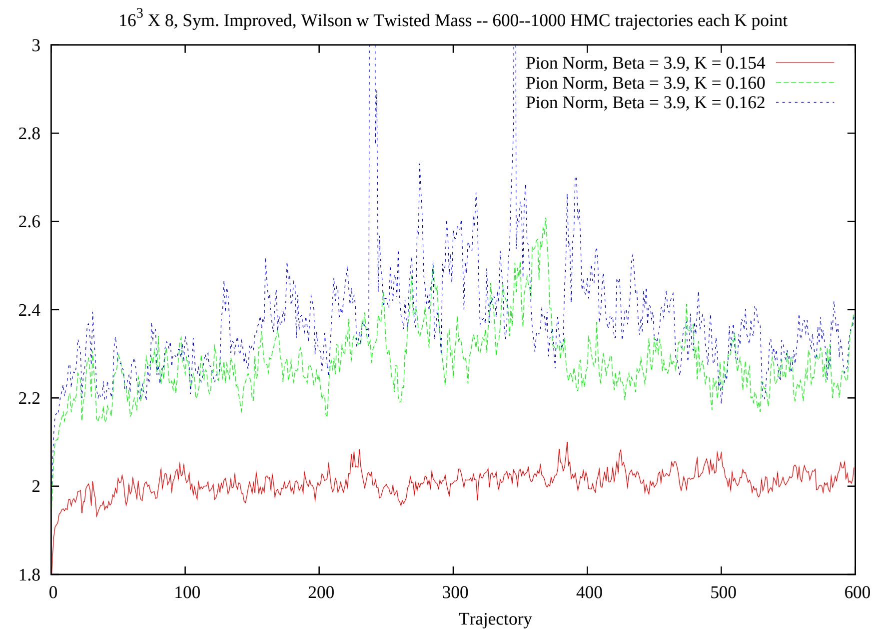












$K_c(\beta = 3.75, T = 0) = 0.1661$ $K_c(\beta = 3.90, T = 0) = 0.1609$

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Importance Sampling and The Positivity Issue

$$\mathcal{Z}(T,\mu,U) = \int dU e^{-(S_{YM}(U) - log(\det M))}$$

$\det M > 0 \to$ **Importance Sampling**

$$M^\dagger(\mu_B) = -M(-\mu_B)$$

$\star$ $\mu = 0$ **:** $-M^\dagger M$ **is positive Particles-antiparticles** **symmetry**

$\star$ **Pseudoreal Group (e.g. SU(2)) :** $-M^\dagger M$ **is positive, Particles-antiparticles** **equivalence**

$\star$ **Imaginary** $\mu \neq 0$ **:** $-M^\dagger M$ **is positive (Real) Particles-antiparticles** **symmetry**

$\star$ **Real** $\mu \neq 0$ **Particles-antiparticles** **asymmetry** $\to \det M$ **is complex in QCD**

QCD with a real baryon chemical potential:
use information from the accessible region

$$Real\mu = 0, Im\mu \neq 0$$

QCD and a Complex μ_B
The Roberge and Weiss analysis

$$\mathcal{Z}(\nu) = Tre^{-\beta H + i\beta\nu N} = e^{-\beta H + i\theta N}$$

1. **$\mathcal{Z}(\theta)$ has a periodicity 2π anyway.**
2. **If only color singlet are allowed, then $N = 0$ mod (N_c) and periodicity becomes $2\pi/N_c$**

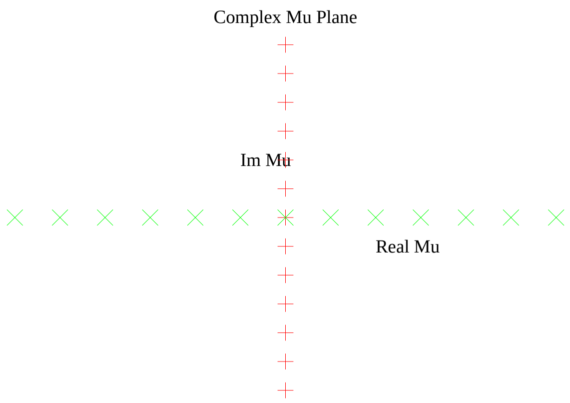
However (Roberge Weiss (1986))

$\mathcal{Z}(\theta)$ has always period $2\pi/N_c$

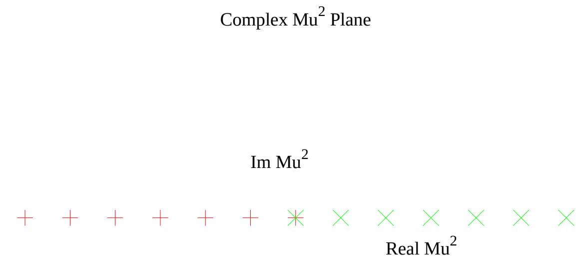
The imaginary chemical potential changes the preferred vacuum for the Polyakov loop from $\phi_P = 0$ to one of its Z_3 images

The strong coupling analysis shows that periodicity is smooth at low temperature, and p.t. theory suggests that it is sharp at high T

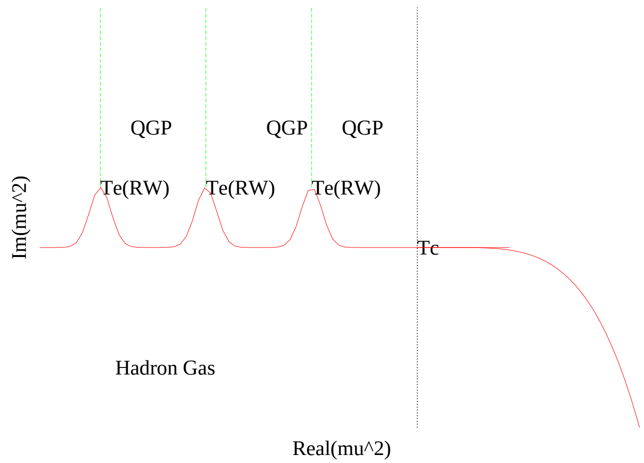
Analytic continuation *along one arc* in the complex μ plane:



$$Z(\mu) = Z(-\mu)$$



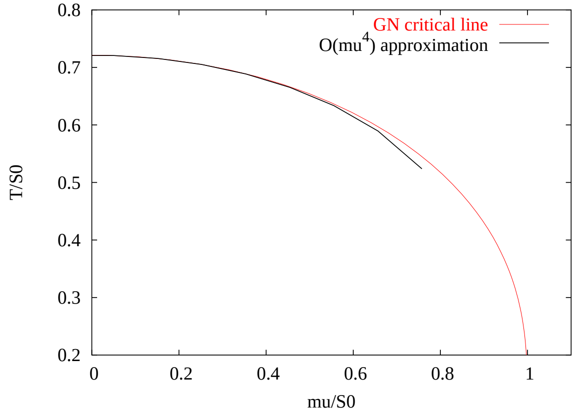
Can be mapped onto the complex μ^2 plane
To define $\mathcal{Z}(\mu^2)$ which is real valued for real μ^2, as in ordinary statistical models in external fields



The Phase Diagram in the T, μ_B^2 Plane

Region accessible to simulations: μ^2 real ≤ 0.

Multiparameter Reweighting ($\mu = 0$):
Fodor, Katz, Csikor, Egri, Szabo, Toth
Derivatives ($\mu = 0$):
Gupta, Gavai; MILC; QCD-Taro
Expanded Reweighting ($\mu = 0$)
Bielefeld-Swansea
Analyitic continuation from Imaginary μ
S.Coupling QCD MpL
Dim. Reduced QCD Laine, Hart, Philipsen
QCD de Forcrand, Philipsen; D'Elia, MpL; Azcoiti et al.; Luo et al.



Analytic continuation of the critical line from an imaginary μ Forcrand, Philipsen
Lessons from the Gross Neveu Model
The critical line of the GN model Hands, Kim, Kogut:

$$1 - \mu/\Sigma_0 = 2T/\Sigma_0 ln(1 + e^{-\mu/T})$$

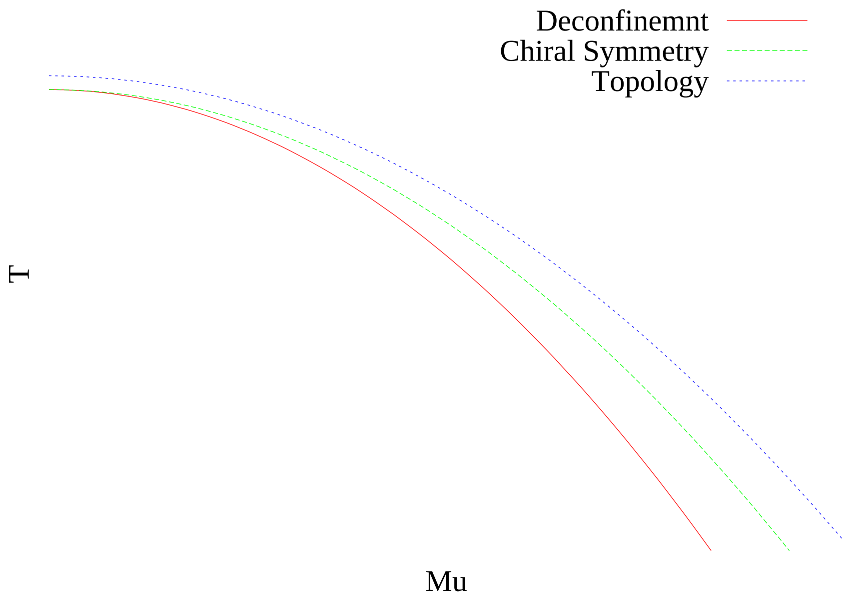
Reduces to:

