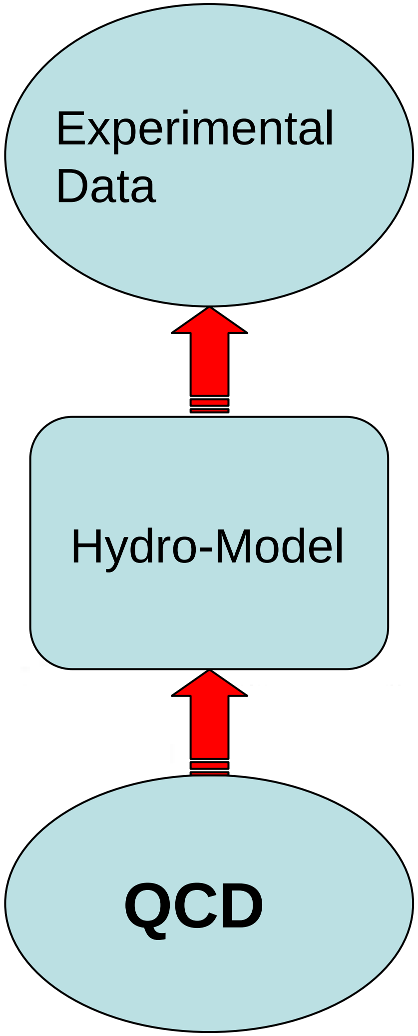
Motivation

Long time ago, when I was young, I was studying in a Lab as a graduate student.

My Supervisor, Prof. Namiki, had studied Landau Hydro-dynamical Model from Field Theory point of view.

It was the only place at that time in Japan, where the hydro was daily discussed.
From the Lab came Muroya, Nonaka, Hirano, Morita … who now actively study the hydro-dynamical model.

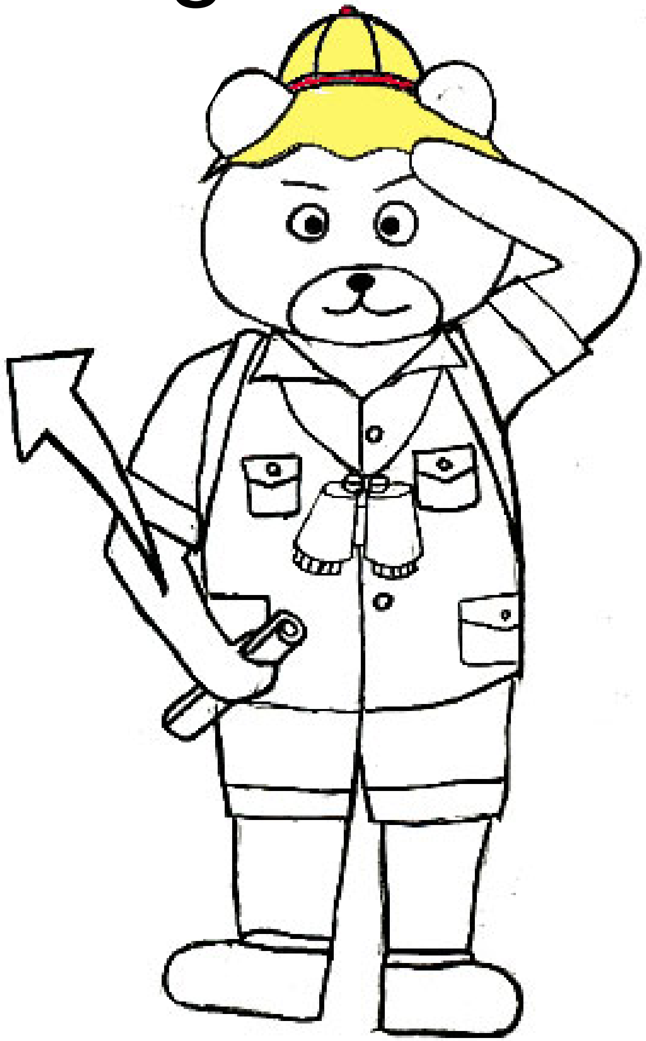
Yes, I will also study the hydro for supporting young friends.



Experimental Data

Hydro-Model

QCD



Viscosities of Strongly Coupled Gluons

scRHIC Physics
Mumbai
Dec.5-9, 2005

Sunao Sakai, Fac.Education, Yamagata Univ.

Atsushi Nakamura, RIISE, Hiroshima Univ.

Contents

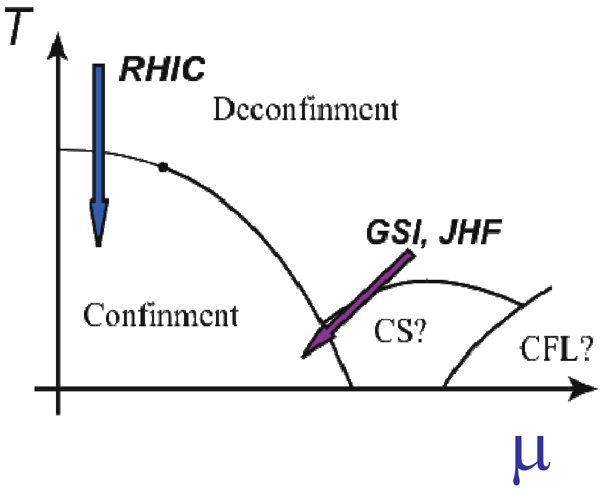
- Introduction
  - New State of Matter at RHIC
- Formulation
- Results
- Summary
- Preparing for the Next: Status Report

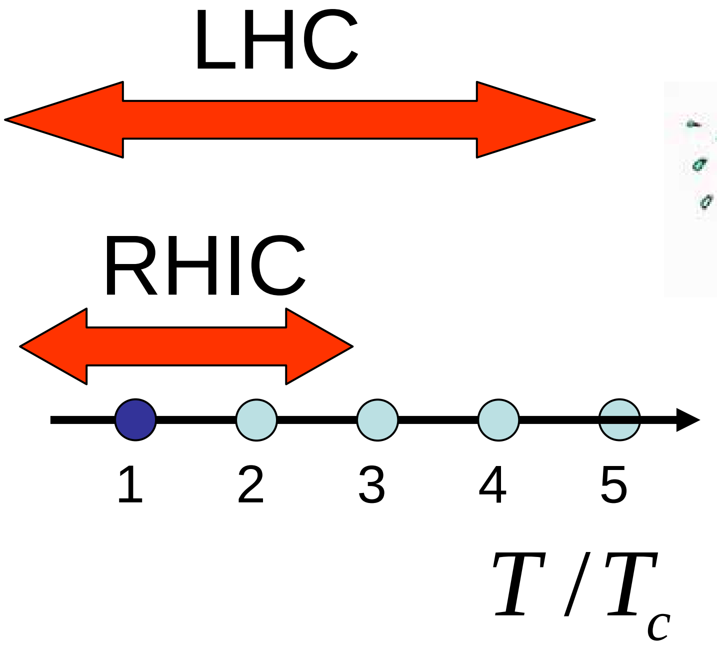
Transport Coefficients

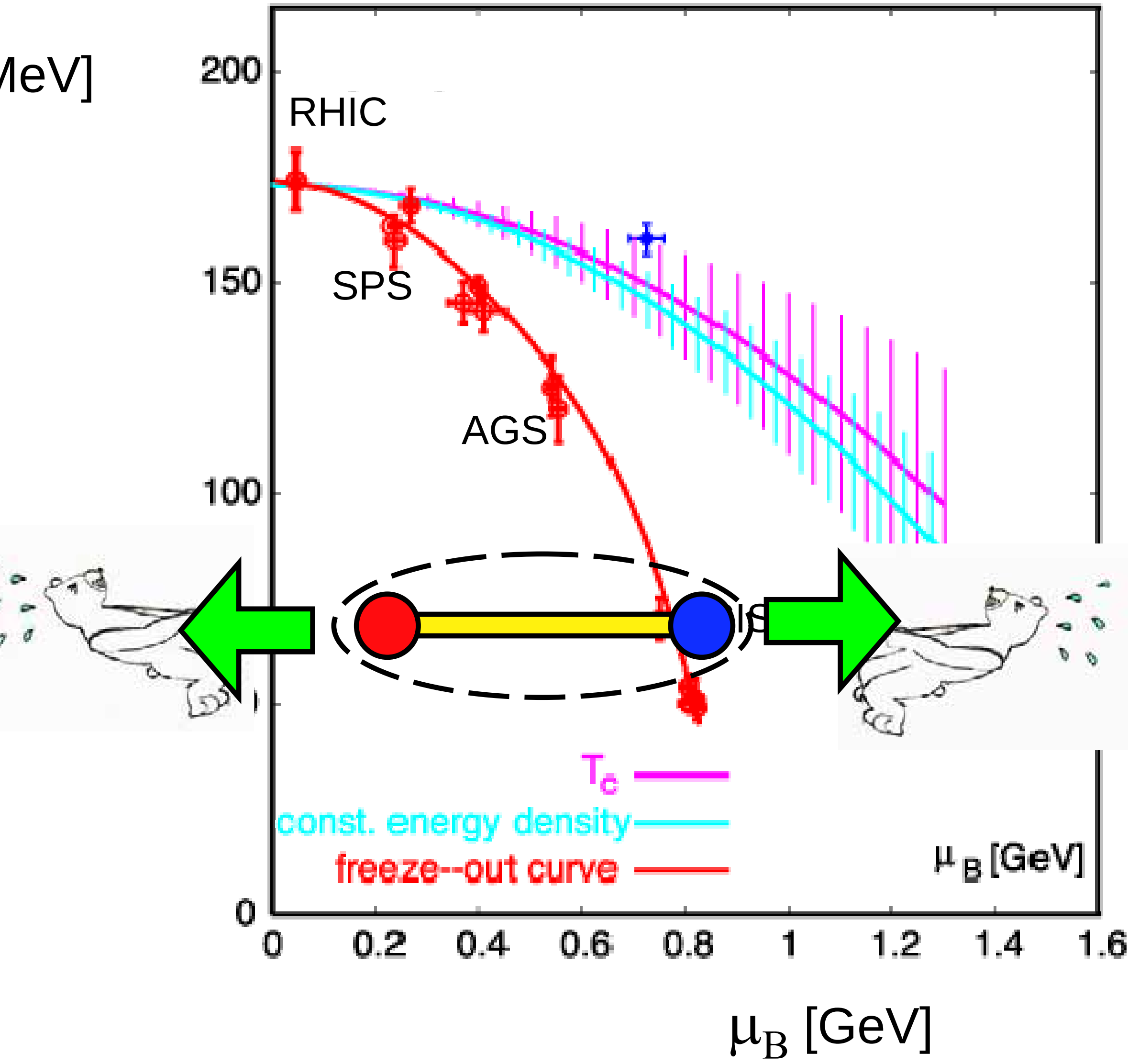
- Important for the Entropy Production
- A Step towards Study of Gluon’s Dynamical Behavior – Parameters for non-equilibrium motion.
- They are (in principle) calculable by Lattice
  - using Kubo Formula
- They are important for understanding “a New State of Matter” which is realized in RHIC and LHC.

A Comparison with Lattice Results

P. Braun-Munzinger, K. Redlich and J. Stachel







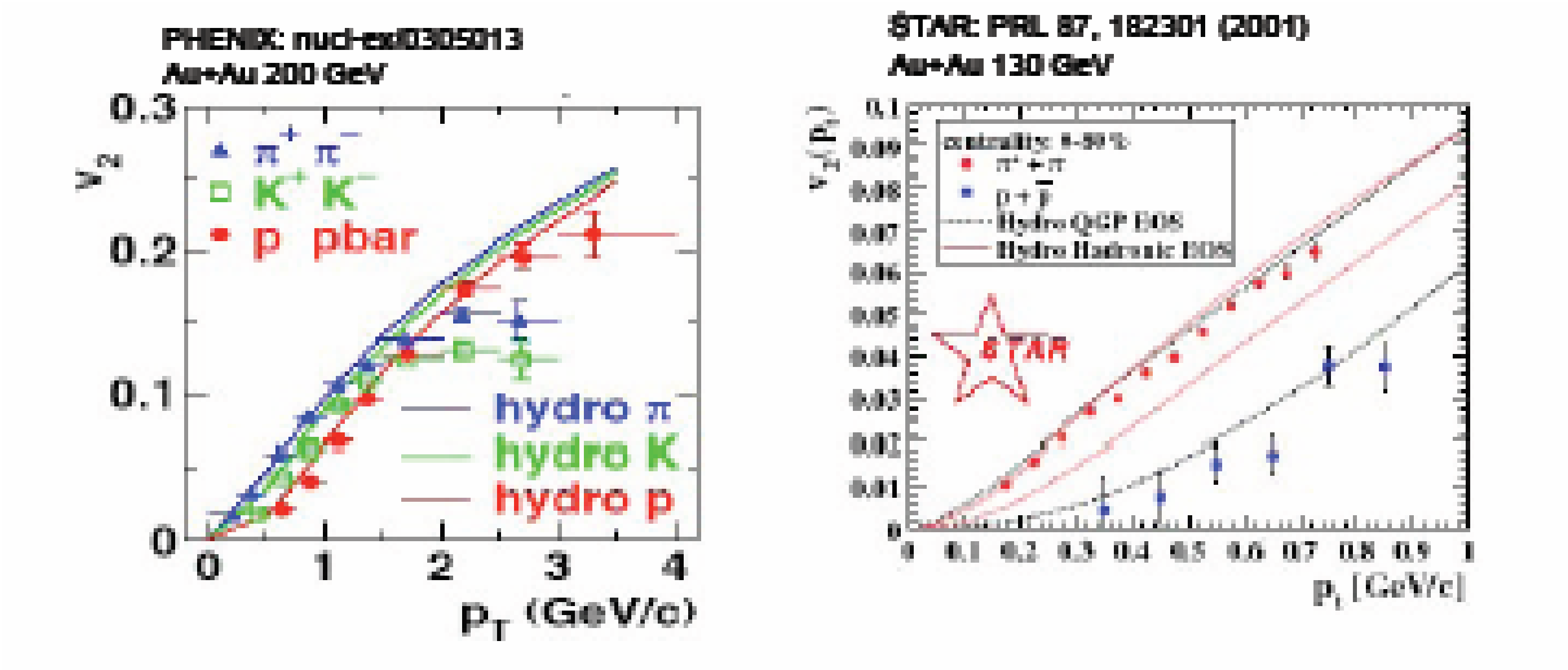
RHIC-data ⇒ Big Surprise !

