Quark Recombination and Elliptic flow

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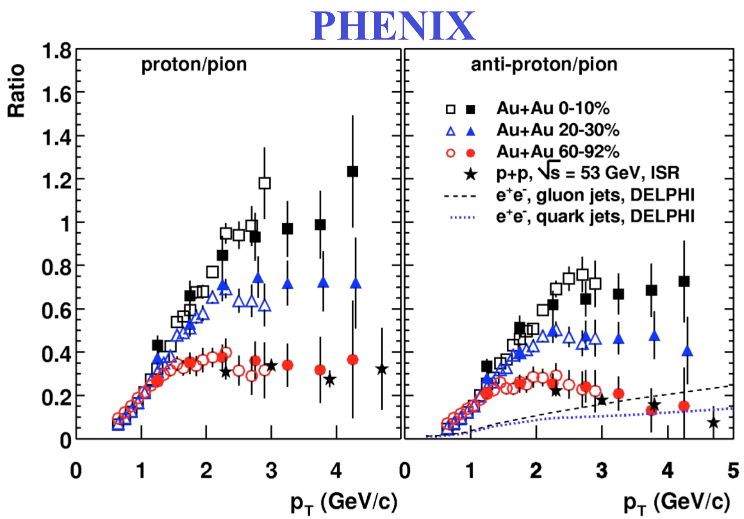
Outline

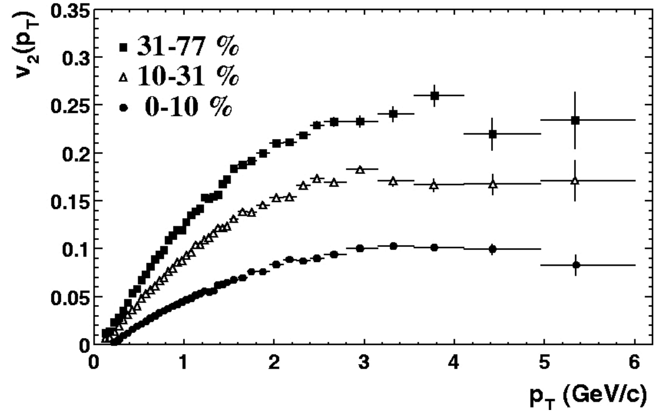
- **Quark Recombination**
  - motivation
  - simple quark recombination formula
  - recombination versus fragmentation
- **Elliptic flow**
  - elliptic flow at RHIC
  - model prediction of elliptic flow
  - quark number scaling of elliptic flow
- **Caveats of quark recombination formalism**
  - phase space density vs volume anisotropy
  - model calculations of quark number scaling
- **Summary**