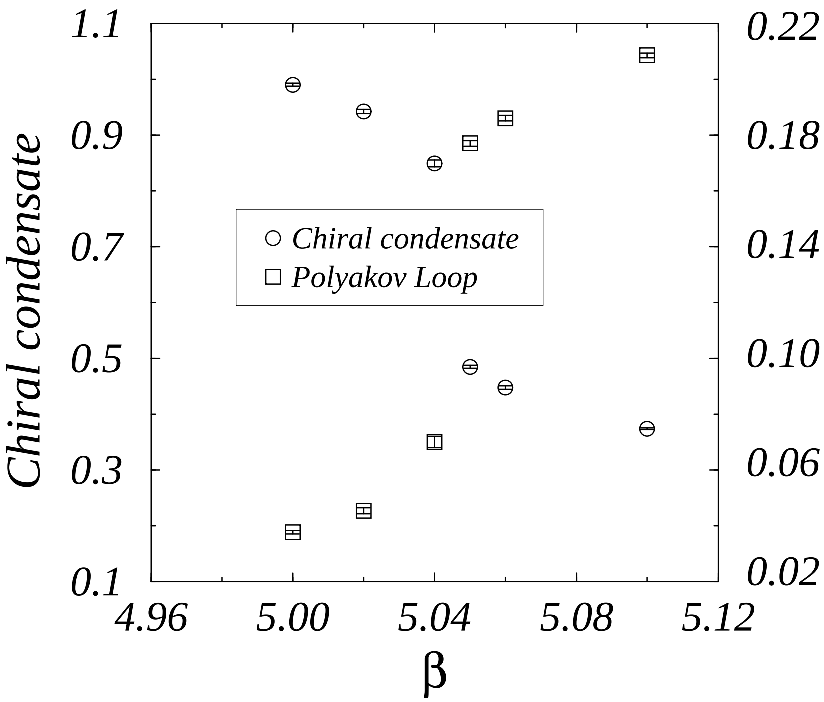
$$T(T - T_c) + \mu^2/(8ln2) = 0$$

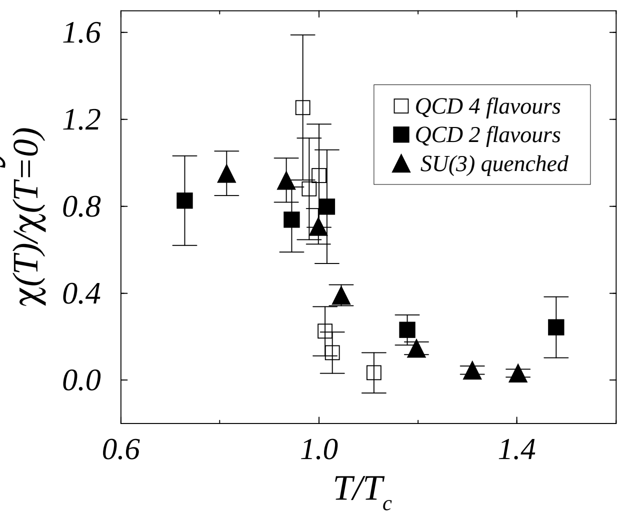
A second order approximation is good up to $\mu \simeq T_c$ D'Elia, MpL

The nature of the critical line

Interrlation among chiral condensate, Polyakov Loop and Chiral Susceptibility at nonzero temperature and density



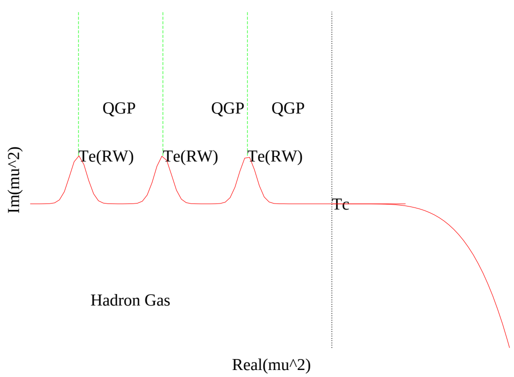


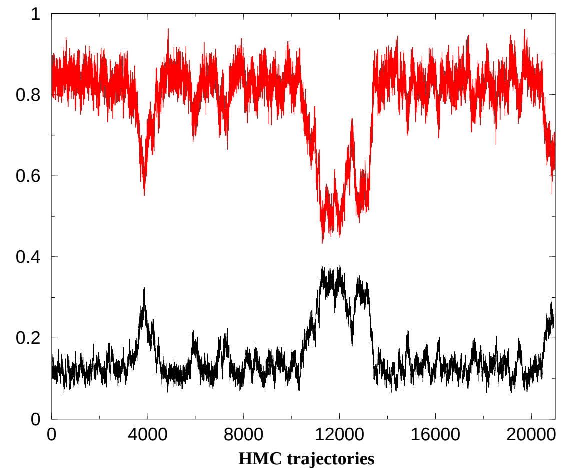


The $\mu = 0$ Transition Pisa group, 2000

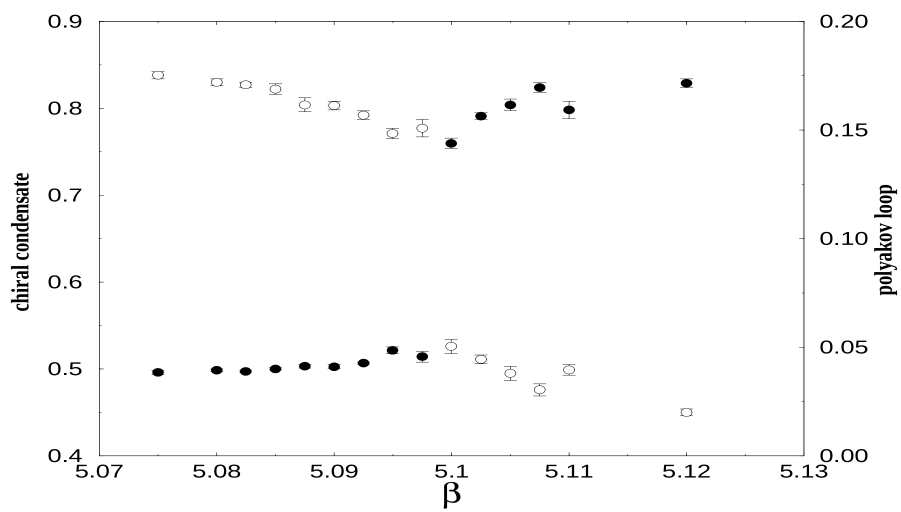
Chiral Symmetry-Confinement From Imaginary Chemical Potential