Hydro-dynamical Model describes RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2


PHENIX: nucl-ex/0305013
Au+Au 200 GeV

STAR: PRL 87, 182301 (2001)
Au+Au 130 GeV

Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

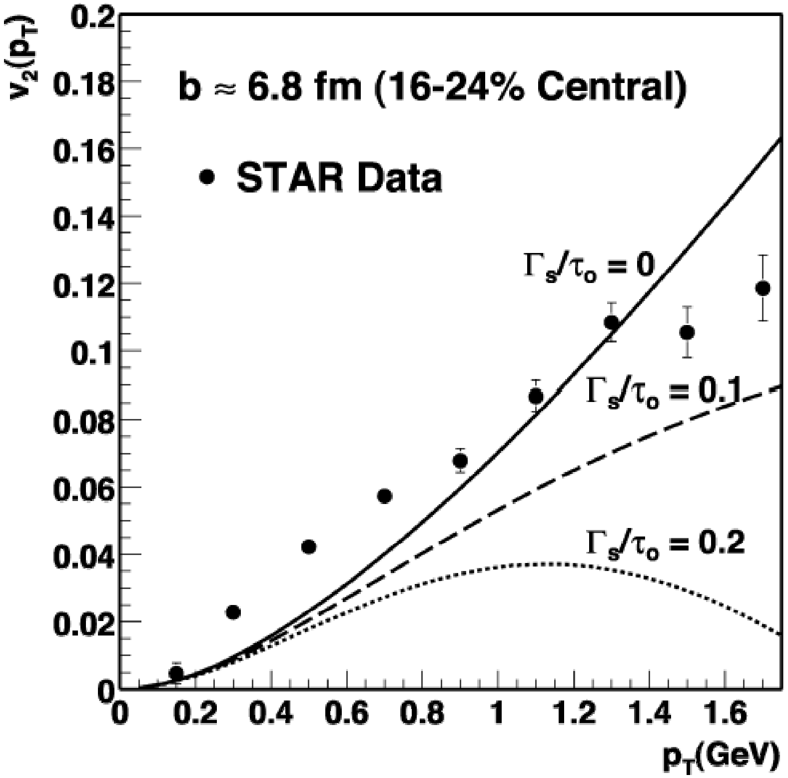
Or not so surprise …

- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen'skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model
    "Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number."

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Teaney, nucl-th/0301099

$$\Gamma_s \equiv \frac{\frac{4}{3}\eta}{sT}$$



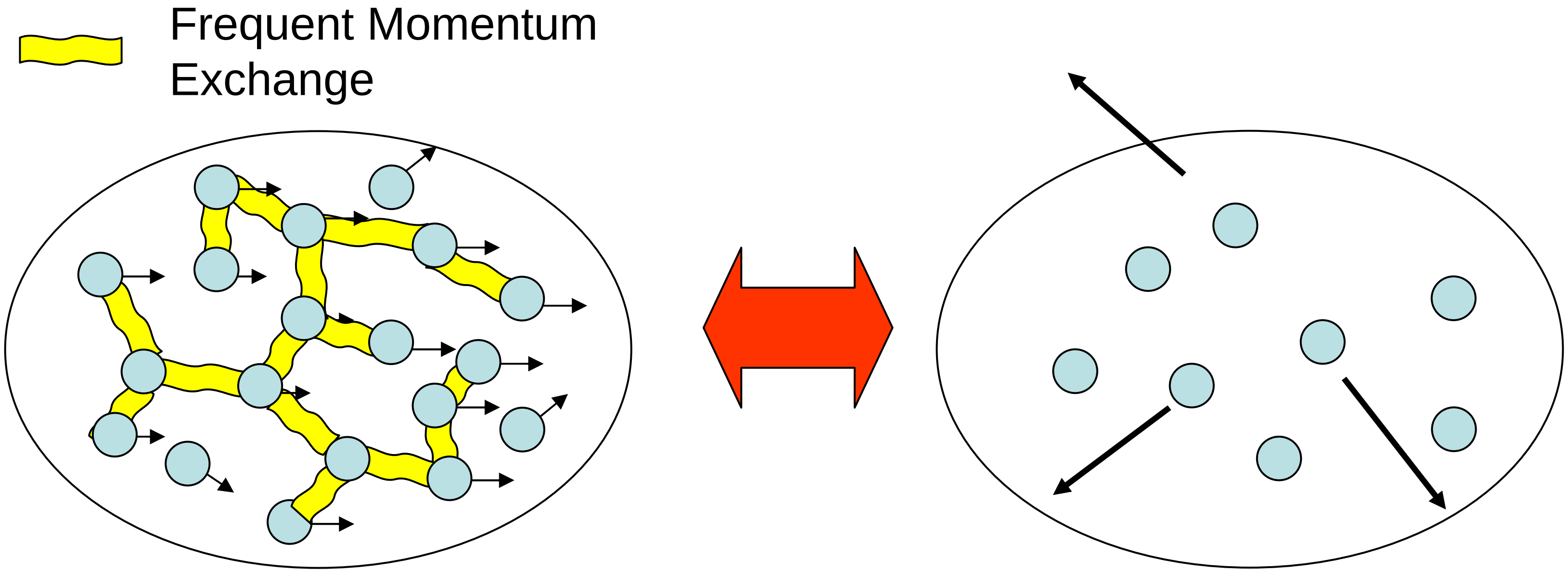
$\eta :$ shear viscosity

$\tau = \sqrt{t^2 - z^2}$: Time scale of the expansion

Another Big Surprise !

- The Hydrodynamical model assumes zero viscosity,
  i.e.,, Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?



Frequent Momentum Exchange

Opposite Situation

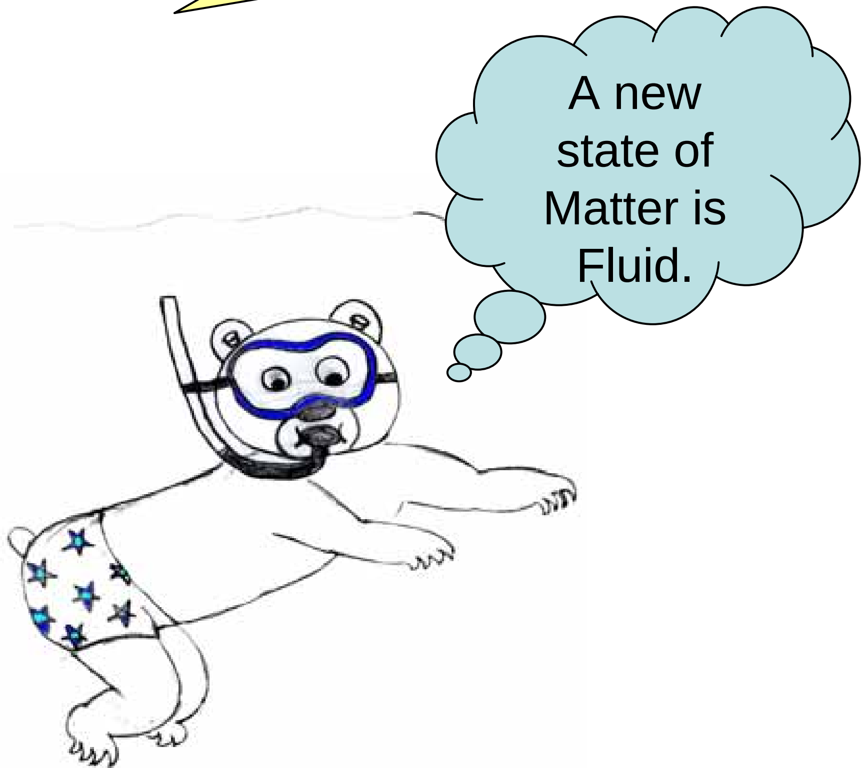
Perfect fluid

Ideal Gas

Literature (1)

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - The first paper to analyze the Hydrodyanamical Model from Field Theory.
  - Applicability Conditions were derived:
    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small

If produced matter at RHIC is (perfect) Fluid, not Free Gas what does it mean ?



Lowest Perturbation
(Illustration purpose only)

Pressure

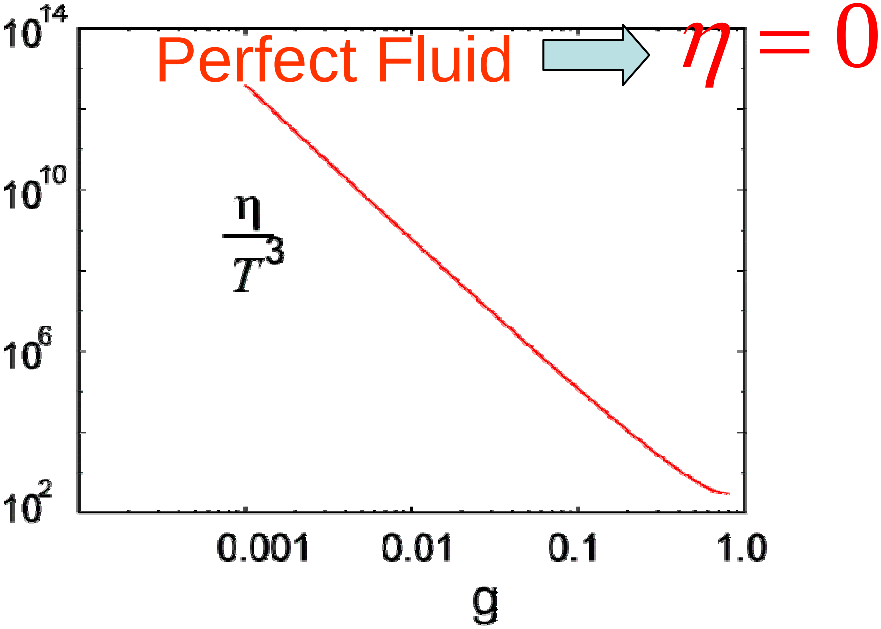
$$P = \underbrace{\frac{\pi^2}{90}T^4}_{\text{Ideal Free Gas}}\left(1 - \frac{15}{8}\left(\frac{g}{\pi}\right)^2 + \cdots\right)$$

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126(N_f = 0),$$
$$86.473(N_f = 2)$$

- At weak coupling, it increases.