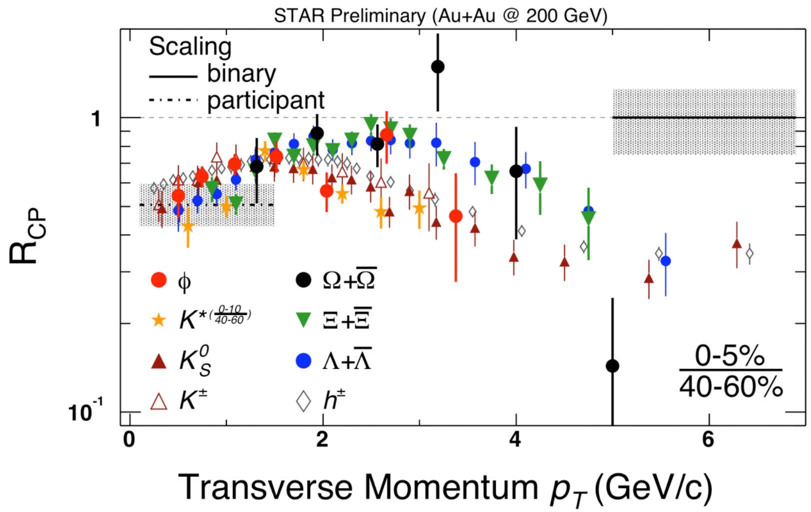
Two RHIC puzzles

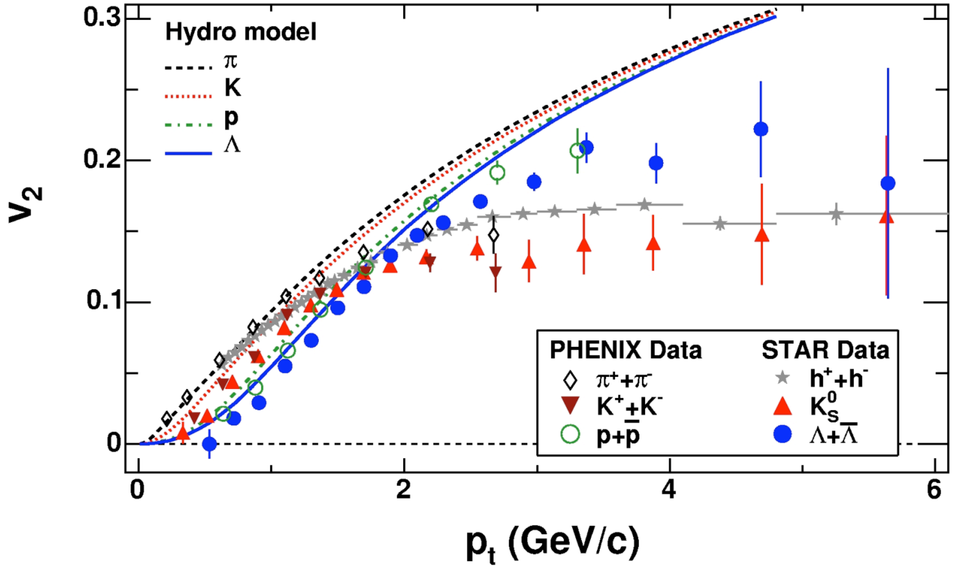
Baryon non suppression: p/π ~ 1

elliptic flow saturates

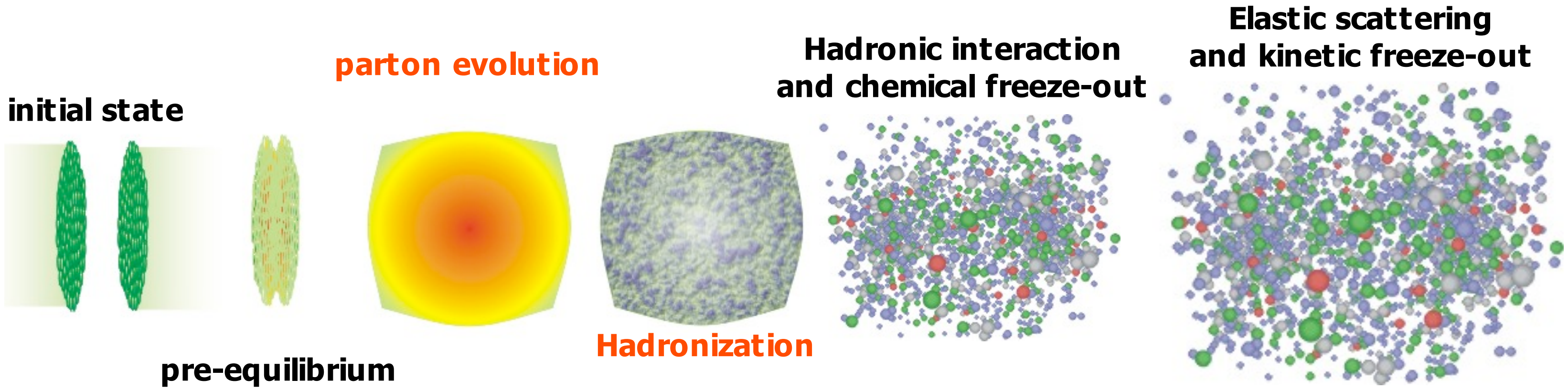




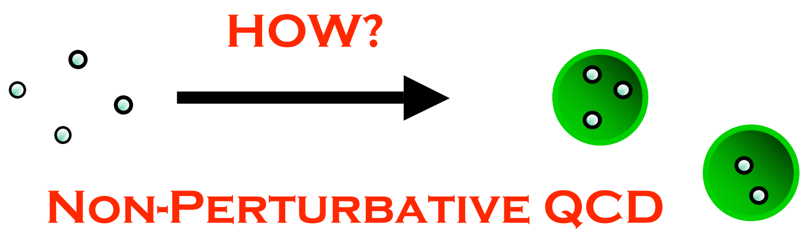




Hadronization problem in HI collision



- ***Free*** partons produced in high energy collisions: $e^+ + e^-$, e+p, p+p, A+A
- Partons in the final state have to be converted into hadrons (confinement)



- Can we learn about the parton phase from a theory of hadronization?

Independent fragmentation in p+p colln

Inclusive hadron distribution – calculable in pQCD

$$dN_h = f_{a/N}(x_a,Q^2)\otimes f_{b/N}(x_b,Q^2)\otimes d\sigma_{ab\to cX}\otimes D_{c\to h}(z_c,Q^2)$$

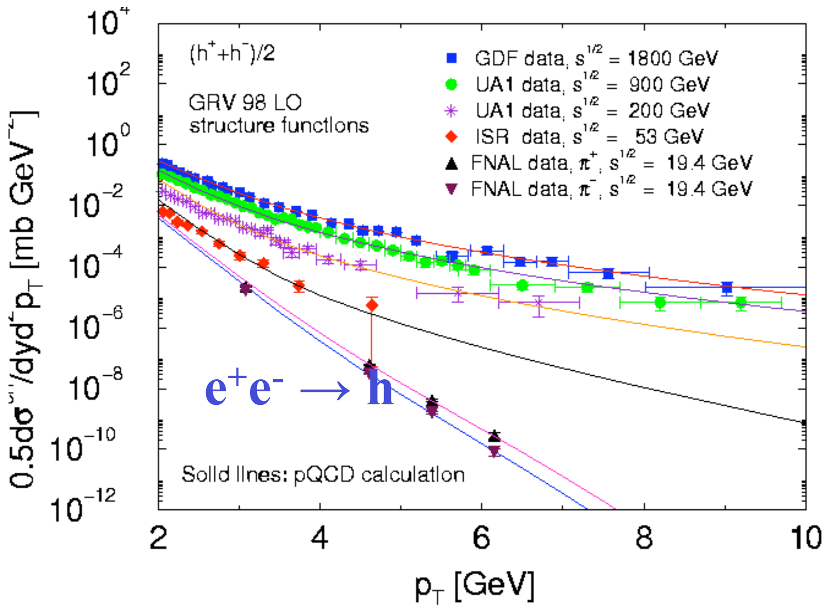
Parton distribution fns

Perturbative cross section

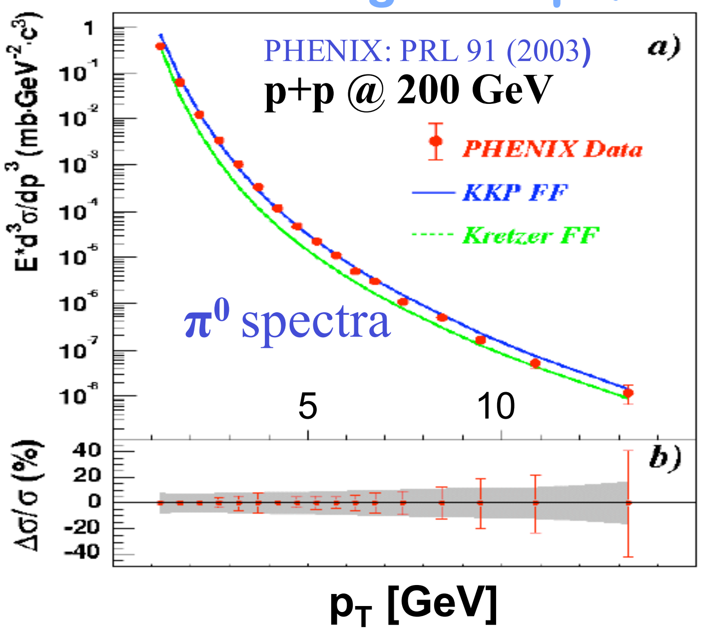
Fragmentation fn

$z_c = p_h/p_q$

Leading order pQCD phenomenology



Next-to-Leading order pQCD

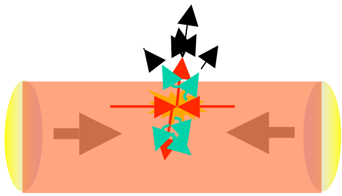


Fragmentation function D works for ***p+p*** *collision at RHIC*

Indep. fragmentation in A+A collision

A+A collision: *inclusive hadron distribution from pQCD*

$$dN_h = f_{a/A}(x_a,Q^2)\otimes f_{b/A}(x_b,Q^2)\otimes d\sigma_{ab\to cX}\otimes \Delta E \otimes D_{c\to h}(z_c,Q^2)$$



Gyulassy, Levai, Vitev NPB594 ('01)

$$\Delta E^{(1)} \approx \frac{C_R\alpha_s}{4}\frac{\mu^2 L^2}{\lambda_g}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots,$$

– Static medium

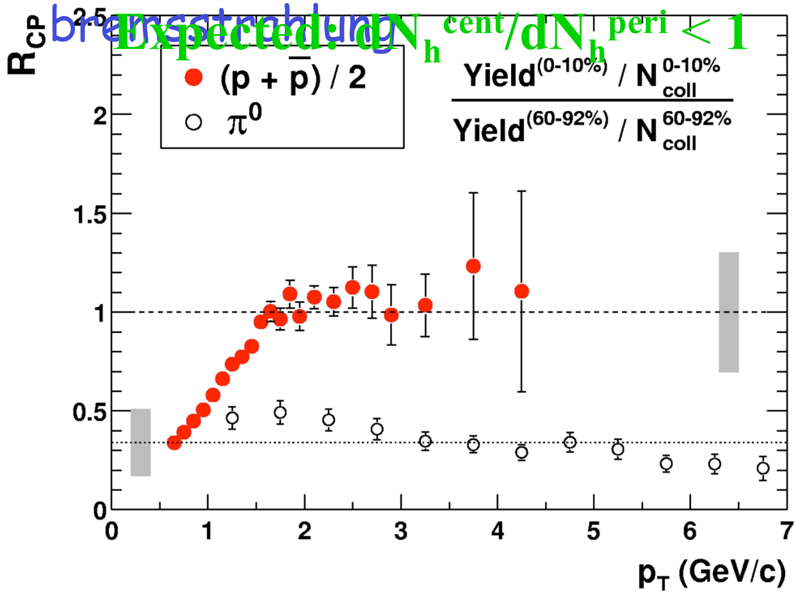
$$\Delta E^{(1)} \approx \frac{9\pi C_R\alpha_s^3}{4}L\frac{1}{A_\perp}\frac{dN^g}{dy}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots,$$