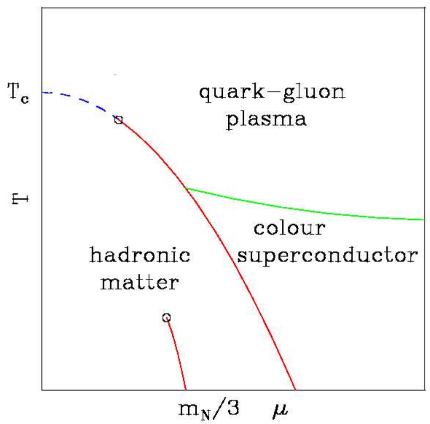
D'Elia MpL 2004

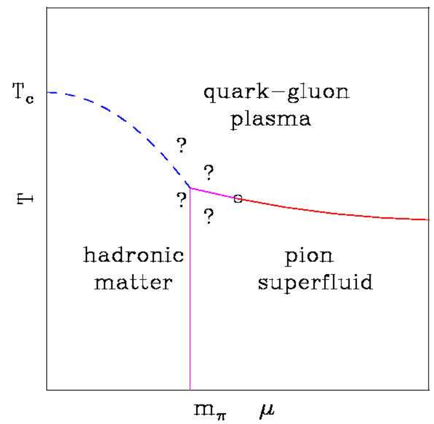




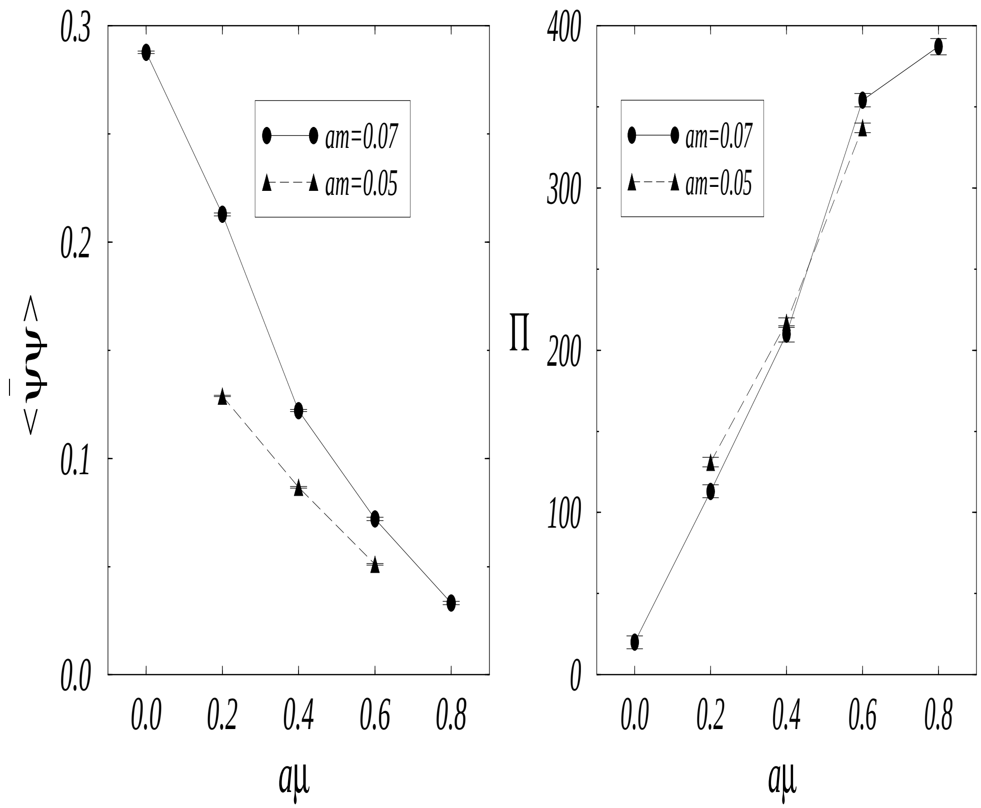
$$\Delta(\beta) = \beta_c^{conf} - \beta_c^{chiral} = 0$$





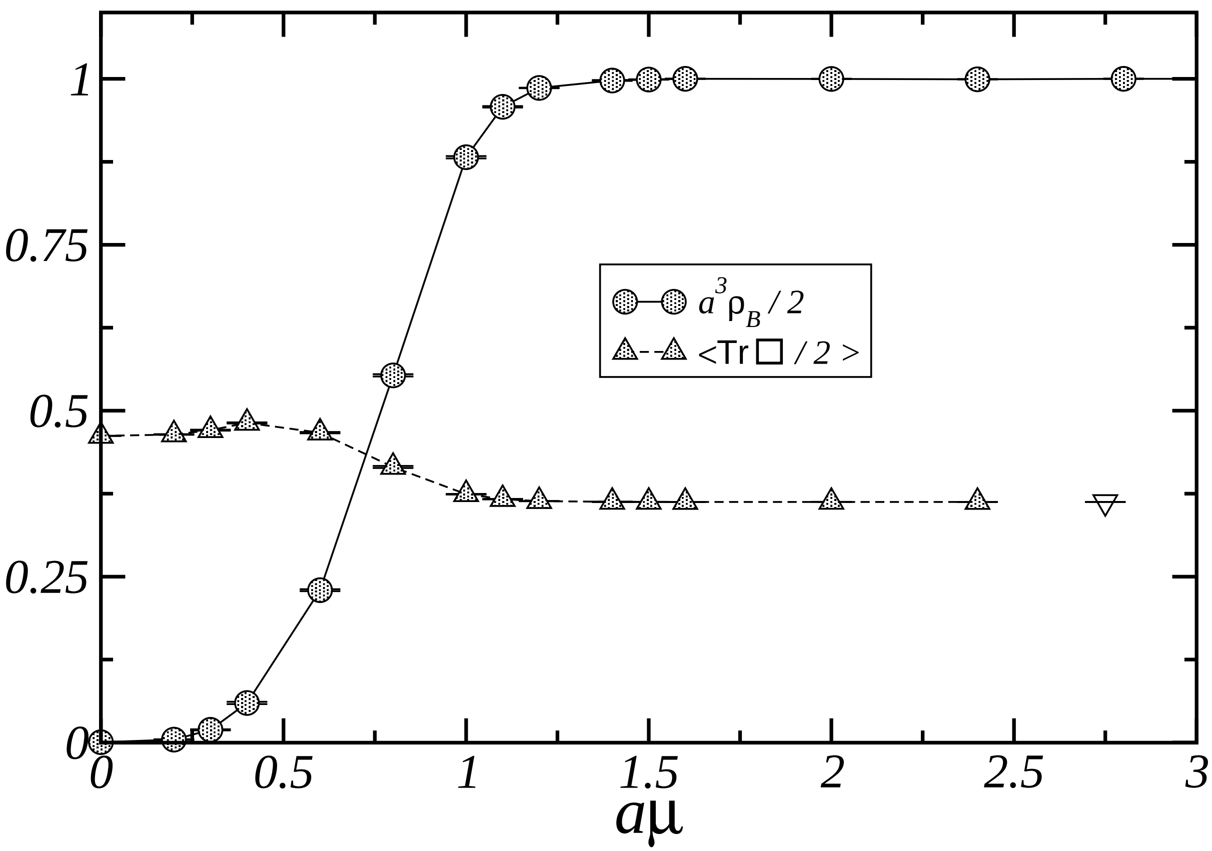


Learning from Two Colors



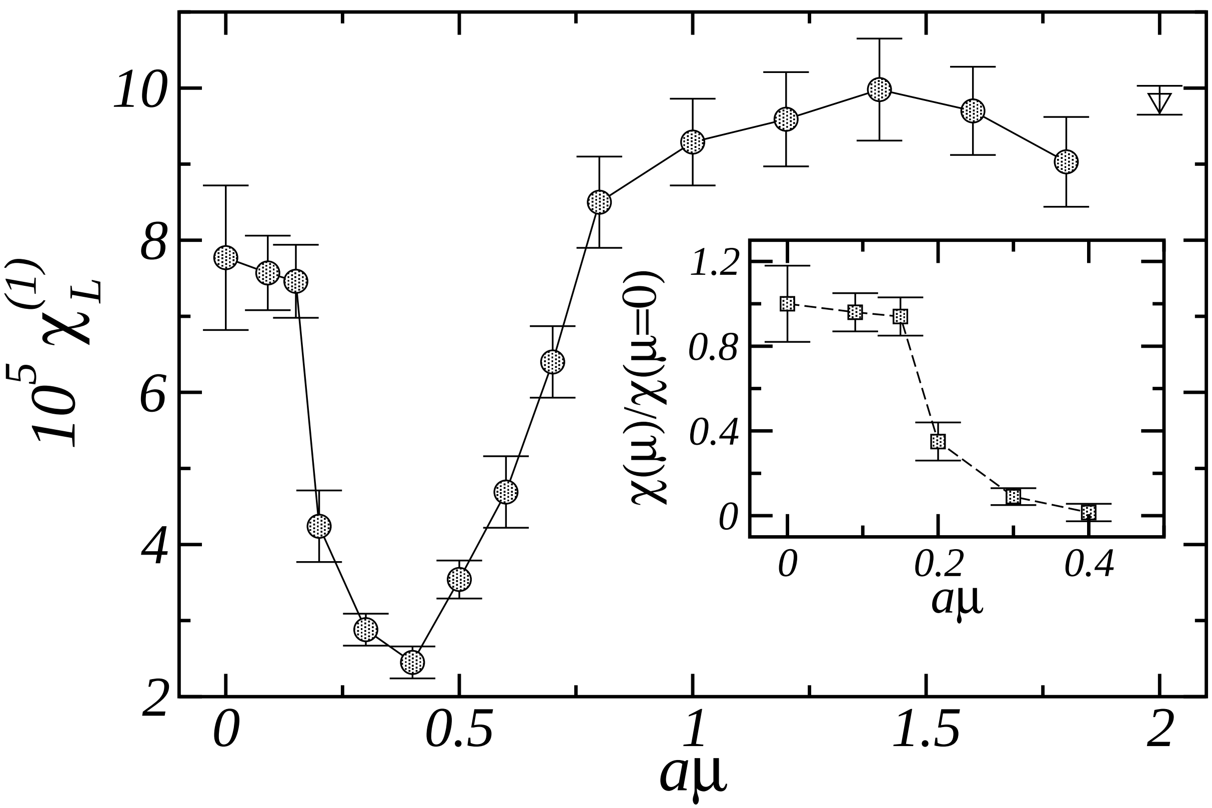
Polyakov Loop and Chiral Condensate

Alles, D'Elia, MpL, Pepe, Lattice2000, Bangalore



Plaquette and number density

Alles, D'Elia, MpL, Pepe; hep-lat/0512xxx



Topological Susceptibility

Alles, D'Elia, MpL, Pepe; hep-lat/0512xxx

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Beyond $\mu/T = 1$

Series representation, Pade' approximants and critical behaviour

Results obtained at imaginary μ_B can be analytically continued to real μ_B.

In principle, rigorous arguments guarantee that the analytic continuation of a function can be done within the entire analytic domain.