Literature (2)

- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
  - Transport Coefficients Formulation
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
  - The first Lattice QCD Calculation
- Aarts and Martinez-Resco, JHEP0204 (2002)053
  - Criticism against the Spectrum Function Ansatz.
- Petreczky and Teaney, hep-ph/0507318
  - Impossible to determine Heavy Quark Transport coefficient

Literature (3)

- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N, Sakai
  hep-lat/0406009

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Kubo’s Linear Response Theory

- Zubarev
  “Non-Equilibrium Statistical Thermo-dynamics”
- Kubo, Toda and Saito
  “Statistical Mechanics”

$\rho \sim e^{-A+B}$: non-equilibrium statistical operator

$$A = \int d^3x \beta(x,t) u^\nu T_{0\nu}(x,t)$$

$$B = \int d^3x \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} T_{\mu\nu}(x,t) \partial^\mu (\beta(x,t) u^\nu)$$

Using: $e^{-A+B} = e^{-A} + \int_0^1 d\tau e^{A\tau} B e^{-A\tau} e^{-A} + \cdots$

$$\rho \approx \rho_{eq} + \int_0^1 d\tau (e^{A\tau} B e^{-A\tau} e^{-A} - \left\langle B \right\rangle_{eq}) \rho_{eq}$$

$$\rho_{eq} \equiv e^{-A} / \mathrm{Tr} e^{-A} \to \exp(-\beta H) / \mathrm{Tr} e^{-A}$$

in the co-moving frame, $u^\mu = (1 \quad 0 \quad 0 \quad 0)$

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{eq} +$$

$$+\int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} \partial^{\rho}(\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \langle T_{\rho\sigma}(x',t') \rangle_{eq} \right) \right\rangle_{eq}$$

$$\langle T^{ij} \rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\langle T^{0i} \rangle = -\chi (\beta^{-1}(x,t) \partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\langle p \rangle - \langle p \rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^{i}{}_{i}$$

- One can show

$$(T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'))_{eq} = -\beta^{-1}\int_{-\infty}^{t'} dt''\left\langle T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'')\right\rangle_{ret}$$

Transport Coefficients are expressed by Quantities at Equilibrium

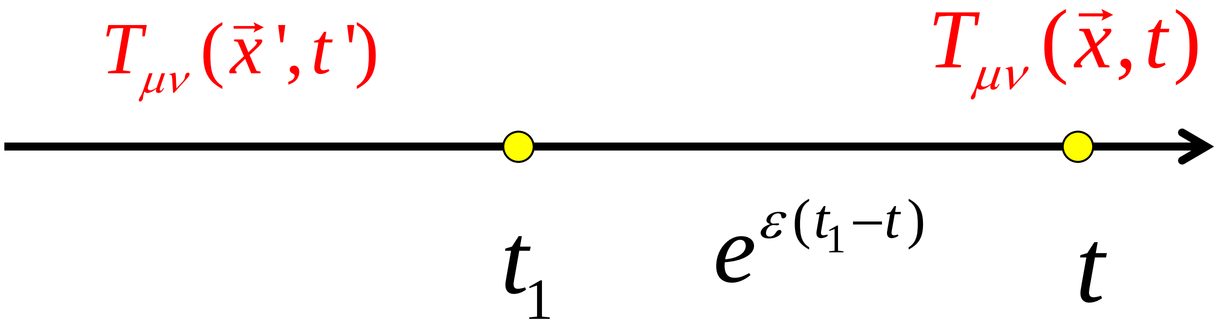
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x}',t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x},t) T_{11}(\vec{x}',t') >$$

$$\chi = -\frac{1}{T}\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x},t) T_{01}(\vec{x}',t') >_{ret}$$

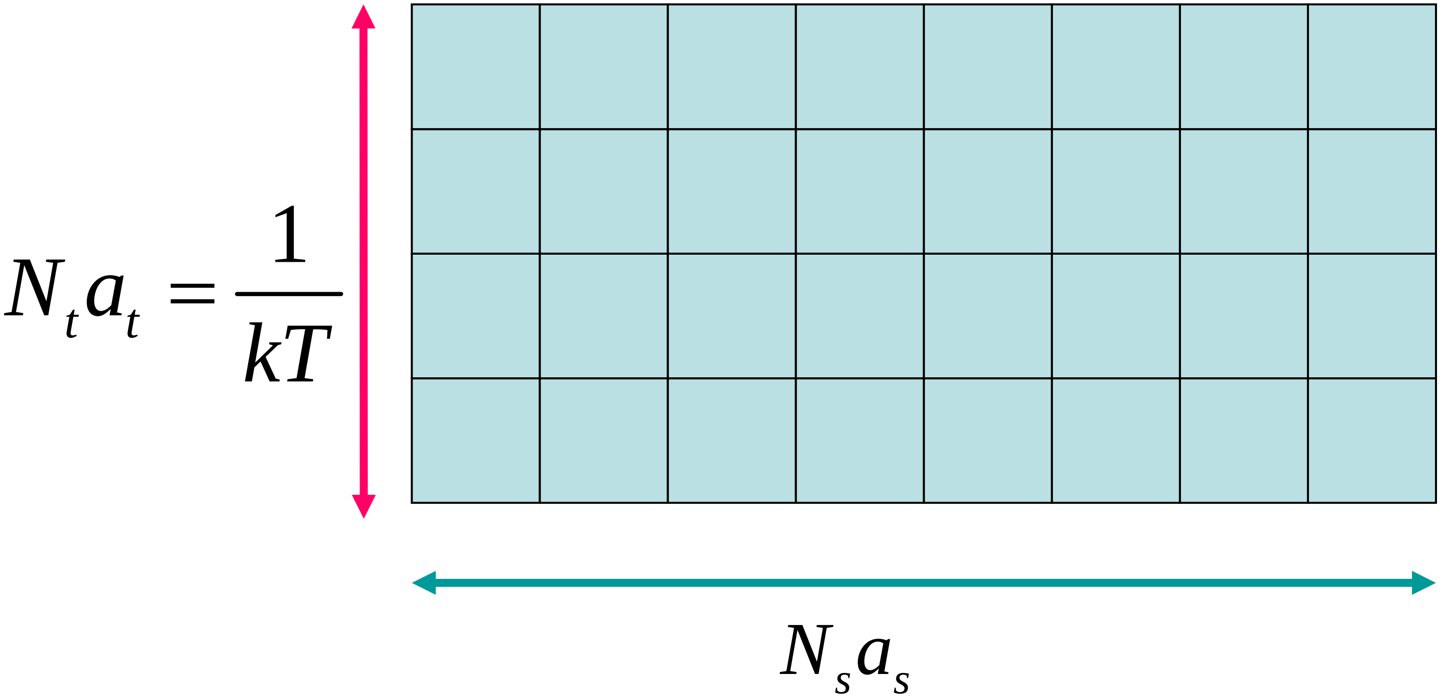
η **: Shear Viscosity** ς **: Bulk Viscosity**

χ **: Heat Conductivity** ⇒ we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Some Special Features of Lattice QCD at Finite Temperature and Density



High Temperature ⇨ $N_t a_t$: small

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right) / 2ia^2 g$$

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu,
Nucl.Phys.B400(1993)267

$$<<\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]>>\equiv\frac{1}{Z}\mathrm{Tr}(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H})$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x})=e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_{\beta}^{ret/adv}(t,\vec{x};t',\vec{x}')=\pm\theta(t-t'/t'-t)<<\ldots>>$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_{\beta}^{ret/adv}(\omega,\vec{p})$$

$$K_{\beta}^{ret/adv}(\omega,\vec{p})=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') =<< T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') >>$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

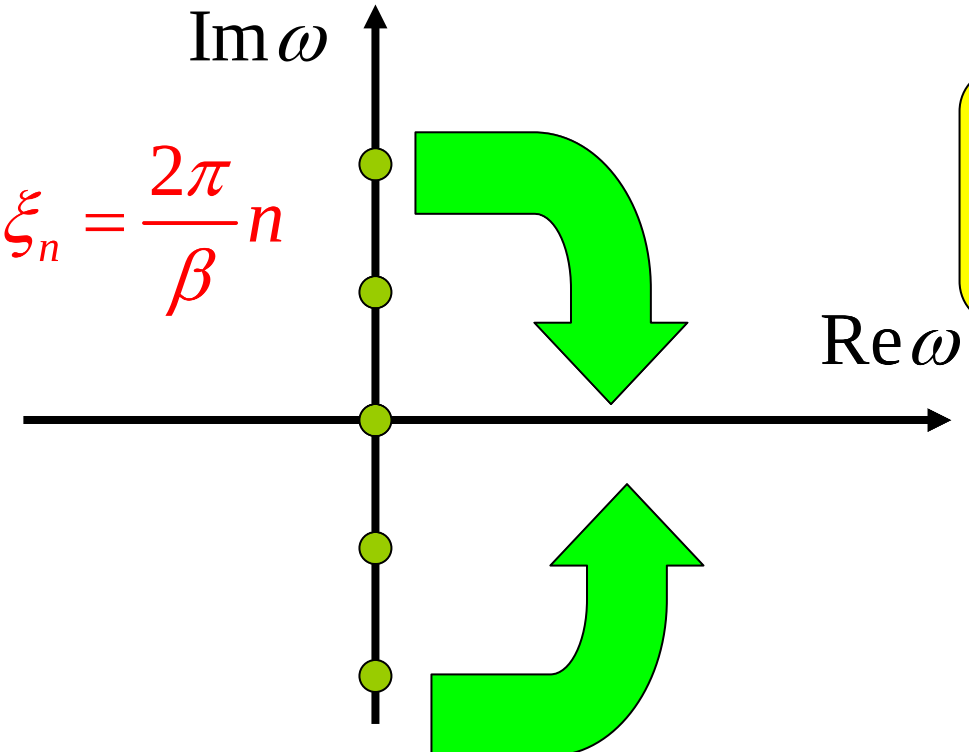
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0,\pm 1,\pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_{\beta}(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_{\beta}(i\xi_n)$$



$$\xi_n = \frac{2\pi}{\beta} n$$

On the lattice, we measure Temperature Green function at

$$\omega = \xi_n$$

We must reconstruct Advance or Retarded Green function.

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$$< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,\vec{x})T_{\mu\upsilon}(0) >= G_{\beta}(t,\vec{x}) = F.T.G_{\beta}(\omega_n,\vec{p})$$

$$G_{\beta}(\vec{p},i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

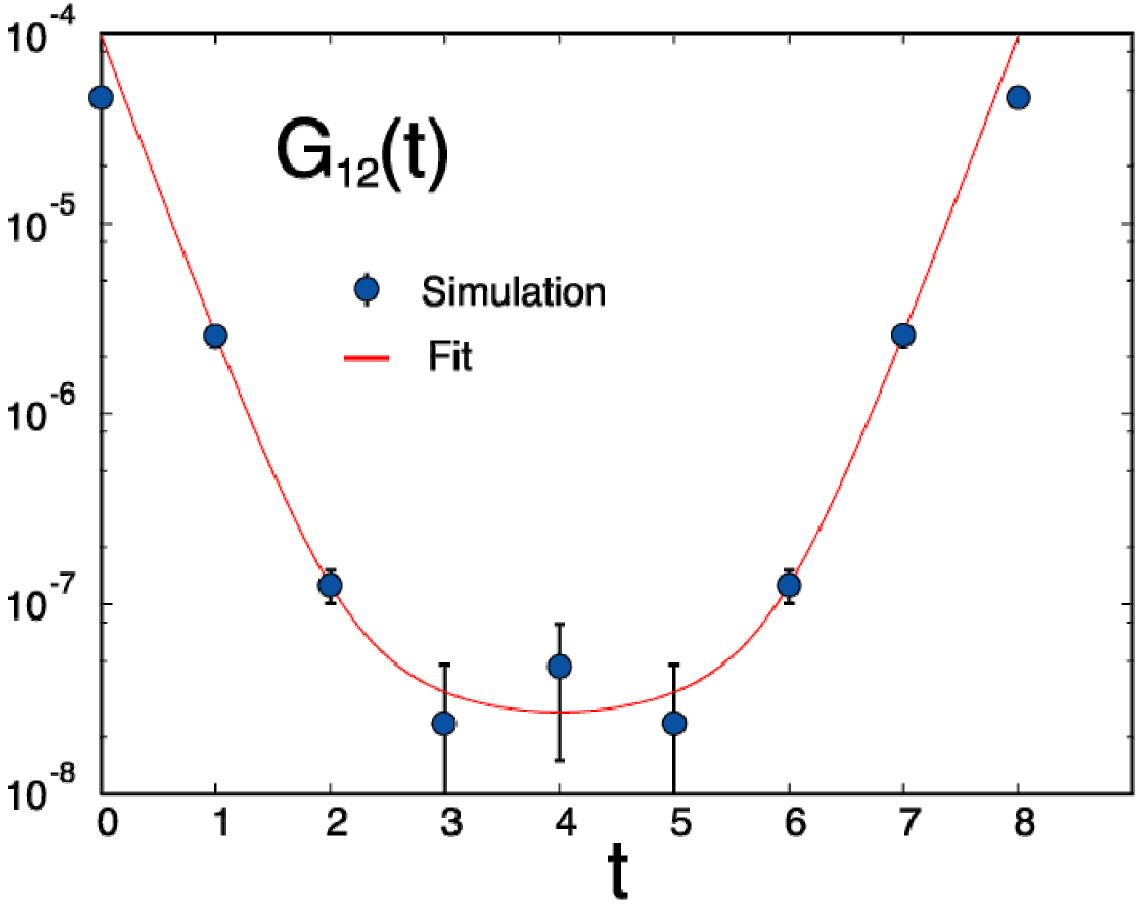
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

and determine three parameters,

A, m, γ.

We need large Nt !