– 1+1D Bjorken

Jet quenching: pQCD energy loss

ΔE of partons by gluon bremsstrahlung

Expected: $dN_h^{cent}/dN_h^{peri} < 1$



PHENIX: Phys.Rev.Lett. 91 (2003)

***A+A* collision with energy loss**

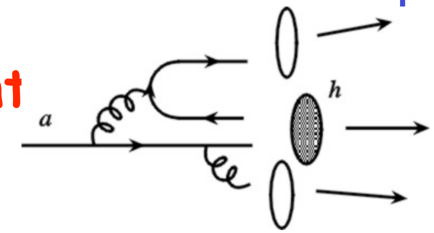
works for pions (suppression) but
fails for baryons (non-suppression)

-Failure of fragmentation function

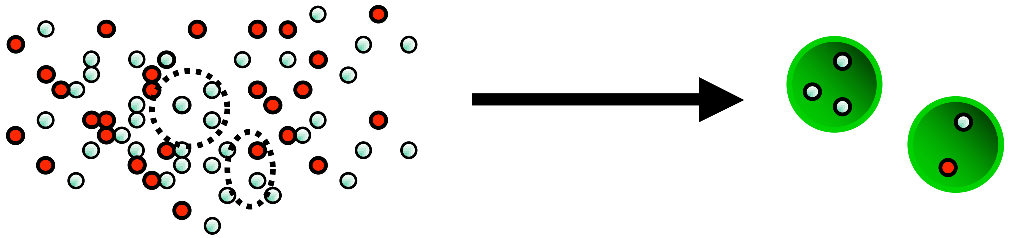
Quark Recombination

- Collision of elementary particles or for A+A at $p_T > 6$ GeV/C

  Jet fragmentation is dominant

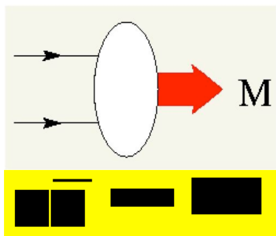


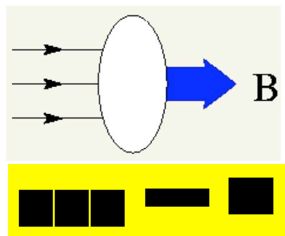
- Heavy ion collisions: large parton density, multi-parton processes possible, additional hadronization channels via **parton recombination**



- **parton ReCo assumption:**

Consider lowest order Fock state, i.e. valence constituent (massive) quarks





Simple recombination formula

- Based on n+p→d

Recombination: $q\bar{q} \rightarrow meson, \quad qqq \rightarrow baryon$

Convolution of Wigner functions

$$\frac{dN_M(\mathrm{p})}{d^3p} = g_M \int (\prod_{i=1,2} d^3x_i d^3p_i)\, f_\alpha(\mathrm{p}_1, x_1) f_\beta(\mathrm{p}_2, x_2)\, W_M(x_1 - x_2, \mathrm{p}_1 - \mathrm{p}_2)\delta^3(\mathrm{p} - \mathrm{p}_1 - \mathrm{p}_2)$$

$$\frac{dN_B(\mathrm{p})}{d^3p} = g_B \int (\prod_{i=1,2,3} d^3x_i d^3p_i) f_\alpha(\mathrm{p}_1, x_1) f_\beta(\mathrm{p}_2, x_2) f_\gamma(\mathrm{p}_3, x_3) W_B(x_{12}, x_{13}, \mathrm{p}_{12}, \mathrm{p}_{13})\delta^3(\mathrm{p} - \Sigma \mathrm{p}_i)$$

Hadron yield   Space-time   Quark distribution fn   Hadron function

❑ **Basic assumptions:**

- **uncorrelated quark distribution function**
- **weak binding**
- **sudden (instantaneous) hadronization**

Das & Hwa, PLB 68 ('77)
Biro et al, PLB 347 ('97)
Voloshin NPA 517 ('03)
Greco, Ko, Levai, PRC 68 ('03)
Fries, Muller, Nonaka, Bass, PRC68 ('03)
Molnar & Voloshin, PRL 91 ('03)

Two approaches to ReCo:

(i) parametrize f_q, (ii) Compute f_q in a dynamical model

Recombination vs fragmentation

Exponential: $f_q \sim e^{-p_T/T}$

$$N_{\text{frag}} = f \otimes D \sim e^{-P_T/\langle z \rangle T} \langle D \rangle$$

$$N_{\text{reco}} = f_\alpha \otimes W_M \otimes f_\beta \sim e^{-P_T/T}$$

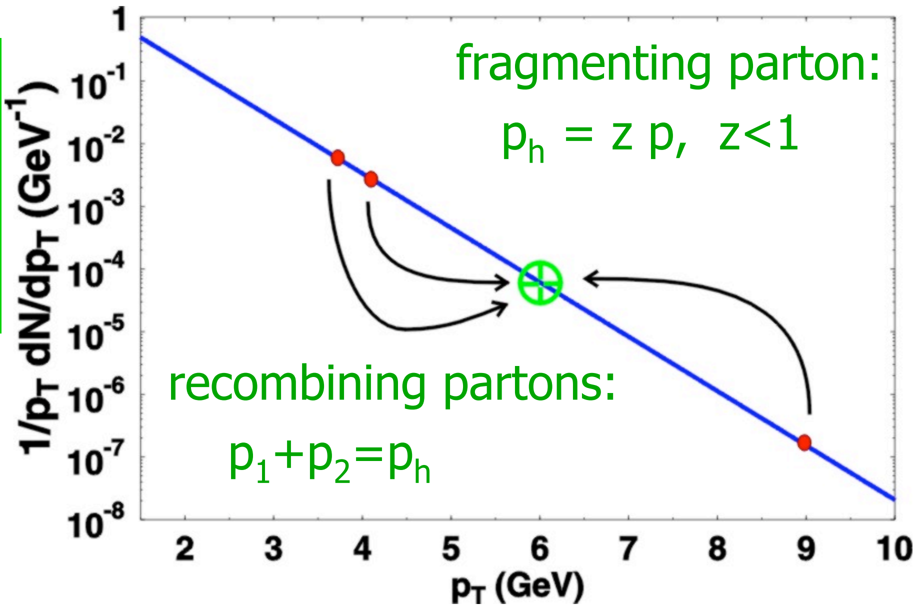
Recombination wins

Power law: $f_q \sim p_T^{-\alpha}$

$$N_{\text{frag}} \sim P_T^{-\alpha}$$

$$N_{\text{reco}} \sim P_T^{-2\alpha}$$

Fragmentation wins



Baryon: $N_B(p_T) = N_q(p_T/3)$

Meson: $N_M(p_T) = N_q(p_T/2)$

- Baryon number dominates meson

Recombination competes with fragmentation and wins at $p_T < 6$ GeV

ReCo by source parametrization

One approach to obtain f_q is via "source parametrization":

- Hard spectra : LO pQCD minijets with jet quenching and indep. fragmentation
- Soft spectra (p_T< 2 GeV): (anti)quark thermalized plasma with radial flow

Total hadron yield $dN_h = dN_h^{coal}(f_q \otimes f_{\bar{q}} \otimes W_h) + dN_h^{frag}(d\sigma \otimes E_{loss} \otimes D)$