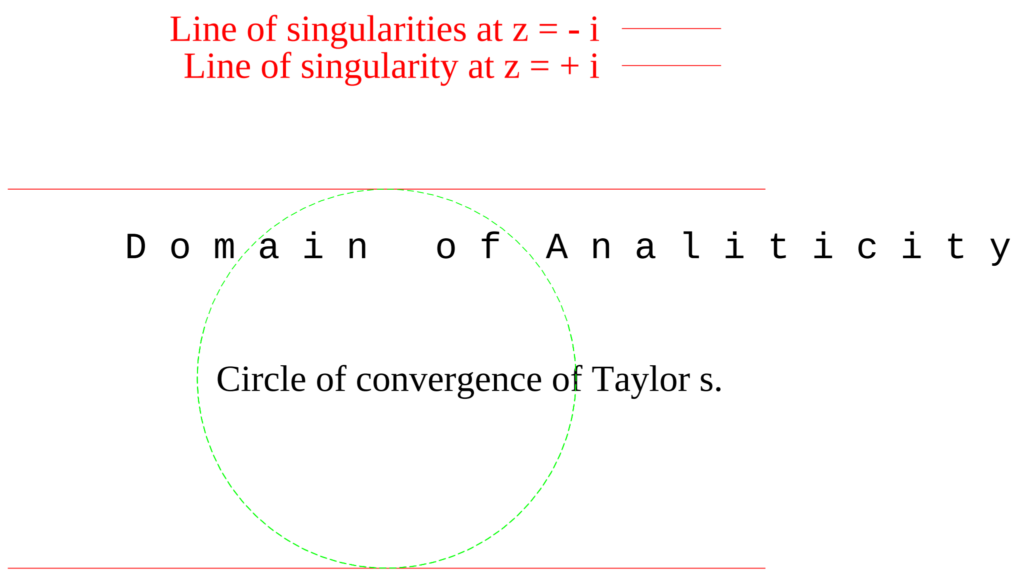
In practice, the exact analytic form is not known, and a systematic procedure relying on the Taylor expansion is only valid within the circle of convergence of the series itself.

Pade' series is one practical way to accomplish analytic continuation beyond the radius of convergence.

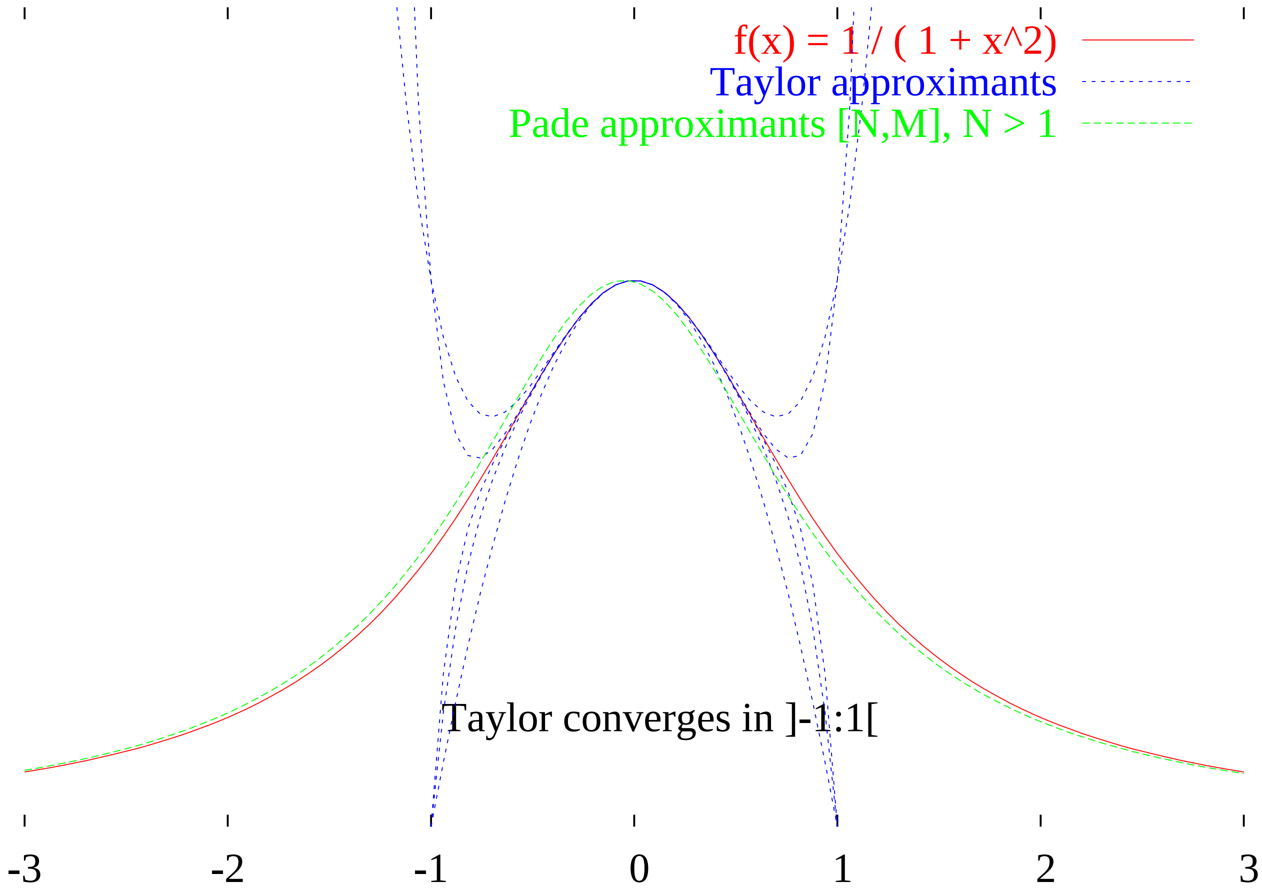
Examples:

$$f(z) = \frac{1}{z^2 + 1}$$

$$f(z) = \frac{e^{-z}}{z^2 + 1}$$

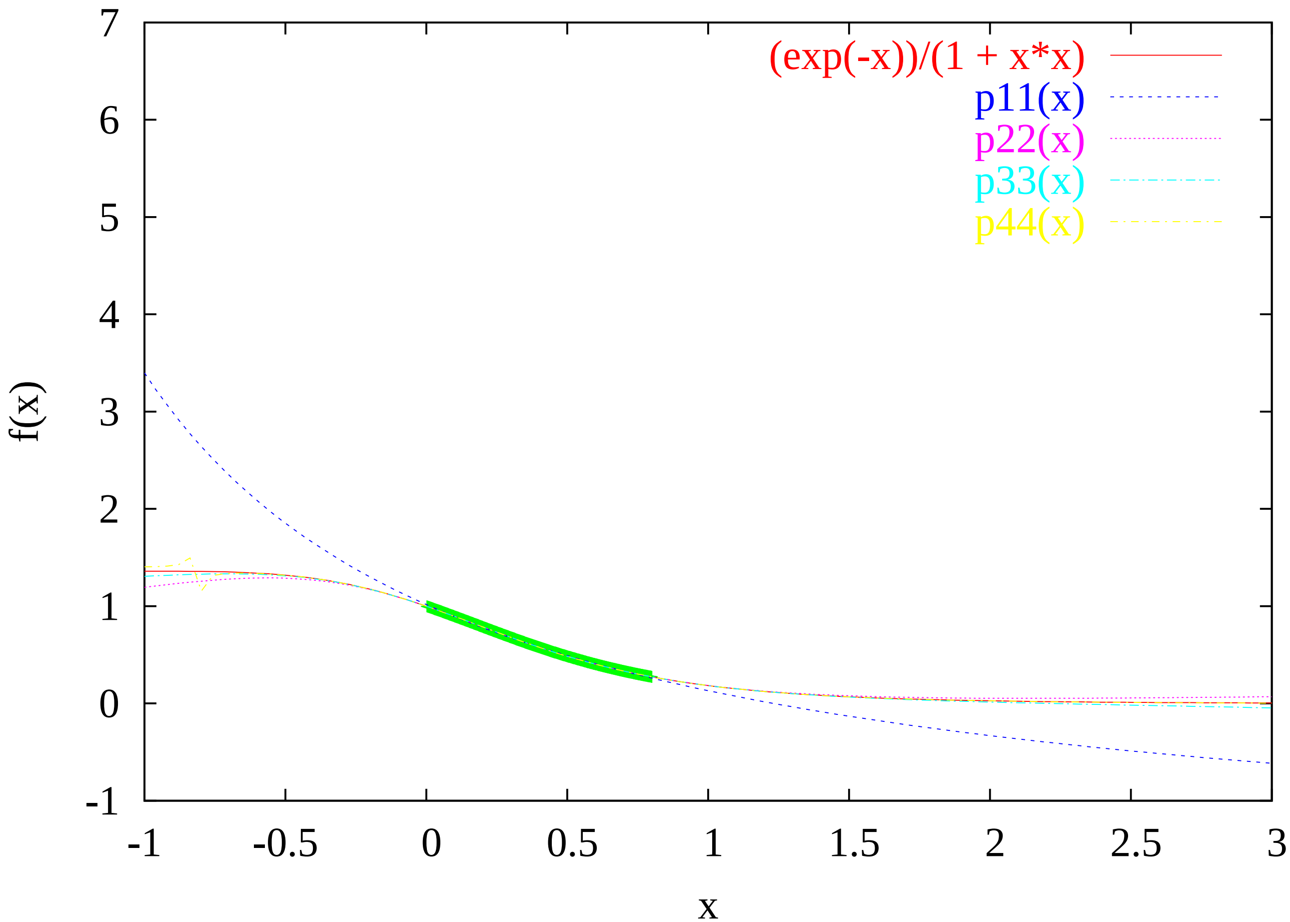


$$f(z) = \frac{1}{z^2 + 1}$$



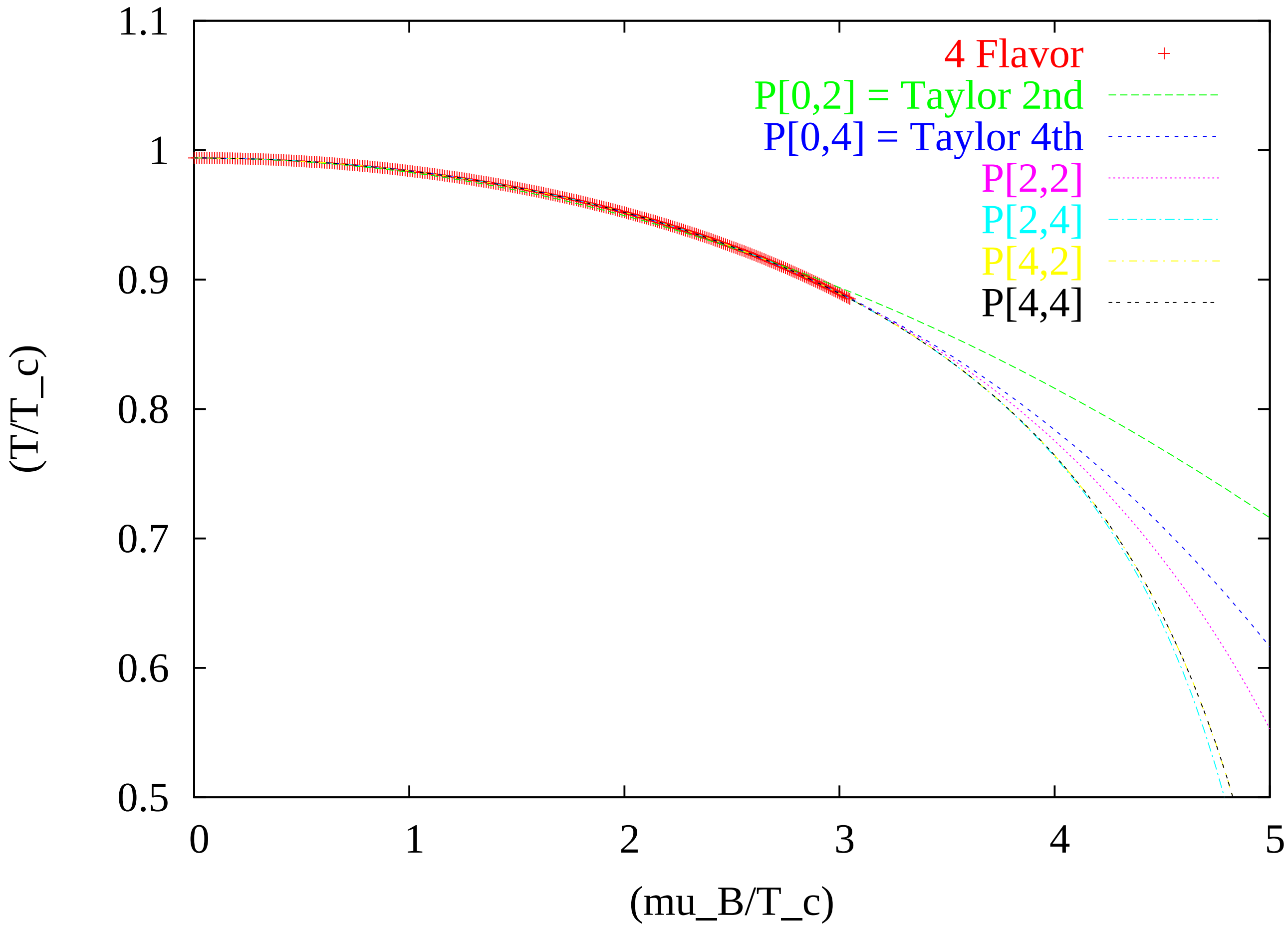
f(x) = 1 / (1 + x^2)
Taylor approximants
Pade approximants [N,M], N > 1
Taylor converges in]-1:1[
-3
-2
-1
0
1
2
3

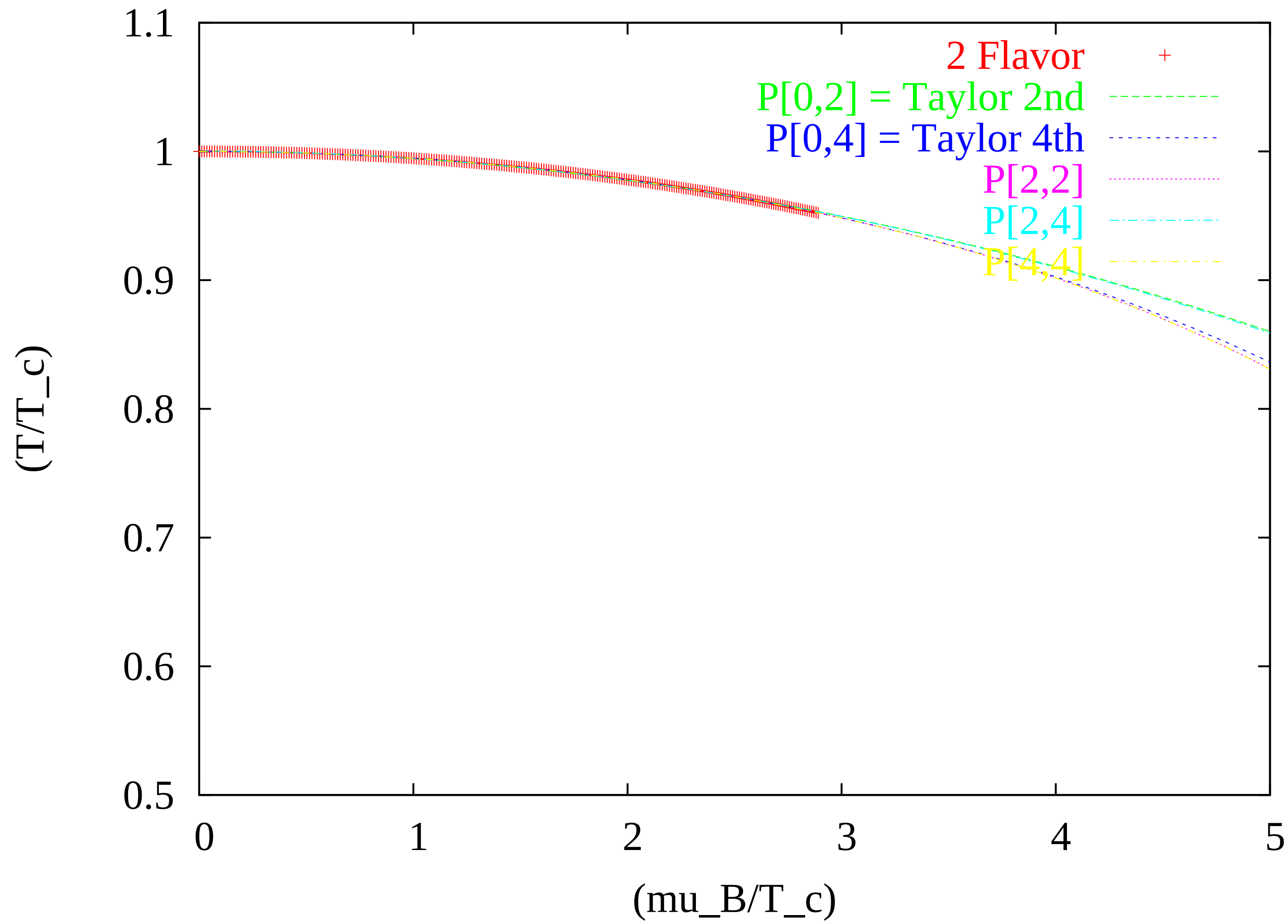
$$f(z) = \frac{e^{-z}}{z^2 + 1}$$



The Critical Line: Pade' analysis vs Taylor

Data from de Forcrand, Philipsen (2 flavour) D'Elia, MpL (4 flavour)





Results seem stable beyond $\mu_B = 500 MeV (\mu_B/T \simeq 1)$, with Pade' analysis in good agreement with Taylor expansion