Nt=8



Lattice and Statistics

Iwasaki Improved Action

$$16^3 \times 8$$

$\beta=3.05$: 1333900 sweeps

$\beta=3.20$: 1212400 sweeps

$\beta=3.30$: 1265500 sweeps

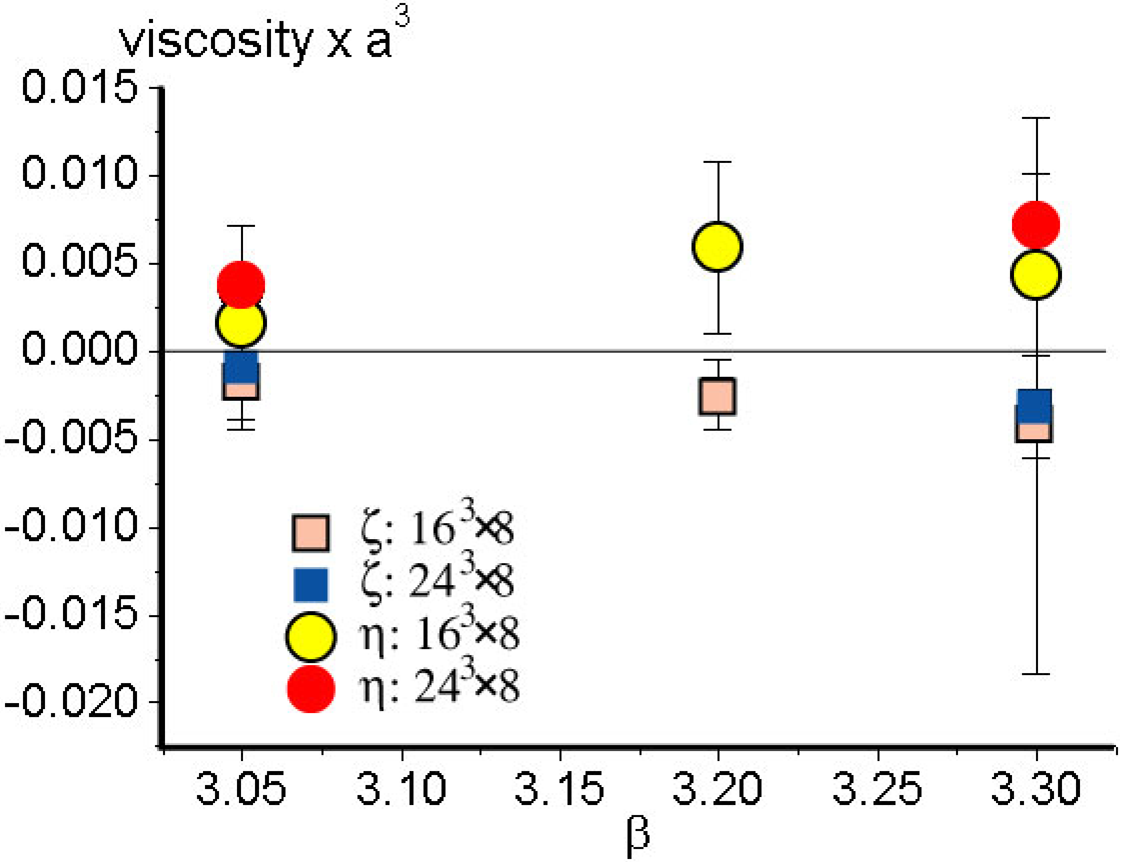
$$24^3 \times 8$$

$\beta=3.05$: 61000 sweeps

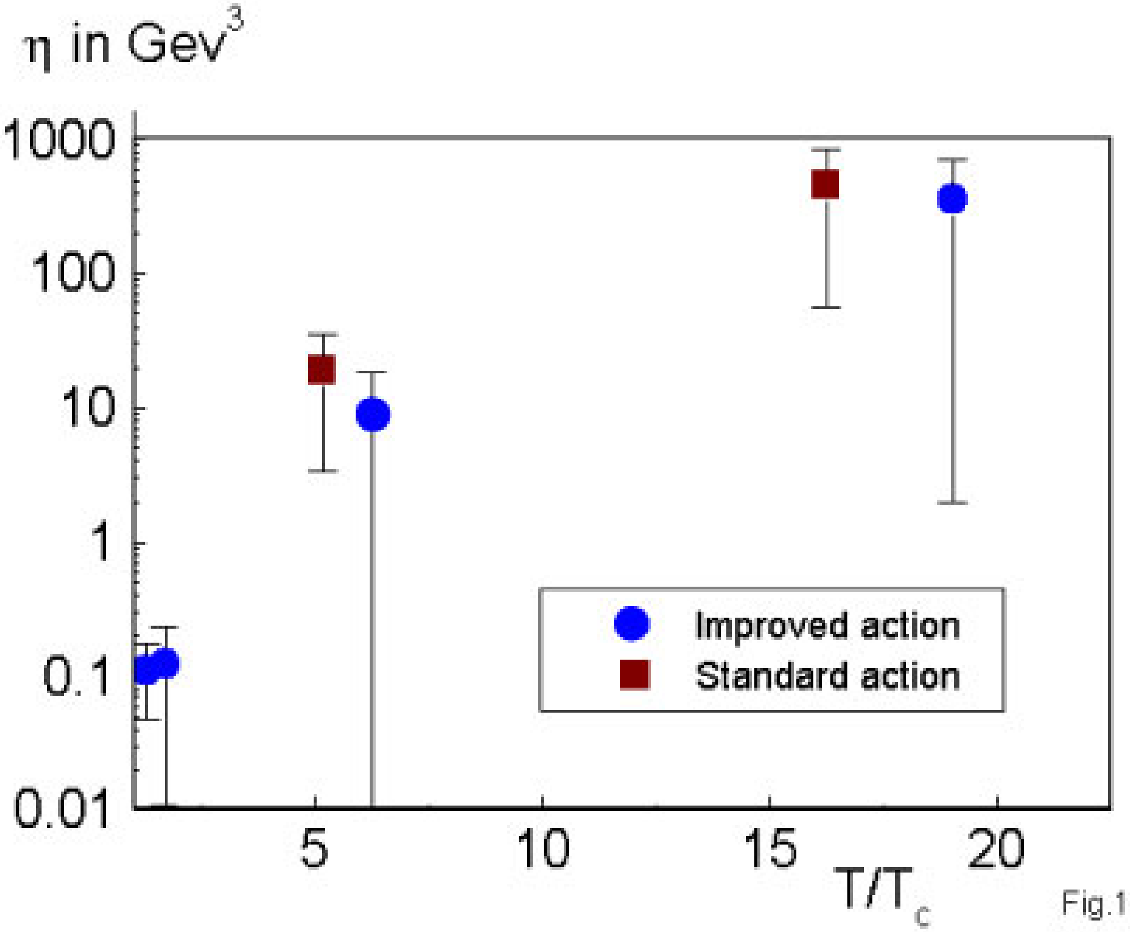
$\beta=3.30$: 84000 sweeps

Quench

Results: Shear and Bulk Viscosities

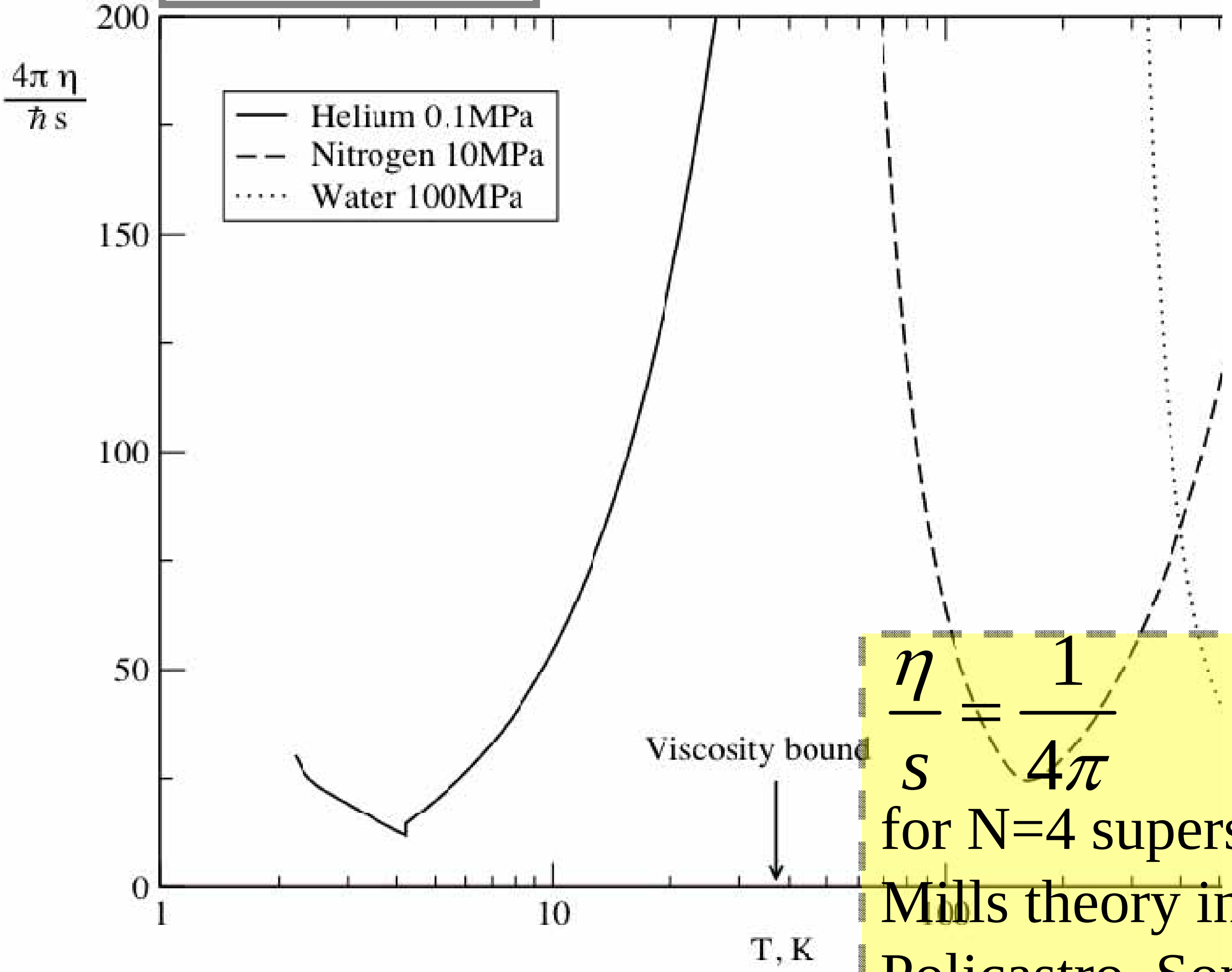


Very high Temperature



$$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}\ !}$$

Kovtun, Son and Starinets, hep-th/0405231



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for N=4 supersymmetric Yang-Mills theory in the large N.

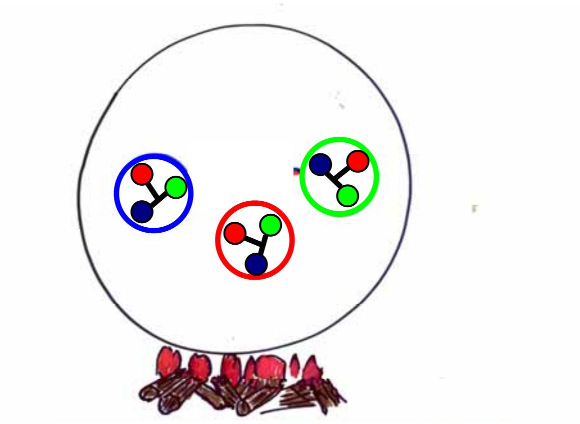
Policastro, Son and Starinets, Phys

Rev. Lett. 87 (2001) 081601

$\frac{\eta}{s}$ can have the lower limit ?

- Counter Example by Prof. Baym
  - We heat up Billiard Balls which have inter-structure. Then Entropy increases. If the surface of the balls does not change, the Viscosity should be the same.