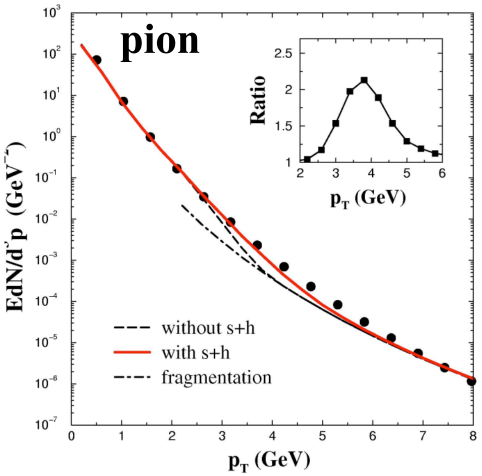
TAMU group: Tq=170 MeV, v_T= 0.5

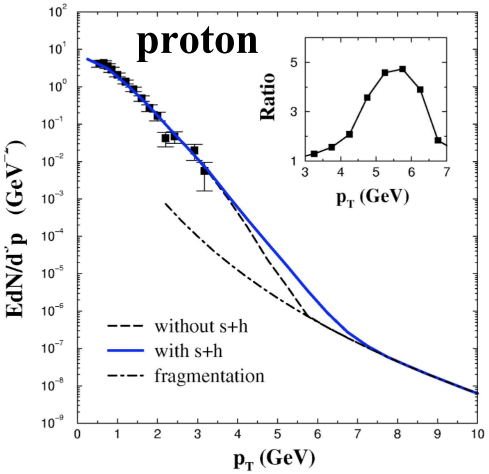
Greco, Ko, Levai, PRC 68 ('03)

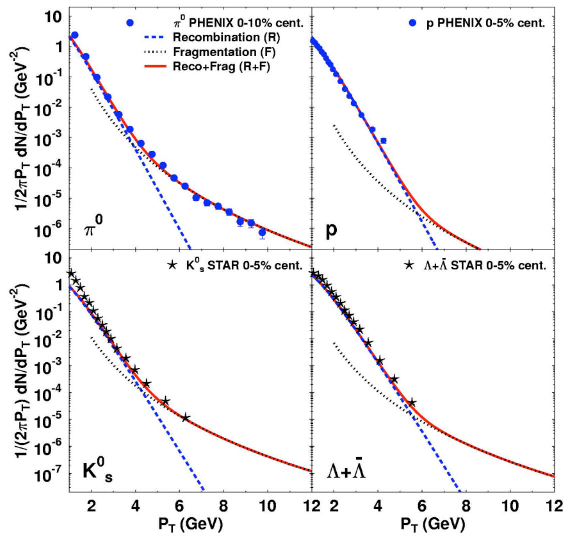
Duke group: Tq=175 MeV, v_T= 0.55

Fries, Muller, Nonaka, Bass, PRC 68 ('03)

Au+Au @ 200AGeV (central)

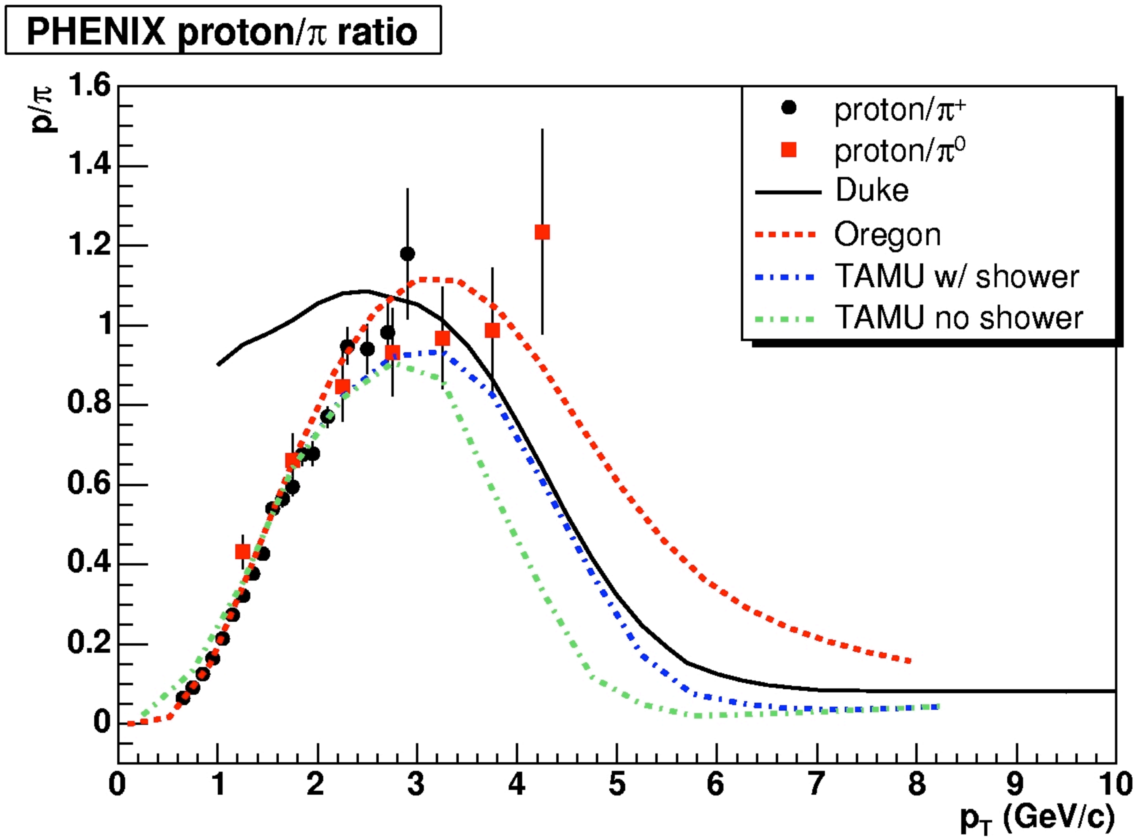






ReCo dominates up to p_T ~ 4 GeV (meson) & to ~ 6 GeV (baryon)

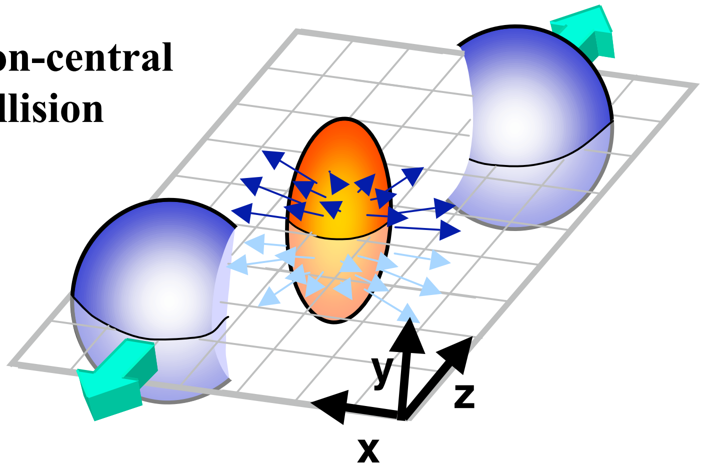
Baryon/Meson anomaly in ReCo

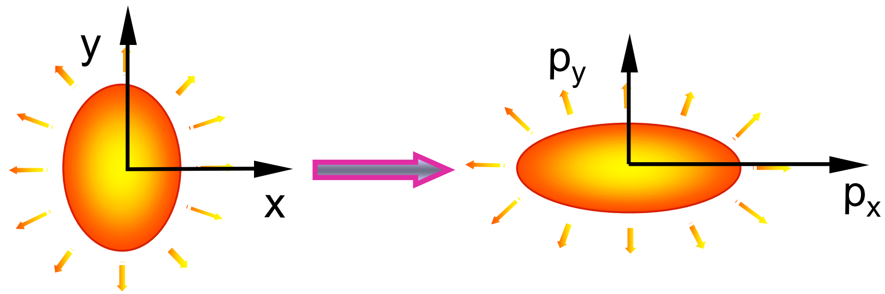


p_T < ~2 GeV recombination via soft partons dominates:

Mesons up to 2x(~2) GeV, Baryons up to 3x(~2) GeV

Elliptic flow





Origin of $v_2 \neq 0$: coordinate-space anisotropy (b > 0) and re-interaction

Anisotropic flows v_n are Fourier coefficients in azimuthal distribution of particles with respect to reaction plane

Ollitrault PRD 46 ('92)

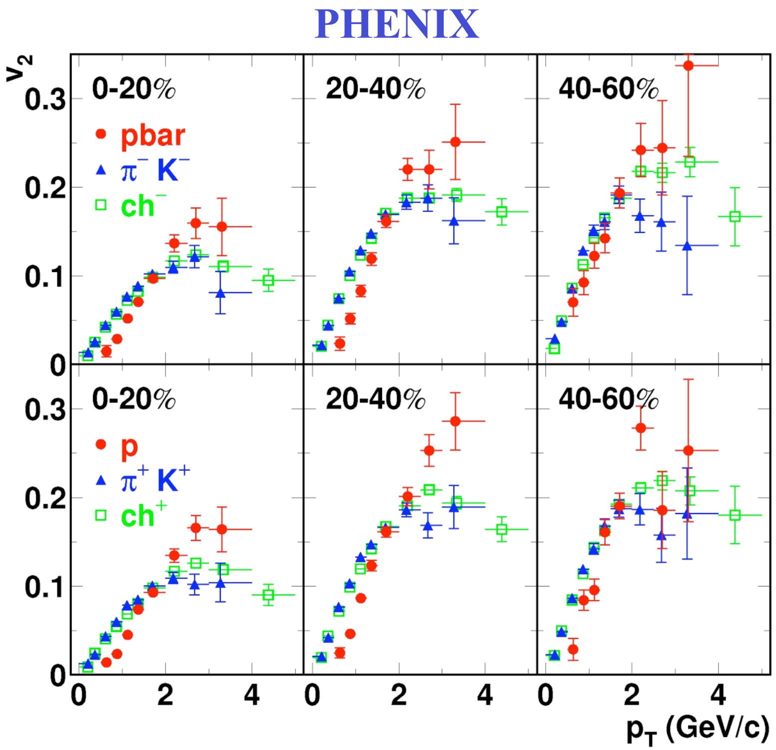
$$\frac{dN}{d\varphi dX} = \frac{1}{2\pi}\frac{dN}{dX}\left[1 + 2\sum_n v_n(X)\cos(n\varphi)\right]$$

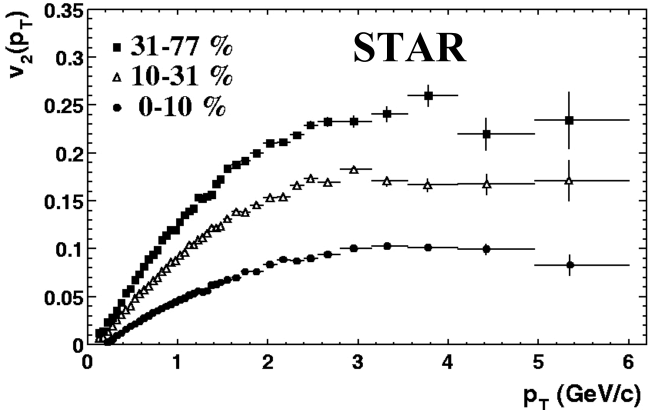
X: particle species and event, eg: centrality, transverse momentum

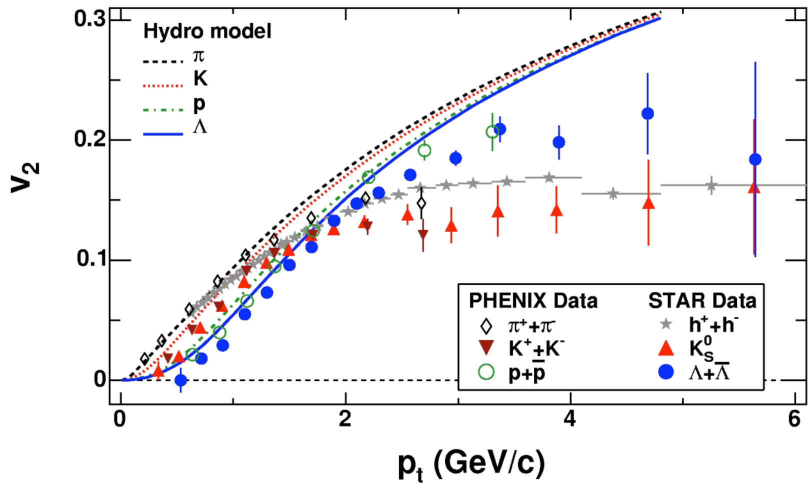
Elliptic flow

$$\Rightarrow v_2(p_T) \equiv \langle\cos(2\varphi)\rangle_{p_T} = \frac{\int d\varphi \cos(2\varphi)\left(dN/dp_T dy d\varphi\right)}{\int d\varphi\left(dN/dp_T dy d\varphi\right)} \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Elliptic flow at RHIC







- Large elliptic flow for all hadron species
- v_2 saturation at different p_T for hadrons
- At $p_T < 2$ GeV, heavier hadrons smaller v_2
- At $p_T > 2$ GeV, heavier hadrons larger v_2

Ideal hydrodynamics & elliptic flow

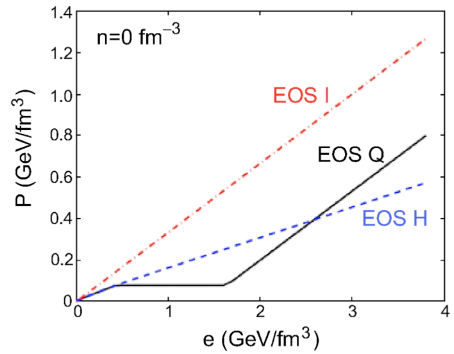
Basic assumption: local thermodynamic equilibrium

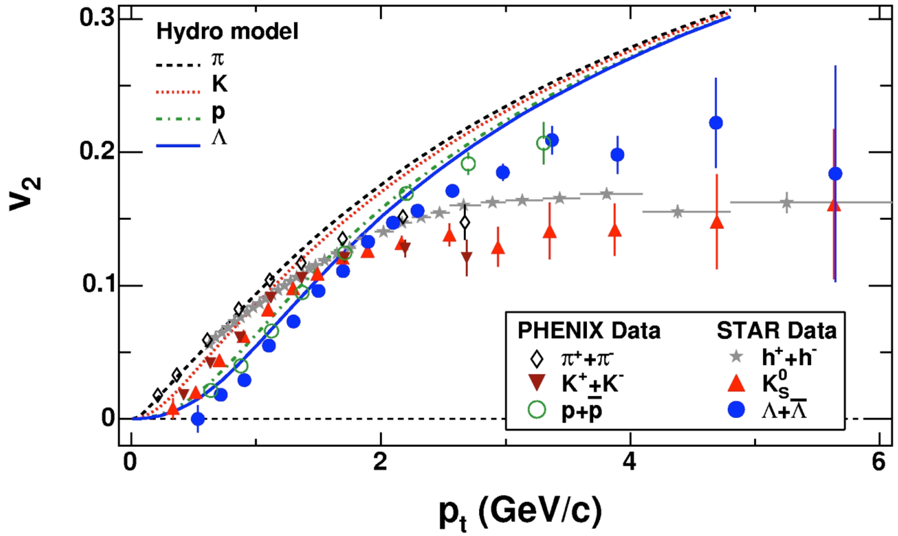
Kolb et al, PRC62 ('00), NPA696 ('01), PLB500 ('01)