The Critical Line from HRG

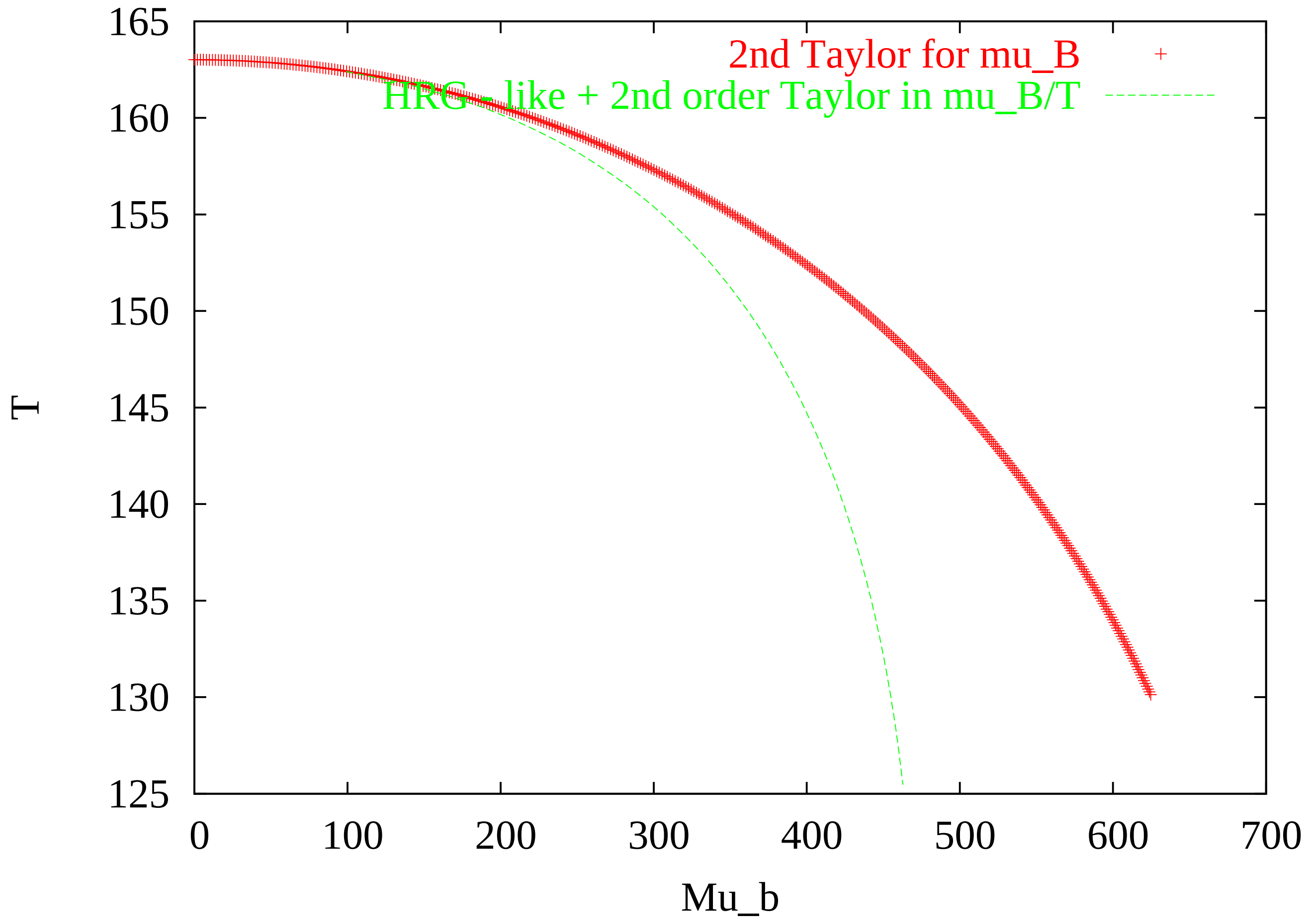
The critical temperature as a function of μ_B is determined by lines of constant energ1 density: $\epsilon \simeq 0.5 - 1.0$ GeV/fm^3. .
Results by: Kogut and Toublan; Karsch, Redlich, Tawfik
A continuation of the critical line using the HRG ansatz plus a fixed energy (or any other quantity determined at $\mu = 0$) suggests this implicit form for the critical line

$$T = f(T) cosh(\mu_B/T)$$
$$\lim_{\mu_B/T \to 0} f(T) cosh(\mu_B/T) = 1 - k\mu^2$$

giving:

$$\begin{aligned} T &= T_c(\mu_B = 0) g(t) \qquad (1) \\ \mu_B &= T_c(\mu_B = 0) acosh(t) g(t) \\ & g(t) = -1 + \frac{\sqrt{(}1 + 8k(t-1)}{4k(t-1)} \end{aligned}$$

HRG and the critical line



Hadronic Phase and Fourier analysis

The *Hadron Resonance Gas* model might provide a description of QCD thermodynamics in the confined, hadronic phase of QCD

$$\frac{P(T,\mu) - P(T,0)}{T^4} \simeq F(T)(\cosh(\frac{\mu_B}{T} - 1)$$

$$F(T) \simeq \int dm \rho(m)(\frac{m}{T})^2 K_2(\frac{m}{T})$$

Karsch, Redlich, Tawfik: match the coefficients of the Taylor expansion to the HRG form.

Hadron Resonance Gas

From real to imaginary μ (D'Elia,MpL, 2002, 2004)

Observables are periodic and continuous for imaginary chemical potential. (Roberge, Weiss, 1986)

$$O_e = ae_F + \sum be_F cos(N_c N_t \mu)$$

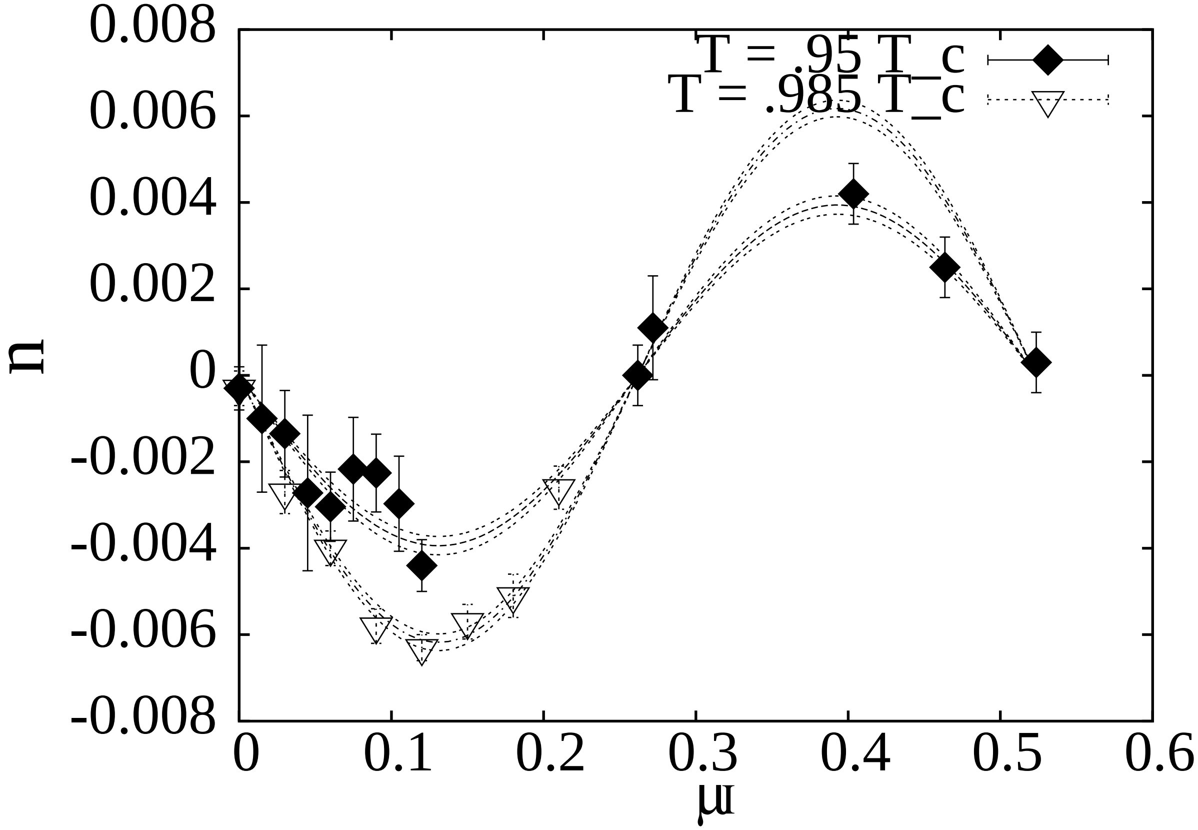
$$O_o = ao_F + \sum bo_F sin(N_c N_t \mu)$$

When HRG holds true, one term in the Fourier series should suffice.