$$\Rightarrow \qquad \frac{\eta}{s} \to 0$$



- We may give Counter-Argument

Entropy Density

$$F = fV$$

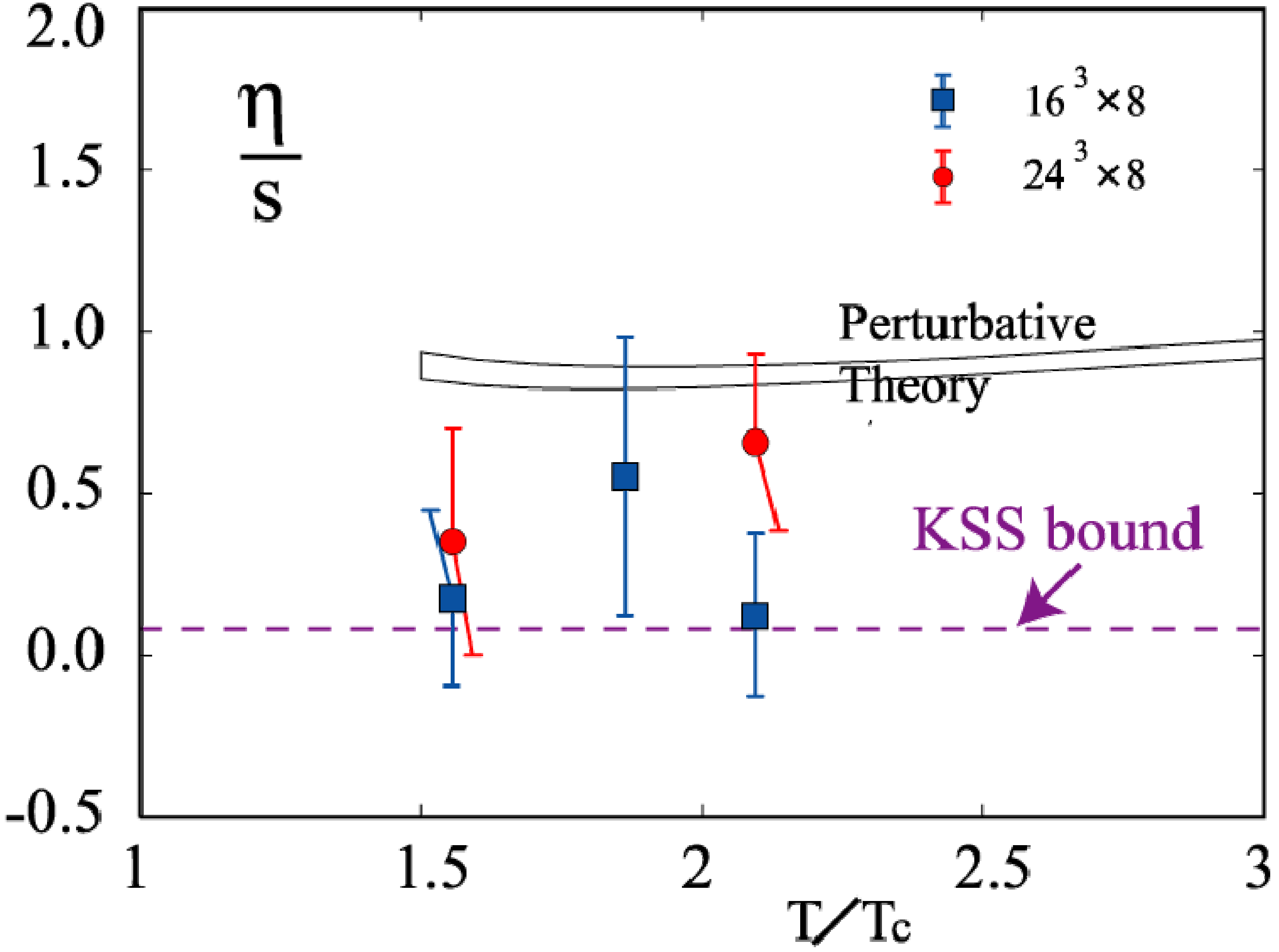
$$f = -p$$

$$U - TS = -T \log Z = F$$

$$\Rightarrow \quad s = \frac{S}{V} = \frac{\varepsilon + p}{T}$$

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \frac{d}{d\beta'} \frac{p}{T^4}$$

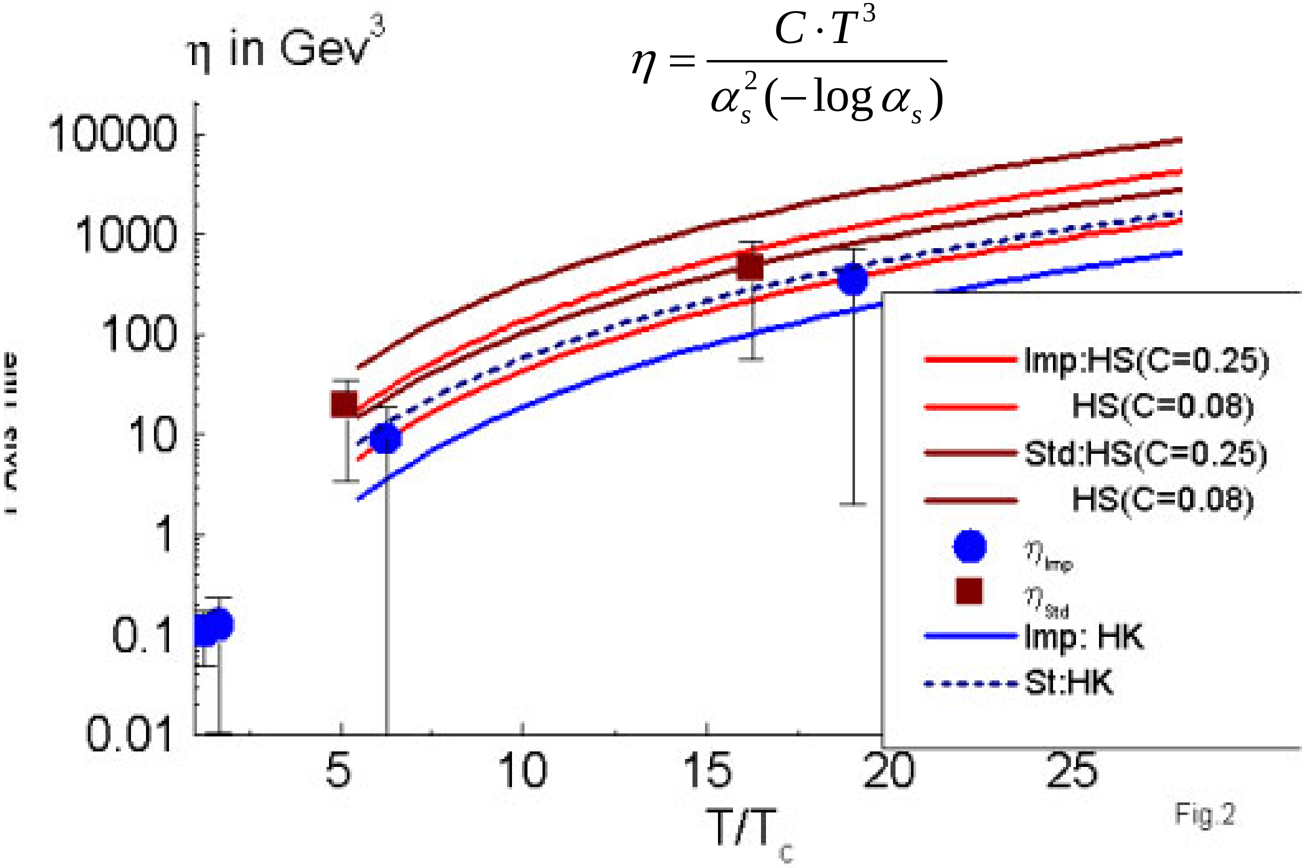
We reconstruct *p* from Raw-Data by CP-PACS
(Okamoto et al., Phys.Rev.D (1999) 094510)



$\frac{\eta}{s}$
2.0
1.5
1.0
0.5
0.0
-0.5
1
1.5
2
2.5
3
T/Tc
$16^3 \times 8$
$24^3 \times 8$
Perturbative Theory
KSS bound

Comparison with Pertubative Calculations

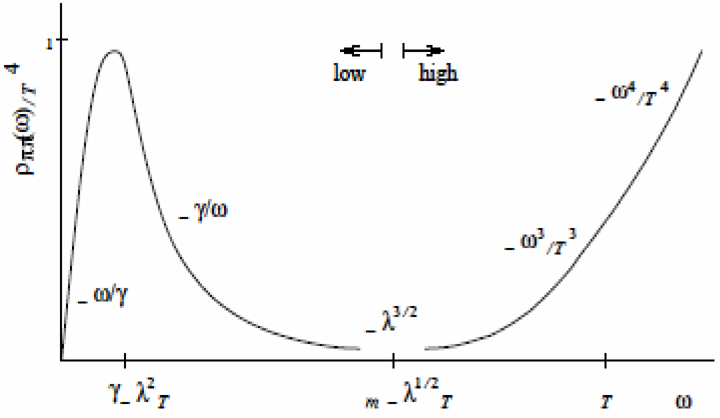
$$\eta = \frac{C \cdot T^3}{\alpha_s^2(-\log \alpha_s)}$$



Good for T/Tc>5

Spectral Function by Aarts and Resco

$$\rho(\omega) = \rho^{\mathrm{low}}(\omega) + \rho^{high}(\omega)$$



$$\frac{\rho^{\mathrm{low}}(\omega)}{T^4} = x \frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots}$$

$$x \equiv \frac{\omega}{T}$$

$$\rho^{high}(\omega) = \theta(\omega - 2m_{th}) \frac{(N_c{}^2 - 1)(\omega^2 - 4m_{th}{}^2)^{5/2}}{80\pi^2\omega}[n(\omega) + 0.5]$$

Fitting with three parameters, $b_1 \ c_1 \ m$

$c_1 < 0$?

Effect of High-Frequency part

$$\rho = \rho^{BW} + \rho^{high}$$

$$\frac{\rho^{\mathrm{low}}(\omega)}{T^4} = x\frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots} \qquad x \equiv \frac{\omega}{T}$$

$$\rho^{BW} = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

β=3.3

ηa^3	m_{th}
0.00225(201)	∞
0.00223(191)	5.0
0.00194(194)	3.0
0.00126(204)	2.0

$$m_{th} = 1.8$$

ρ^{high} contribution is larger than ρ^{BW} at t=1.

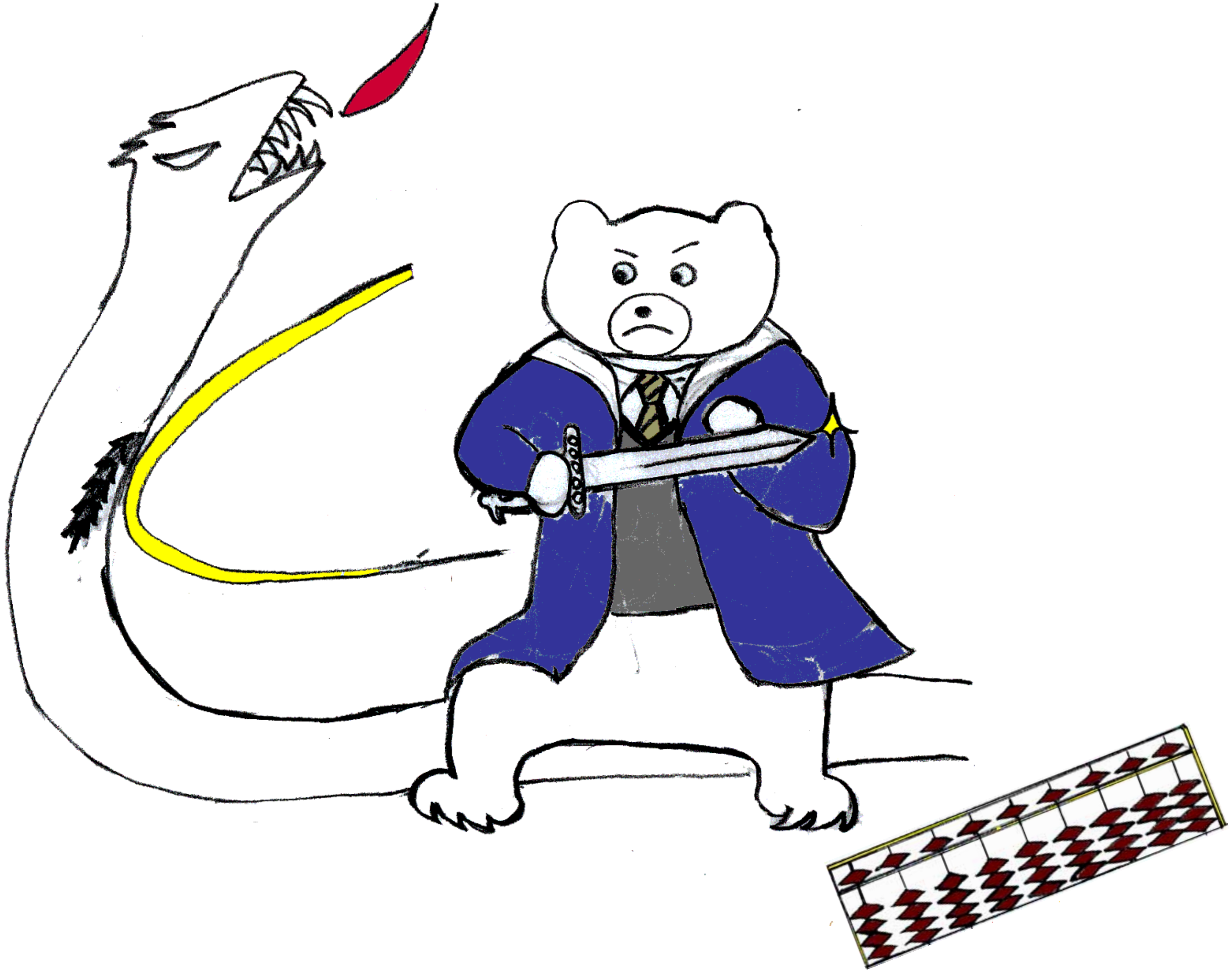
Summary

- We have calculated Transport Coefficients on Nt=8 Lattice:
  - Quench Approximation
  - We can fit three parameters in the Spectral Function:

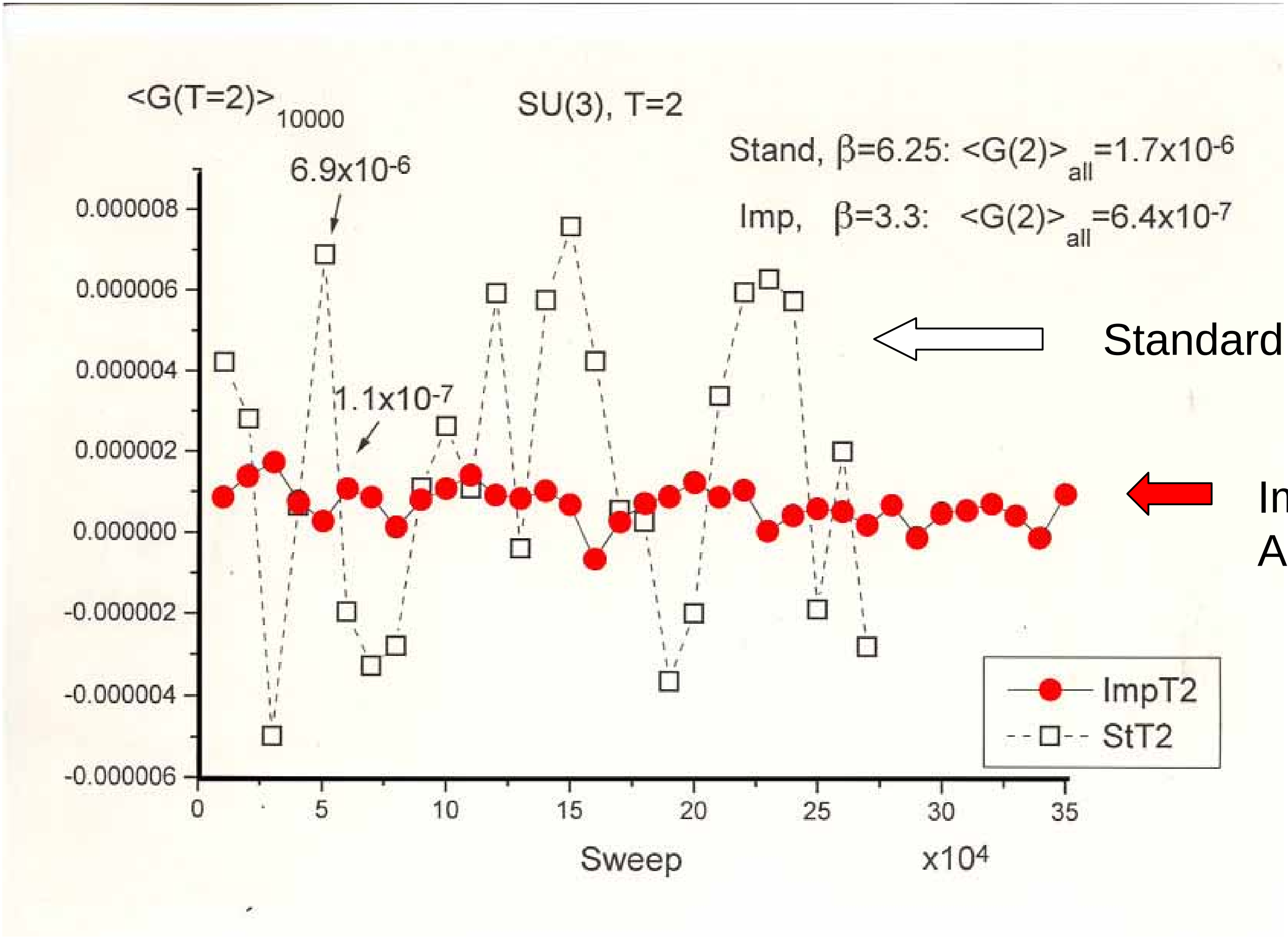
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2} \right)$$

- Shear Viscosity
  - Positive $\eta / s \sim 0.1$
- Bulk Viscosity ~ 0
- Improved Action works well to get good Signal/Noise ratio. $T_{\mu\nu}$

Fighting against Noise



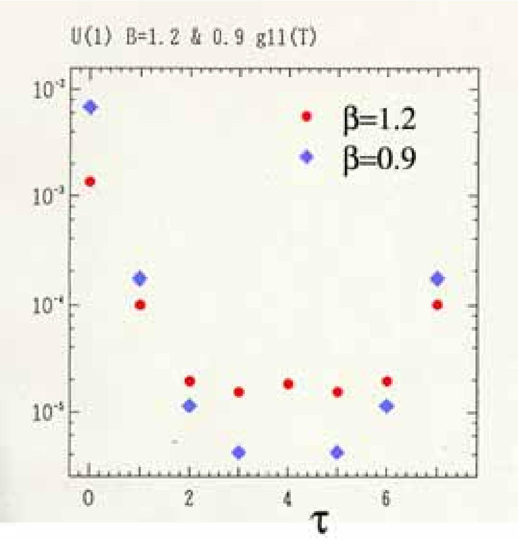
Fluctuations in MC sweeps



Standard Action

Improved Action

Correlators



U(1)
Coulomb and Confinement Phases

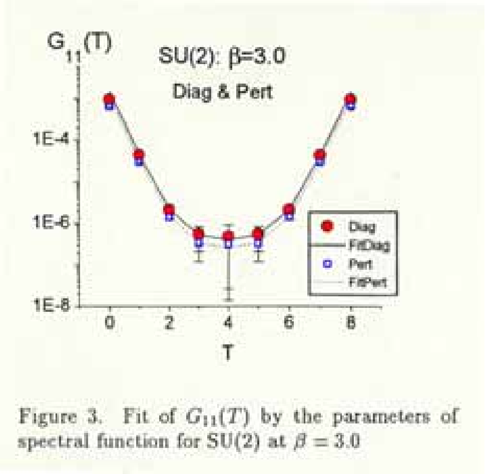


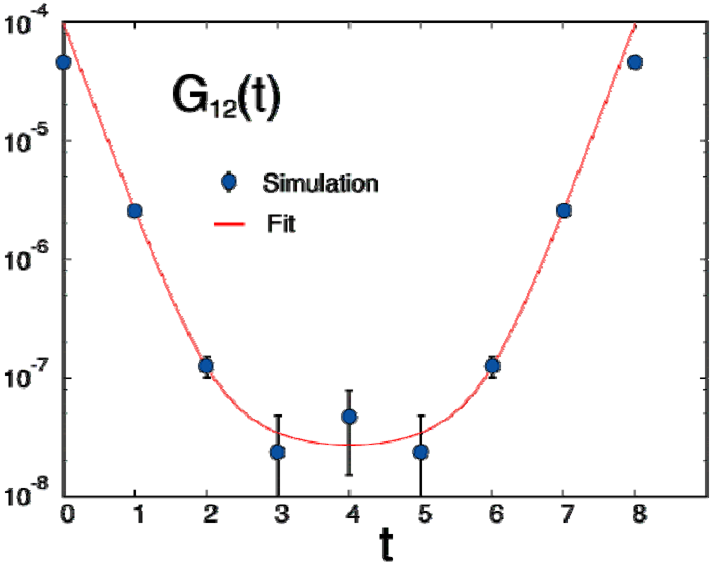
Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$

SU(2)

Two Definitions:

F=log U

F=U-1



SU(3)

Improved Action

Errors in U(1), SU(2), SU(3) standard and SU(3) improved

1995 U(1)

1997 SU(2)

1998 SU(3) preliminary

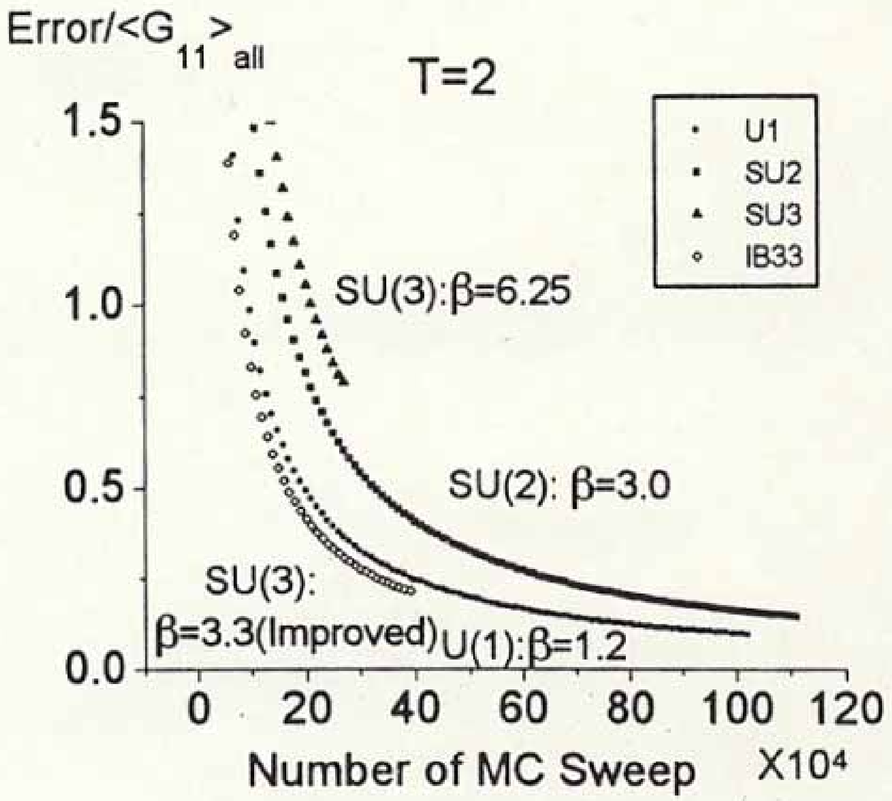


Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3) $\beta = 3.3$.

Low Frequency Region in Spectral Function $\rho(\omega)$ is Important

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

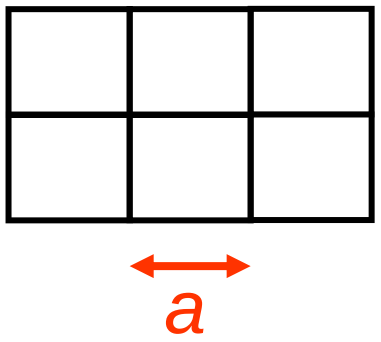
Horsley and Shoenmaker

$(\varepsilon \longrightarrow 0)$ after the Thermo-Dynamics Limit

Long Range in τ of Thermal Green Function $< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$ on the Lattice should be precisely determined.

➡ The finite volume scaling will be required.

Why they are so noisy ?



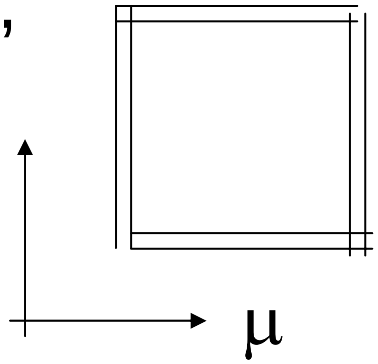
- RG improved action helps lot.
  - Noise from Lattice Artifact ?
(Finite *a* correction ?)
  - Once we checked that there is not so much difference between

$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right)/2i$ and $F_{\mu\nu} = \log U_{\mu\nu}/i$

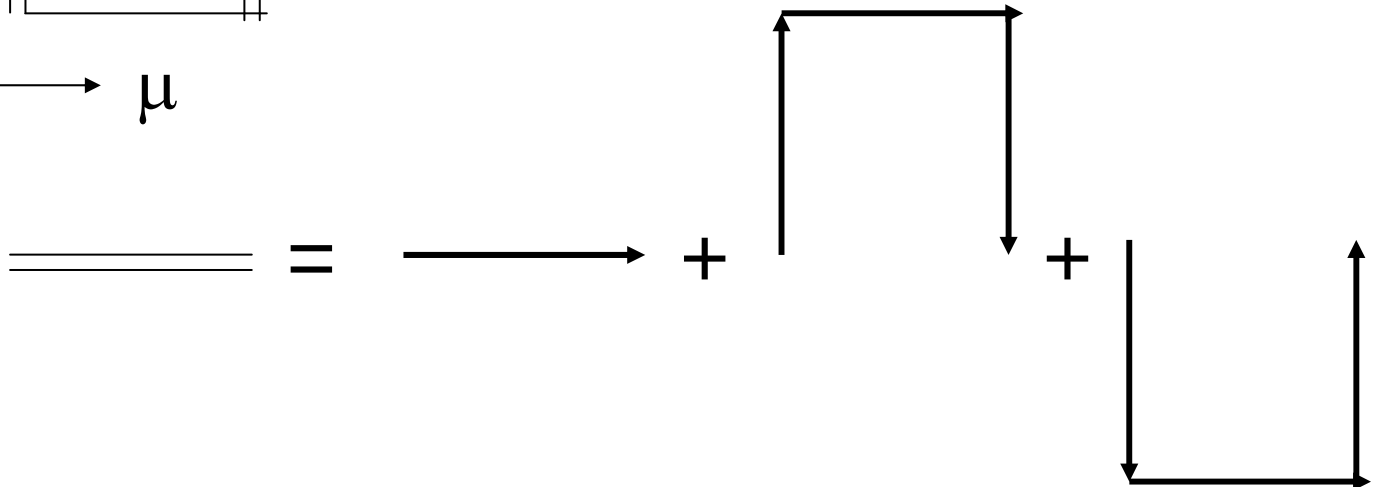
for SU(2). But we should check it again.