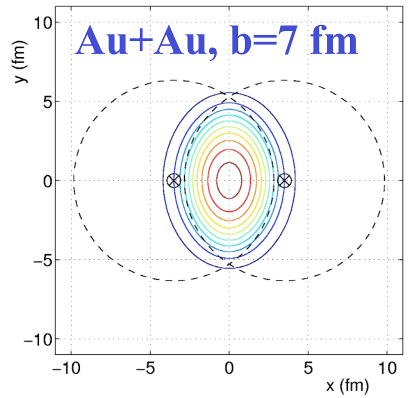
Evolution equations are local conservation of energy-momentum and net baryon number

$$\partial_\mu T^{\mu\nu}(x) = 0, \quad \partial_\mu N^\mu(x) = 0$$

5 eqns, 6 unknowns (ε,n,p,**u**) requires an EOS: p = p(ε,n)







- ❖ Hydro explains v_2 systematics with a QGP EOS for $p_T < 2$ GeV
- ❖ Mass ordering at $p_T < 2$ GeV - massive hadrons smaller v_2
- ❖ mass ordering violated at large p_T and v_2 saturates in data

Elliptic flow from recombination

If all partons have same momentum distribution

$$\frac{dN_M(p_T)}{d^3p_T} \propto \left[\frac{dN_q(p_T/2)}{d^3p_T}\right]^2, \quad \frac{dN_B(p_T)}{d^3p_T} \propto \left[\frac{dN_q(p_T/3)}{d^3p_T}\right]^3$$

$$v_2^{(M)}(p_T) \approx 2v_2^{(q)}(p_T/2), \quad v_2^{(B)}(p_T) \approx 3v_2^{(q)}(p_T/3)$$

At low p_T<2 GeV: $v_2^{(B)}(p_T) < v_2^{(M)}(p_T) < v_2^{(q)}(p_T)$

→ mass ordering

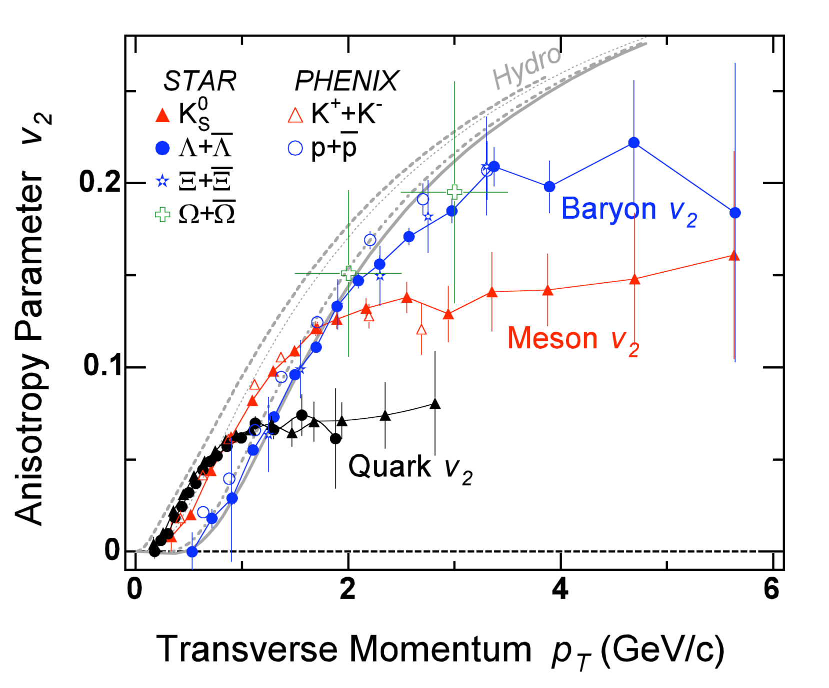
At high p_T>2 GeV: $v_2^{(B)}(p_T) > v_2^{(M)}(p_T) > v_2^{(q)}(p_T)$

→ hadron flow amplified

Amplification: 3x for baryons

2x for mesons

→ **50% larger baryon flow**



➢ ***By measuring v_2 for various hadrons, quark flow can be extracted***

Quark number scaling of v_2

$$v_2^{(H)}(p_T) \approx n\, v_2^{(q)}(p_T/n), \quad n = 2,3 \text{ for mesons, baryons}$$

$$v_2^{(H)}(p_T)/n \rightarrow \text{universal}$$

If $v_2^{(q)} \neq v_2^{(s)}$ (light vs. strange)

Mesons:

$$v_2^{(\partial)}(p_T) \approx 2v_2^{(q)}(p_T/2)$$

$$v_2^{(K)}(p_T) \approx v_2^{(q)}(p_T/2) + 2v_2^{(s)}(p_T/2)$$

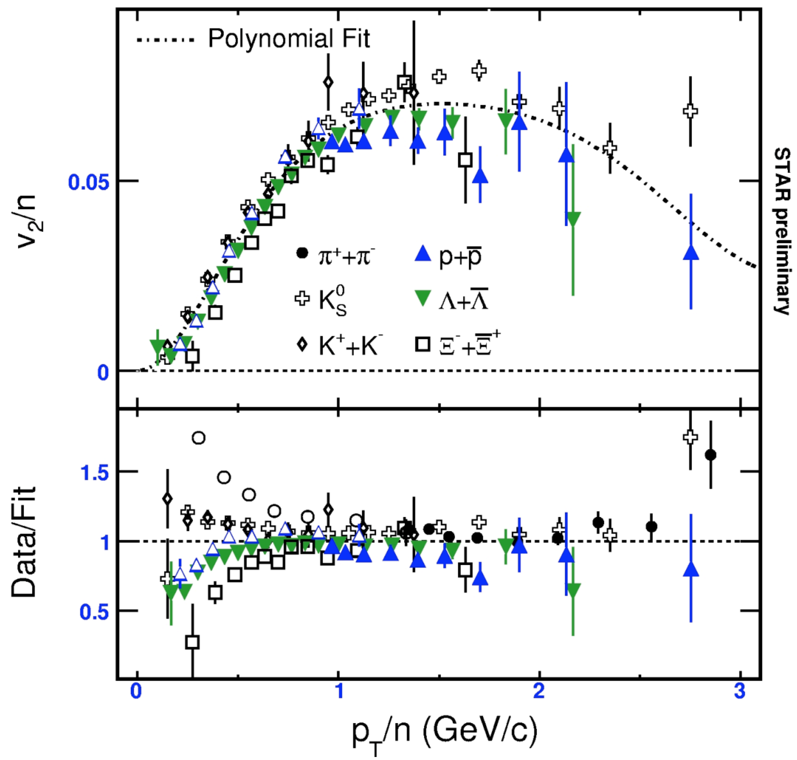
Baryons:

$$v_2^{(p)}(p_T) \approx 3v_2^{(q)}(p_T/3)$$

$$v_2^{(\Lambda,\Sigma)}(p_T) \approx 2v_2^{(q)}(p_T/3) + v_2^{(s)}(p_T/3)$$

$$v_2^{(\Xi)}(p_T) \approx v_2^{(q)}(p_T/3) + 2v_2^{(s)}(p_T/3)$$

$$v_2^{(\Omega)}(p_T) \approx 3v_2^{(q)}(p_T/3)$$



- ❑ **Coalescence prediction approx. verified**
- ❑ **RHIC data shows $v_2^{(q)} = v_2^{(s)}$**