$sinh(x) \to sin(x)$

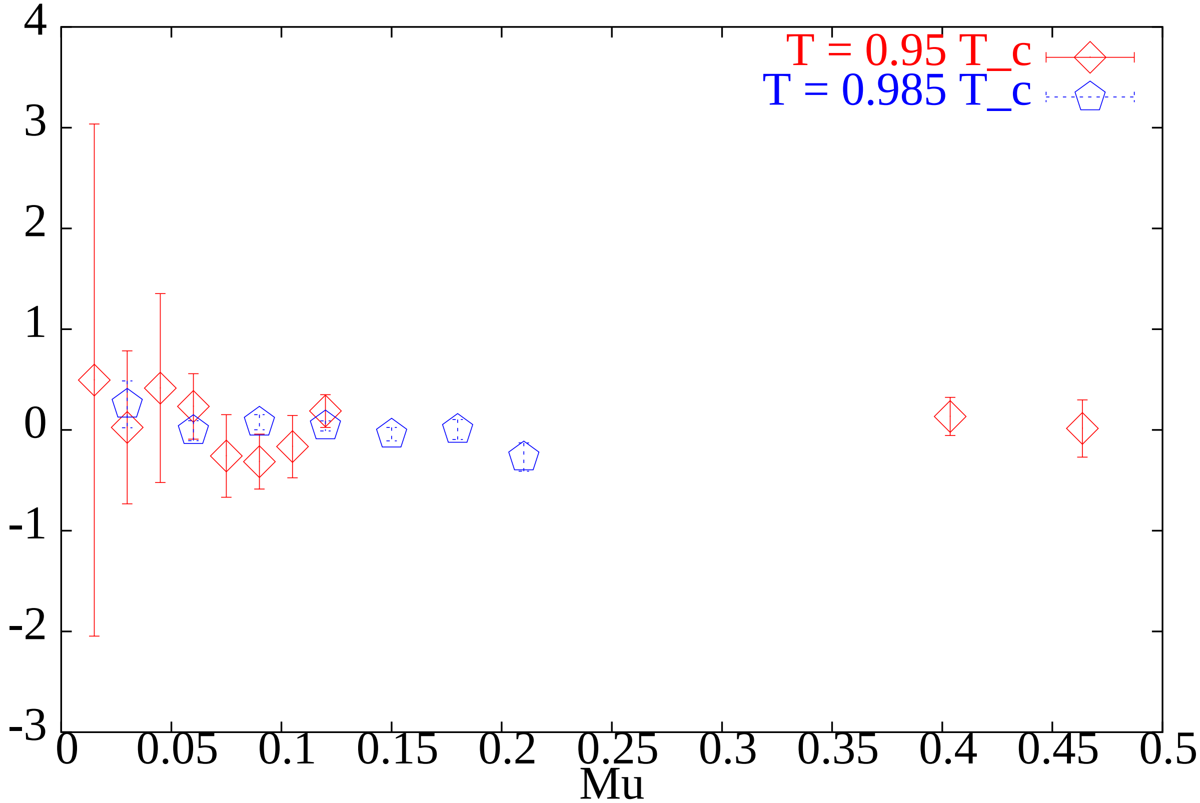
$$n(\mu) = \frac{\partial P(\mu)}{\partial \mu} = K sin(N_c N_t \mu)$$

HRG accurate up to $T \simeq .985T_c$: Fits

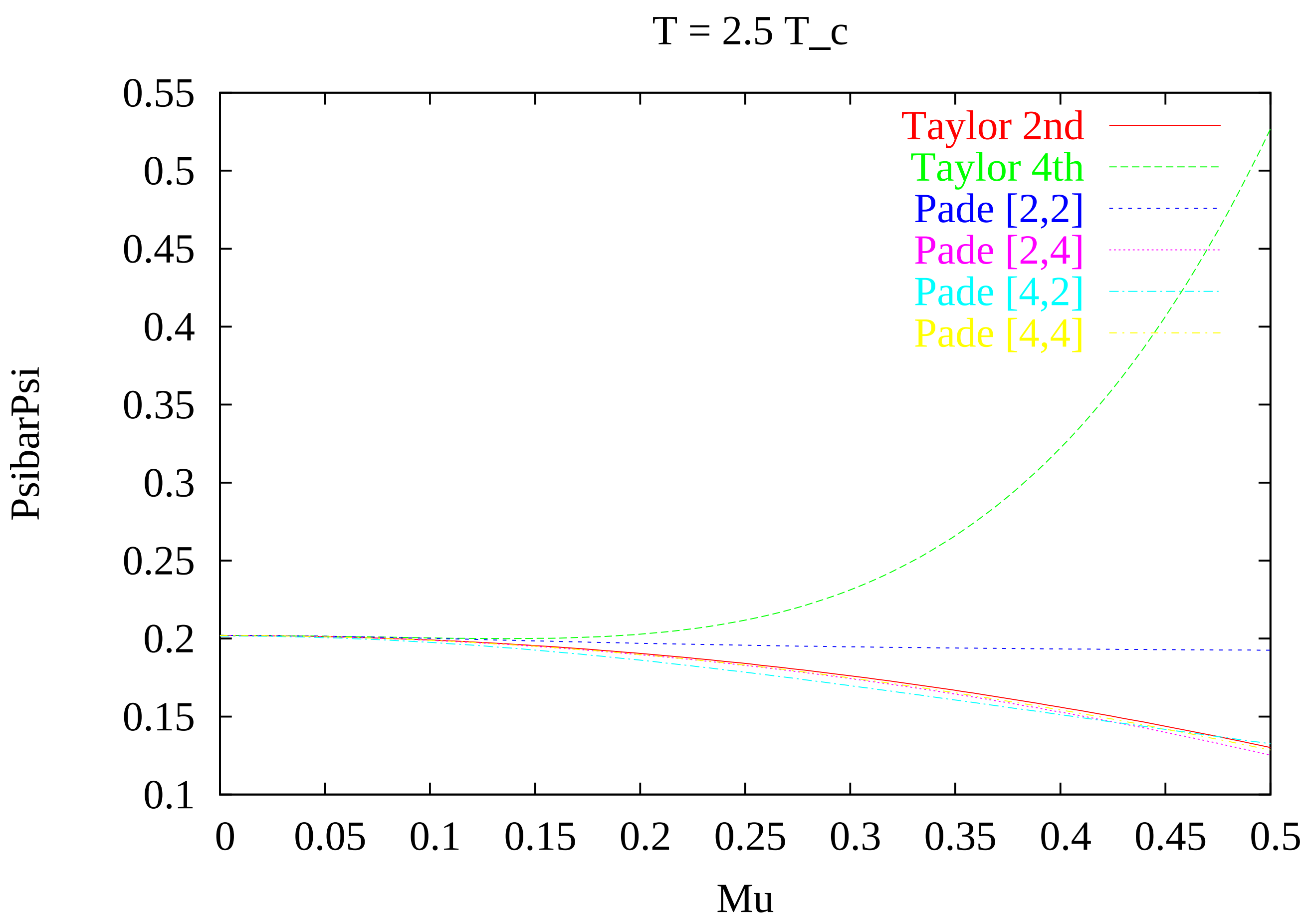


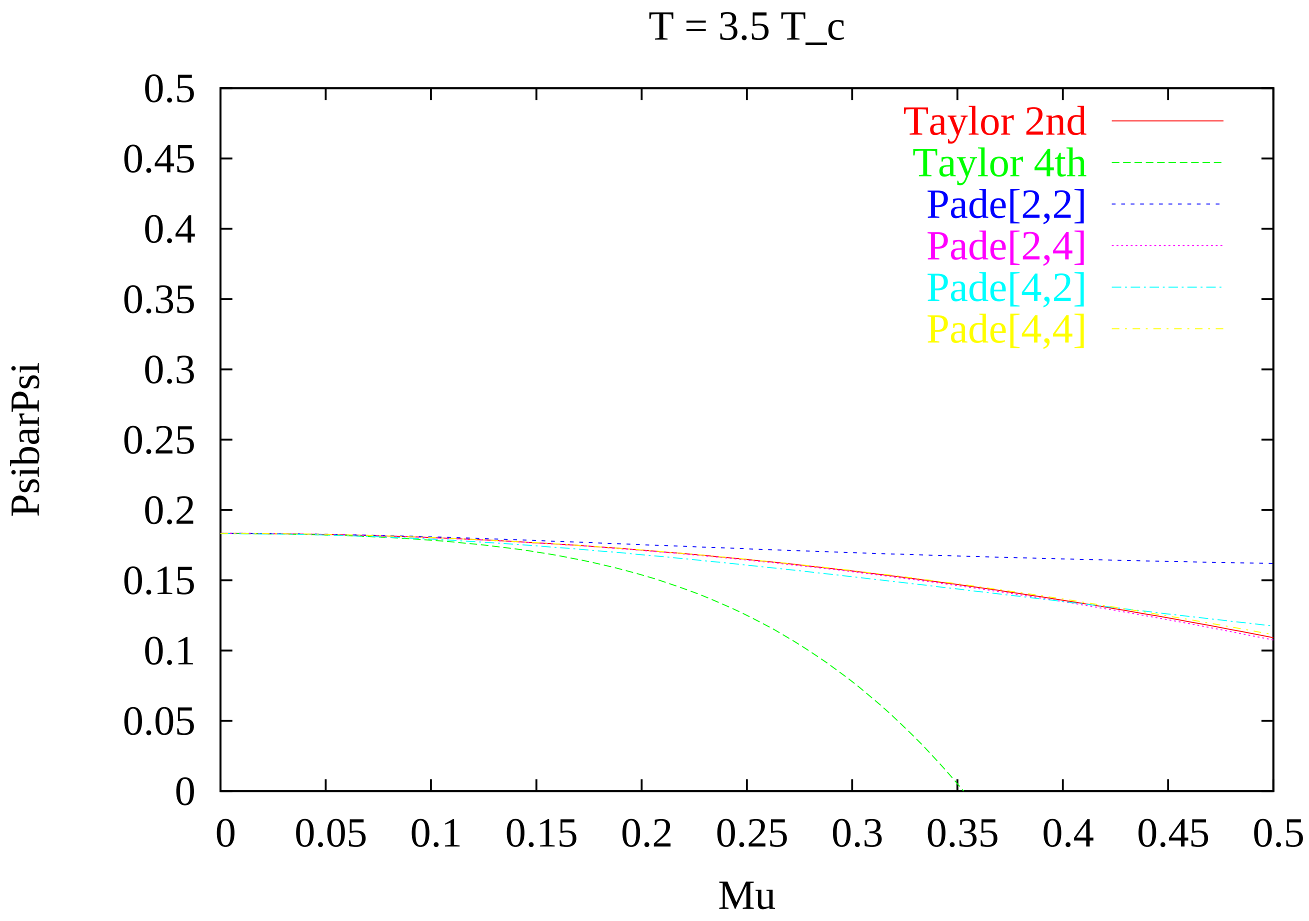
HRG accurate up to $T \simeq .985 T_c$: Direct check in an 'effective mass analysis' style:

$$Mismatch = n(\mu)/sin(N_c N_t \mu) - k$$



Thermodinamics at $\mu_B/T > 1$





1. **Again, Pade' produces stable results.**
2. **Taylor might do the same at T = 3.5 T_c.**

Radius of convergence and critical behaviour

(At least) Three different critical behaviours: second, first, weak first order:

$$f1(z) = A(z)(1 - z/z_c)^{-\lambda}$$

$$f2(z) = A(z)\theta(z - z_c)$$

$$f3(z) = A(z)\theta(z - z_c)(1 - z/z^*)^{-\lambda}$$

Correspondingly, different radius of convergence of the Taylor series:

$r_1 = |z_c|, r_2 = \infty, r_3 = z^*$

The radius of convergence of the Taylor expansion for the critical line might be infinite as well a finite, depending on the nature of the Roberge Weiss transition.