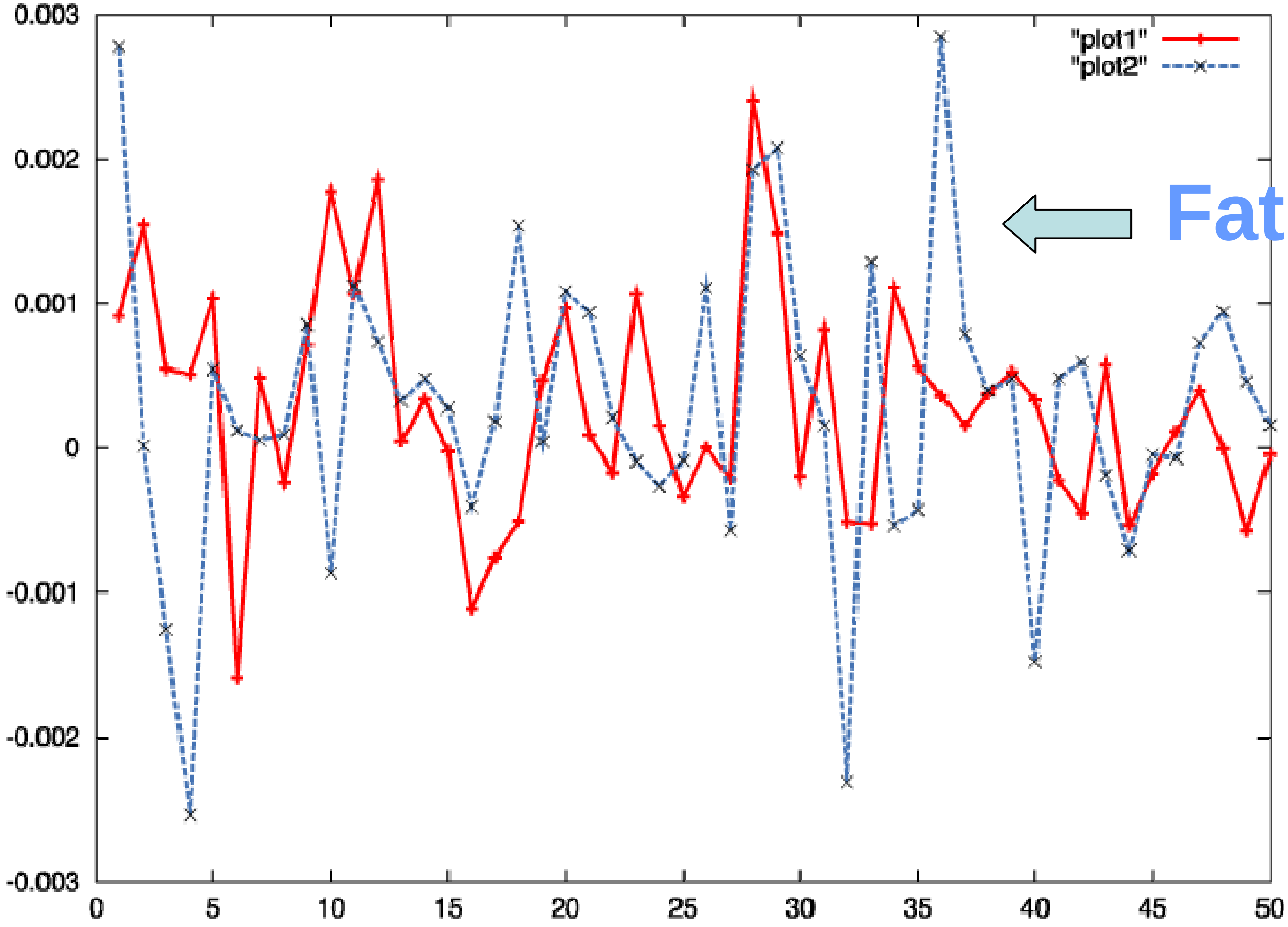
The situation reminds us Glue-Ball Case. (I thank Ph.deForcrand for discussions on this point.)

- Glue-Ball Correlators = $\left\langle \square(\tau)\,\square(0) \right\rangle$
- Large (extended) Operators work better, e.g.,



where

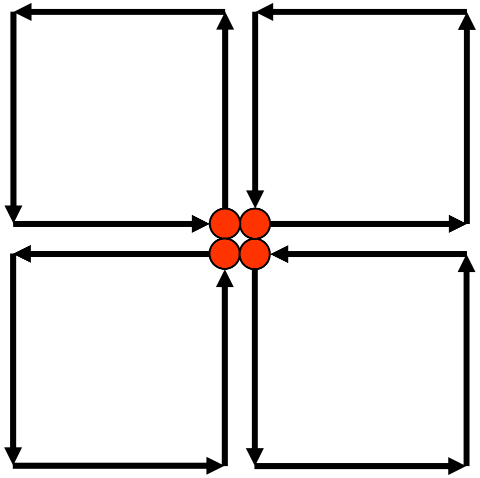




Fat Link

- Mmmm… not works …

Another Extended $F_{\mu\nu}$



(I thank Rajan Gupta for convincing me by stressing this operator so strongly.)

A Crazy method
Source method + Langevin (Parisi)

Source Method

$$Z(J) = \int D\phi e^{-S+J\phi}$$

$$\left\langle \phi(x)\phi(y) \right\rangle = \frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} \log Z(J)$$

Langevin Update

$$\frac{d\phi(x)}{dt} = -\frac{\partial S}{\partial \phi(x)} + \eta$$

Deterministic
No Accept-
Reject step

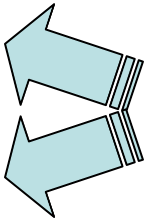
t : Langevin time,

η : Gaussian Random Numbers

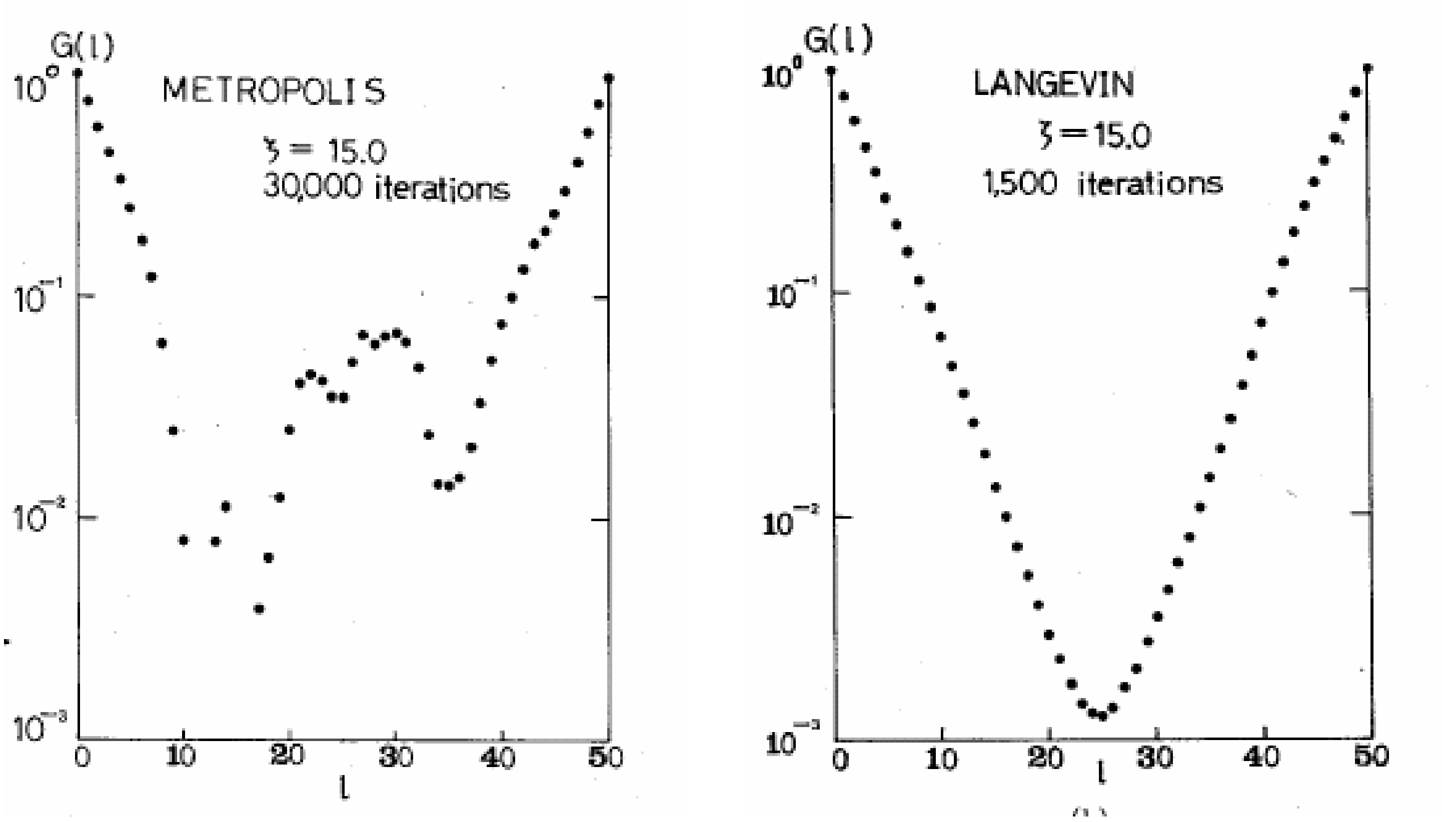
$$\left\langle \phi(x)\phi(y) \right\rangle = \frac{\delta}{\delta J(x)} \left\langle \phi(y) \right\rangle_J = \frac{\left\langle \phi(y) \right\rangle_{\varepsilon J} - \left\langle \phi(y) \right\rangle_0}{\varepsilon}$$

$$\left\langle \phi(y) \right\rangle_{\varepsilon J}$$

$$\left\langle \phi(y) \right\rangle_0$$



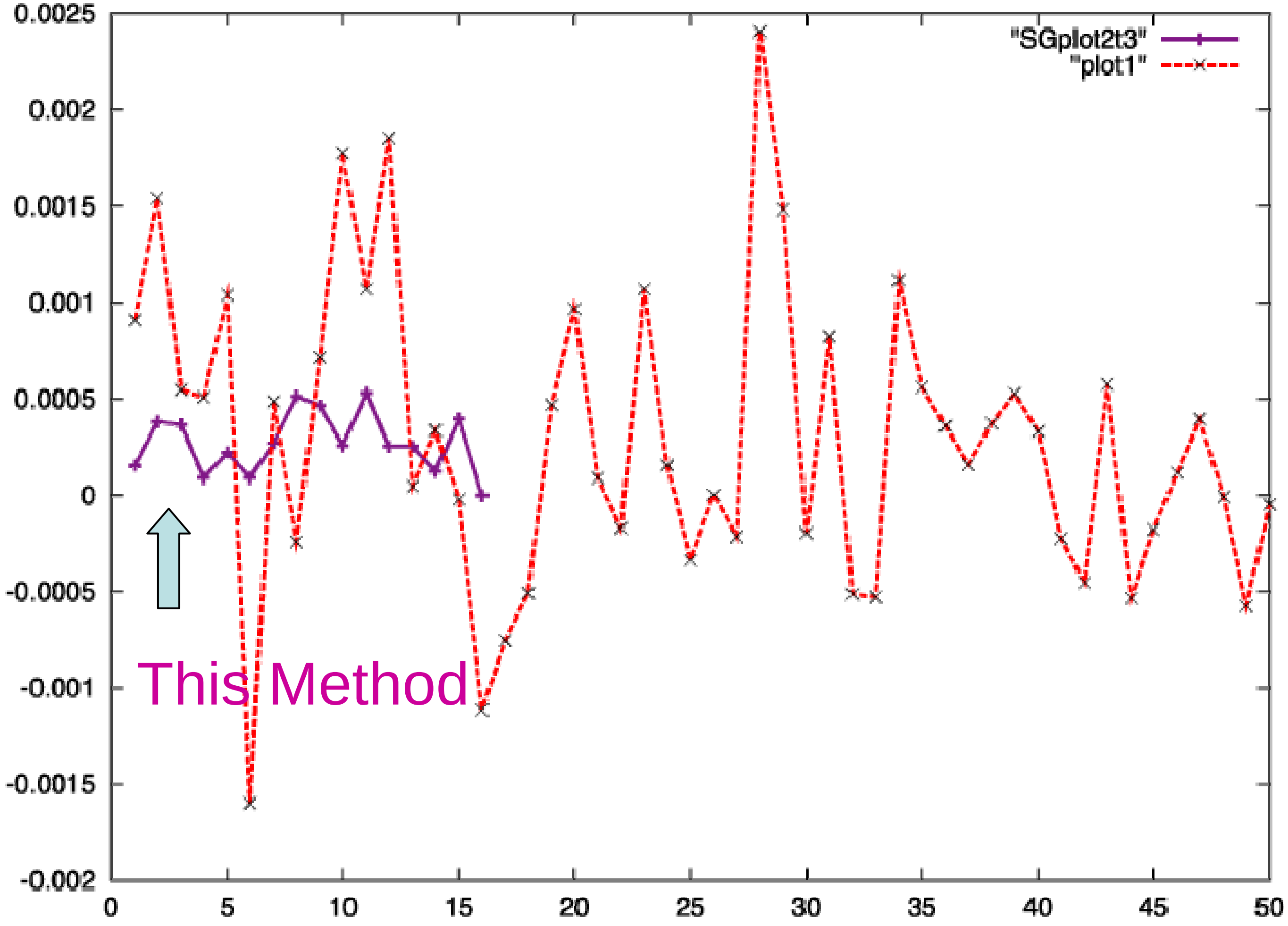
Calculate by Langevin
by the same Random Numbers