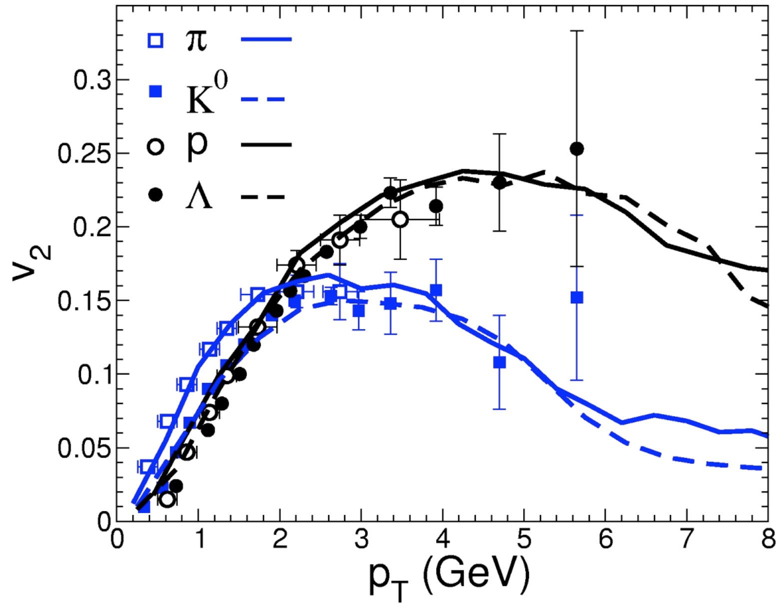
v_2 from source parametrization

Quark transverse momentum anisotropy included:

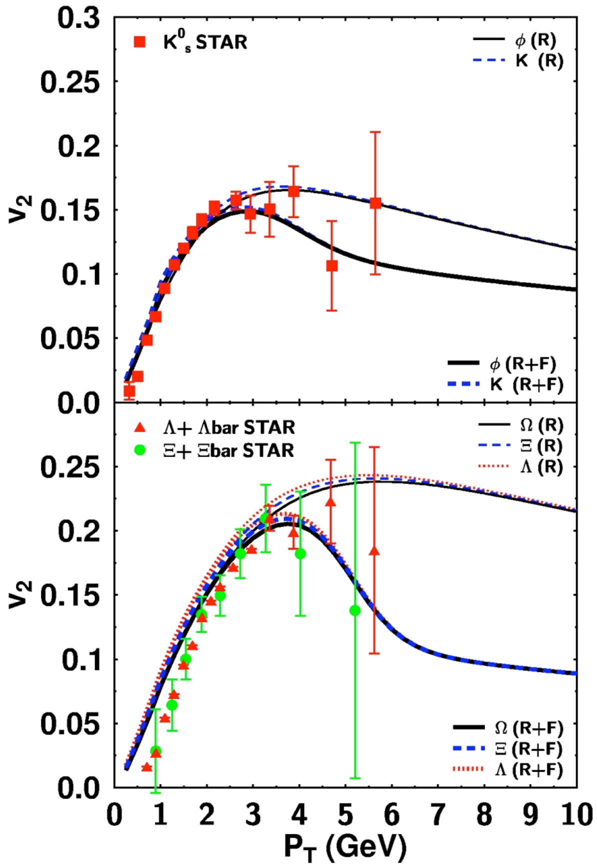
$$\frac{dN_q}{d^2p_T} = \frac{dN_q}{p_T dp_T}[1+2v_2\cos(2\phi)]$$

→ **Quark elliptic flow extracted from pion v_2**

Greco, Ko, Levai, PRC 68 ('03)



Nonaka, Fries, Bass, PLB 583 ('04)



Covariant transport model

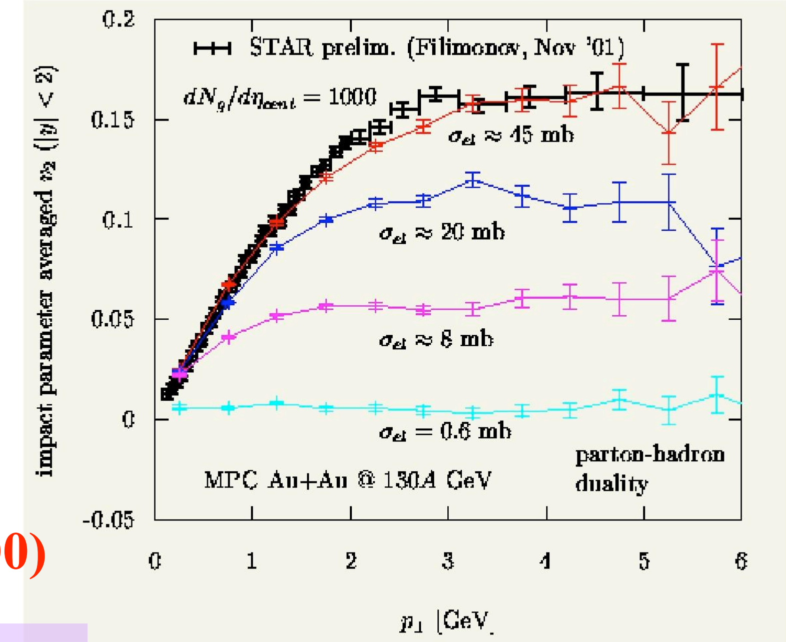
- Interpolates between ideal hydro λ = 0 and free streaming λ = ∞

MPC: kinetic theory of parton gas (Boltzmann)

Molnar & Gyulassy, NPA 697 ('02)

$$p^{\mu}\partial_{\mu}f_i(x,\mathrm{p}) = S_i(x,\mathrm{p}) + \sum_{jkl} F^{2\to 2}_{ij\to kl}[f_i, f_j, f_k, f_l]$$

- **Minijet initial conditions in $S_i(x,p)$**
- **Elastic 2→2 cross section**
- **Debye screened 2→2 pQCD $\sigma \sim 1/(t-\mu^2)^2$**
- **1 parton → 1 hadron hadronization**



Require large opacities:

σ_{el} x dNg/dη ≈ 45000 mb » pQCD (3 mb x 1000)