In either case, the analytic continuation can be accomplished via Pade' approximants.

Thermodynamics of the Hot Phase:

Monitoring the approach to a free gas of quarks and gluons
M. D'Elia and MpL, 2004

We can contrast results at imaginary chemical potential with analytic predictions by analytically continue from Real to Imaginary μ

Corrections to Free Field

A. Vuorinen 2004:

$$P(T,\mu) = \frac{\pi^2}{45}T^4\left(8 + 7N_c\frac{n_f}{4}\right) + \frac{n_f}{2}\mu^2T^2 + \frac{n_f}{4\pi^2}\mu^4 + ...$$

Alternatively (Rafelski, Letessier 2003)

$$P(T,\mu) = f(\mu)(\frac{\pi^2}{45}T^4\left(8 + 7N_c\frac{n_f}{4}\right) + \frac{n_f}{2}\mu^2T^2 + \frac{n_f}{4\pi^2}\mu^4)$$

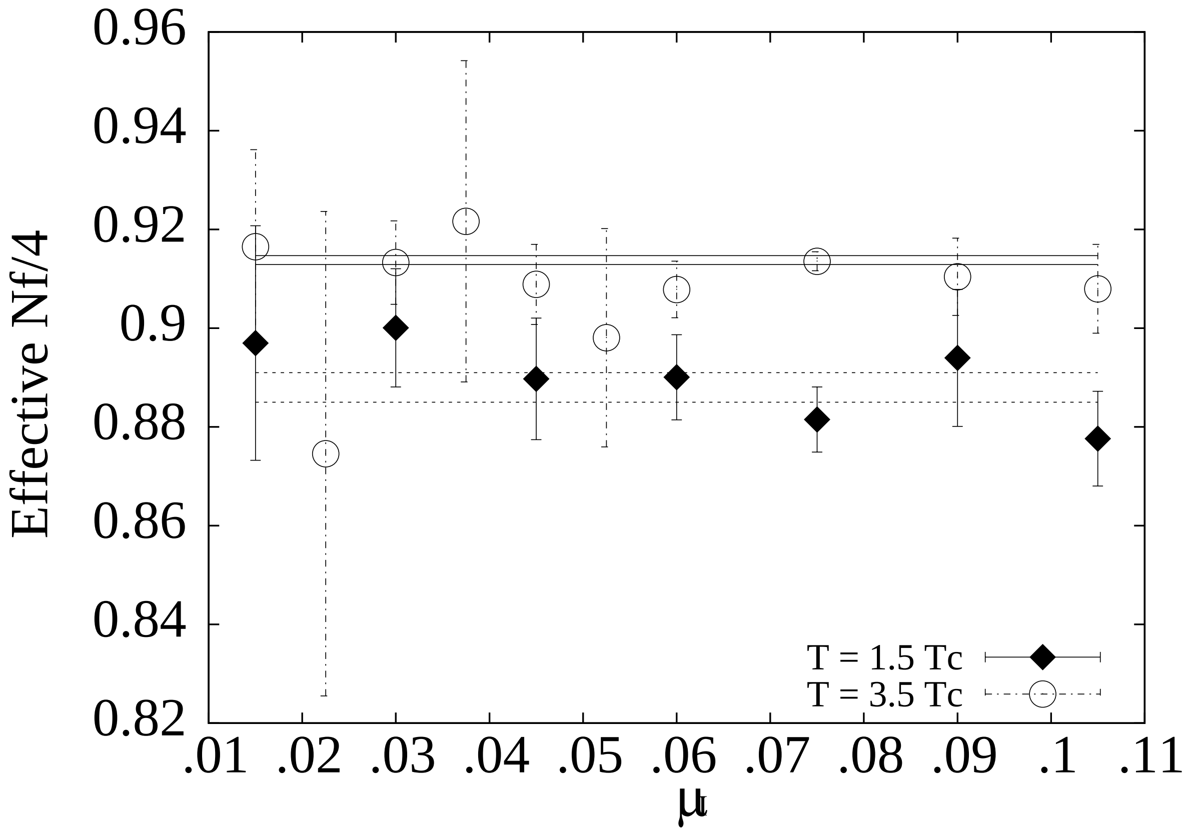
Trivial possibility: $f(\mu)$ is a constant .

Possible interpretation: a free field with an effective number of flavors N_{eff} different from N_f

$$P(T,\mu) - P(T,0) = (N_{eff}/n_f)(\frac{n_f}{2}\mu^2T^2 + \frac{n_f}{4\pi^2}\mu^4) = (\frac{N_{eff}}{2}\mu^2T^2 + \frac{N_{eff}}{4\pi^2}\mu^4)$$

Nearly Free Field for $T > 1.5T_c$

$f(\mu) = (N_{eff}/N_f)$ **estimated on the lattice appear to be a constant for** $T \geq 1.5T_c \neq 1$ **i.e.** $N_{eff} \neq N_f$

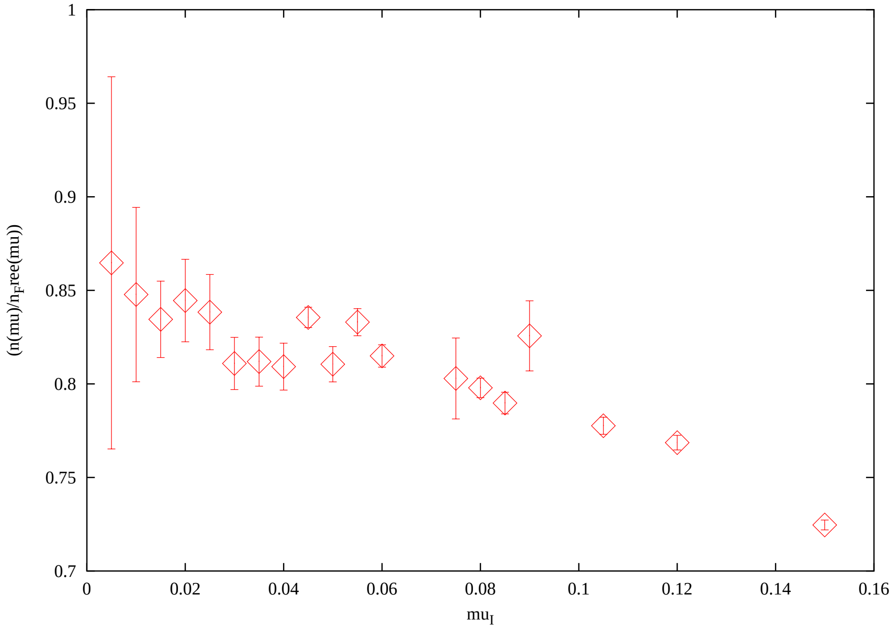


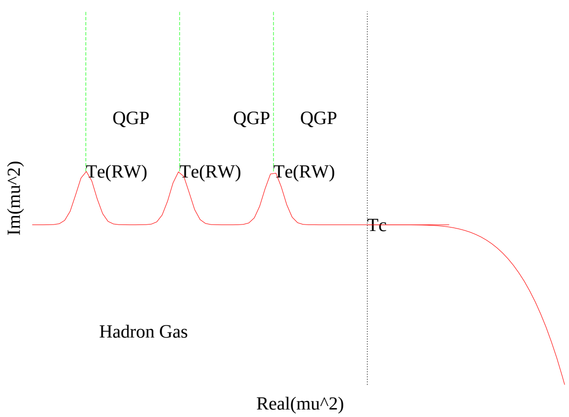
RW Transition Strong First Order ?

$T_c < T < 1.5T_c$: a Strongly Coupled Quark Gluon Plasma?

Influence of the chiral transition at imaginary μ ?

Francesco Di Renzo, Massimo D'Elia, MpL, hep-lat/0511029, and in progress





Summary

Wilson Fermions Thermodynamics towards to continuum limit: $\mu = 0$ ***simulations are in progress with*** $N_t = 8$***, Symanzik improvement for the gauge fields.***

Chiral Symmetry, Confinement, Topology remain correlated at non–zero baryon density.

Results for imaginary μ ***can be analytically continued beyond*** $\mu/T \simeq 1$.
.The curvature of the critical line increases at lower temperature.