Namiki et al., Prog.Theor.Phys. 76 (1986) 501

O(3) Non-linear σ-model

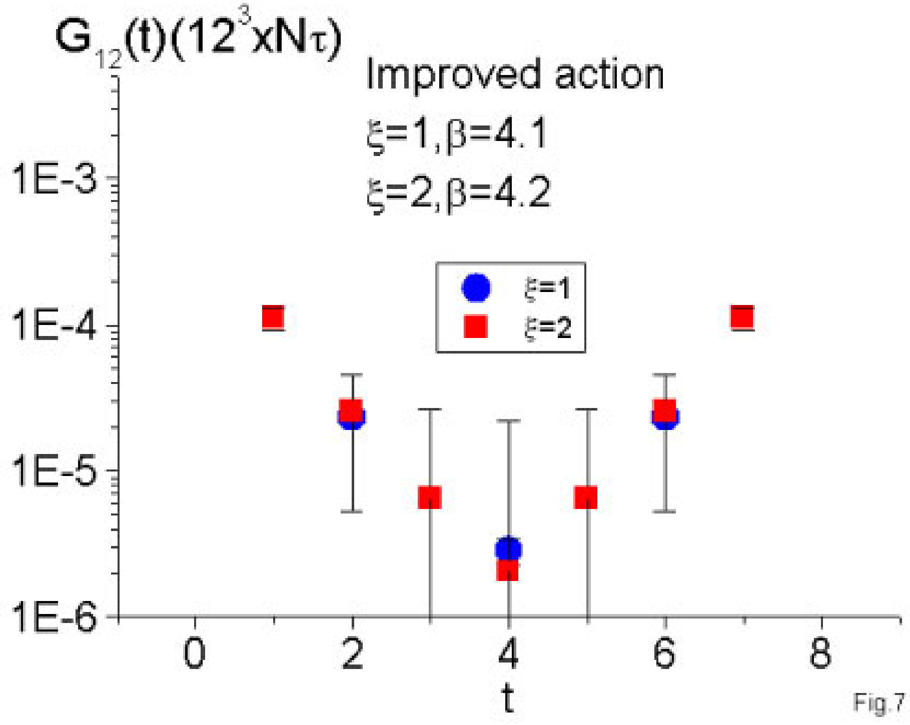
In our case, …
(Very very preliminary)

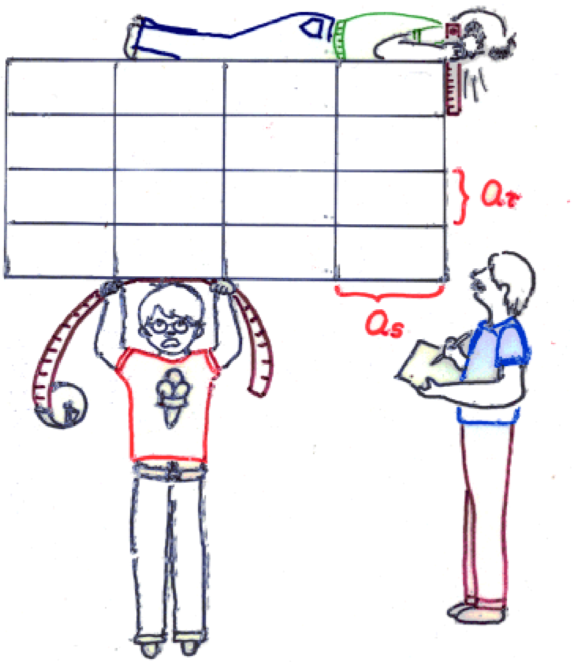


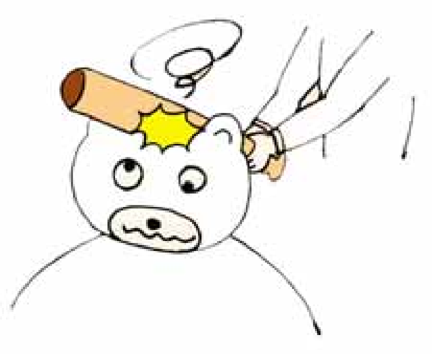
This Method

Anisotropic Lattice ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.







Aarts and Martinez-Resco, JHEP0204 (2002)053

Criticism against the Spectrum Function Ansatz.

Petreczky and Teaney, hep-ph/0507318

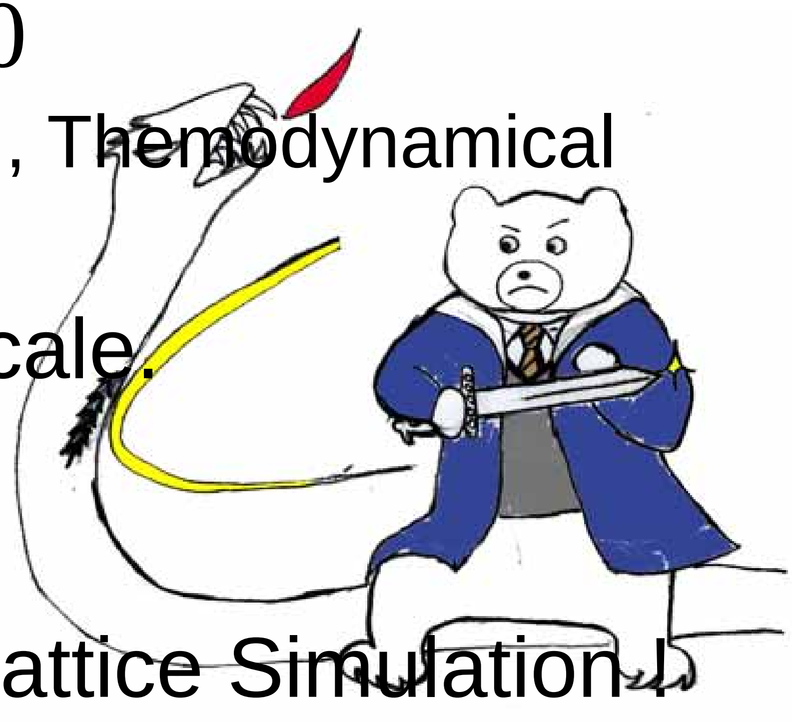
Impossible to determine Heavy Quark Transport coefficient

Note that

Non-Equilibrium Calculations are in general subtle.

- Important Regions : $\omega \sim 0$
  - Physics is in Infra-Red i.e., Themodynamical Limit
- 1/ε is Coarse-Graining Scale.
  - ε > 1/L

    $L \rightarrow \infty, \varepsilon \rightarrow 0$
- But this is Challenge of Lattice Simulation !

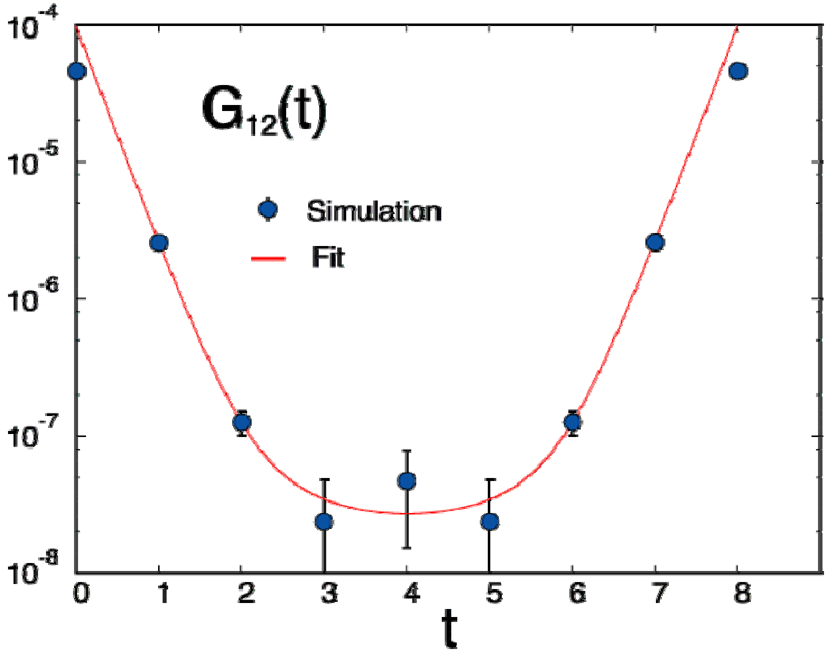


Future direction ?

- If we can extract the Spectral Density $\rho(\omega)$ we can get the Transport Coefficients.
  - Maximum Entropy Method by Asakawa, Nakahara and Hatsuda
  - S. Gupta’s method hep-lat/0301006
- We need (probably)
  - Anisotropic Lattice
  - Finite size scaling analysis
- Full QCD ?
  or
  with Quark Sector even in quench ?

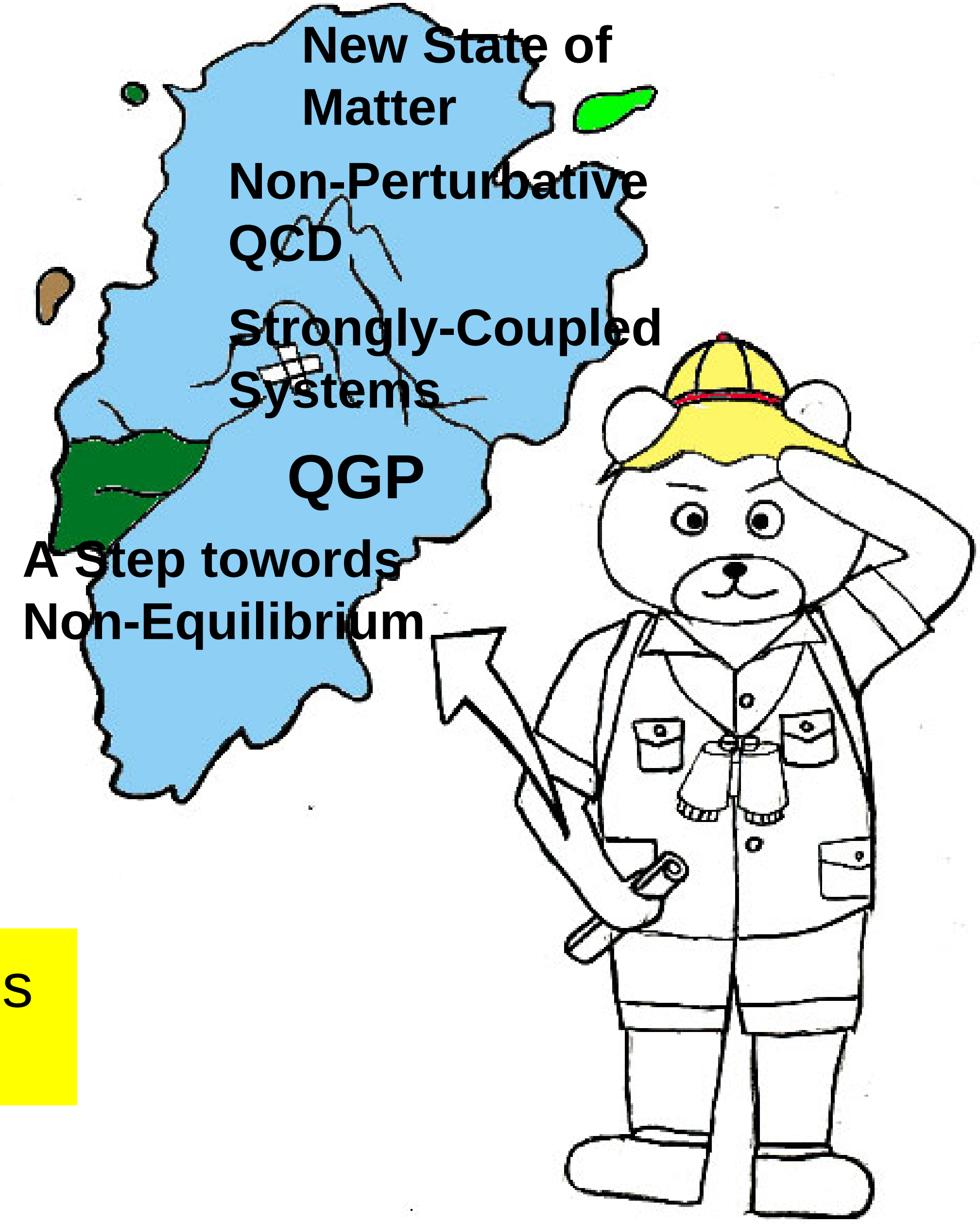
We need data at large τ (small ω) with $O\left(\frac{1}{10}\right)$ Errors

- Brute Force ?
  - Not so crazy because the next Super-Computer is Peta-Flops Order.
- Good Operator
  - Extended
  - Renormalized



Anyway

- Lattice Study of Transport Coefficients is a Challenge !
- And I will appreciate any your comments for the next step.



New State of Matter

Non-Perturbative QCD

Strongly-Coupled Systems

QGP

A Step towords Non-Equilibrium

- What we showed today is a Starting Point.