Solution to opacity puzzle: Recombination

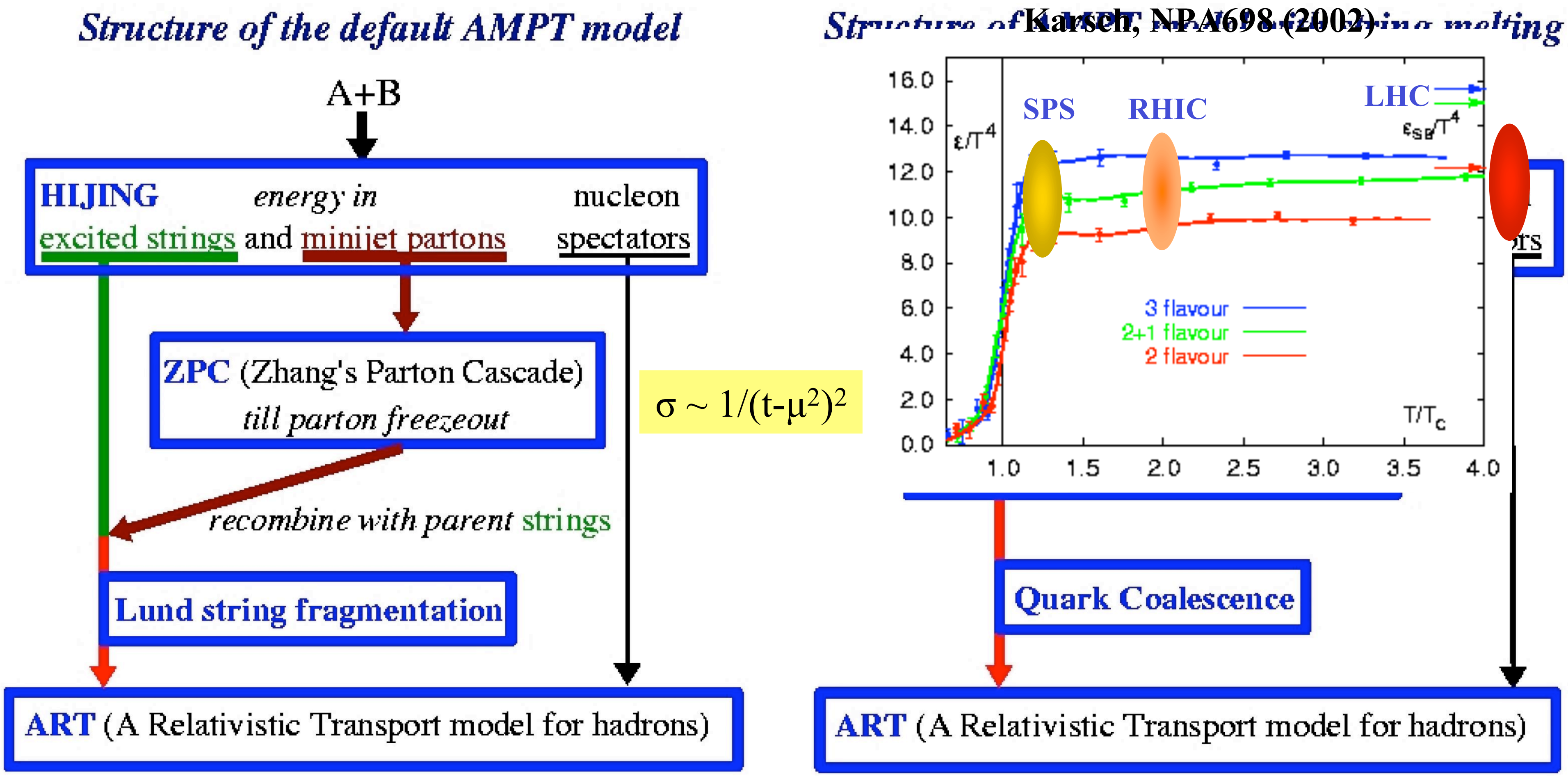
ReCo dynamics: f_q, $f_{\bar{q}}$ and hypersurface

Molnar & Voloshin, PRL 91 ('03)

Elliptic flow fit with ReCo requries smaller σ_{el} x dNg/dη (b=0) ≈ 7000-15000 mb

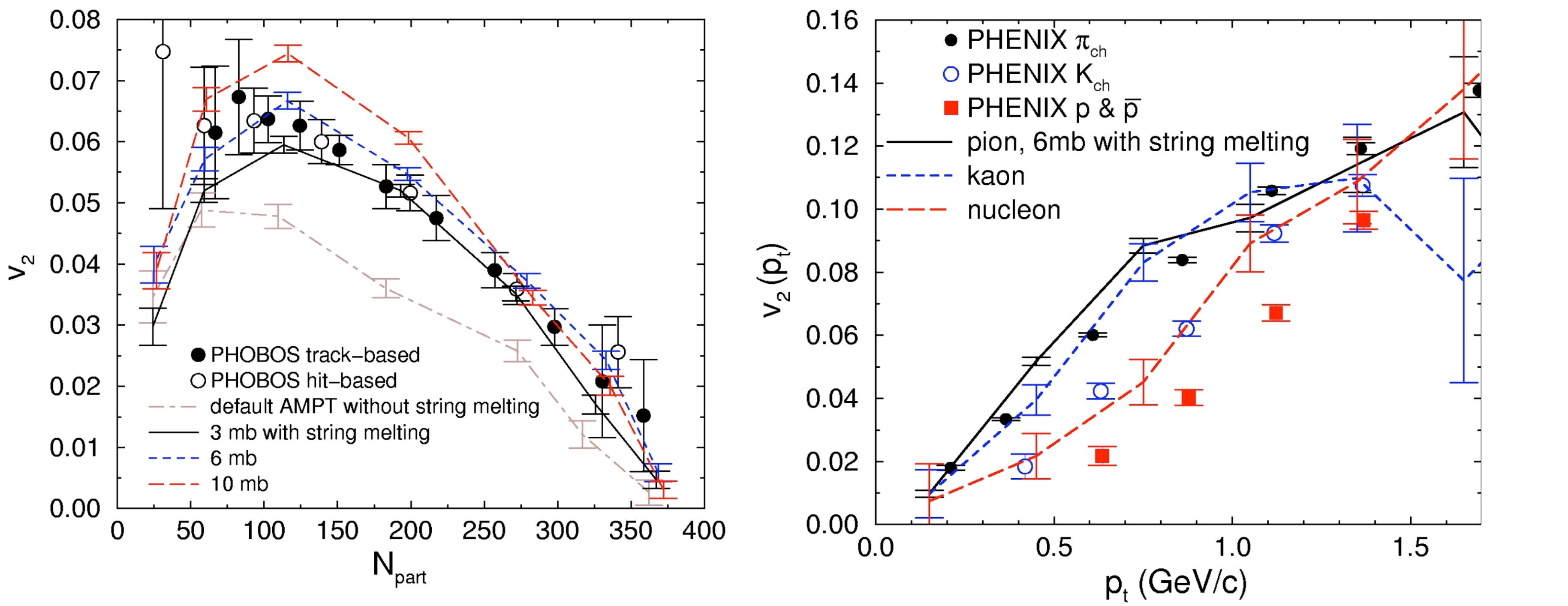
A MultiPhase Transport model (AMPT)

Lin, Ko, Li, Zhang, S.P, nucl-th/0411110



v_2 in AMPT model

Au+Au @ 200A GeV (min bias)



- String melting enhances parton density
- v_2 well explained with recombination & string melting
- v_2 sensitive to parton elastic $\sigma_{qq \to qq}$ (= 6 mb best fit)

Recombination revisited

$$\frac{dN_H(\mathrm{p})}{d^3p} = g_H \int (\prod_i^n d^3x_i d^3p_i)\, \Phi_q(x_i,\mathrm{p}_i)\, W_H(x_{i\ldots n},\mathrm{p}_{i\ldots n})\, \delta^3(\mathrm{p} - \sum_i^n \mathrm{p}_i)$$

Assumptions in $\mathrm{v}_2^{(\mathrm{H})}(p_T) \approx n\,\mathrm{v}_2^{(\mathrm{q})}(p_T/n)$

S. Pratt, S.P, PRC 71 ('05)

No spread in relative coordinate and momenta in hadron wave fn.

$$W_H(x,\mathrm{p}) = \delta(x)\delta(\mathrm{p})$$

- **Uncorrelated quarks** $\Phi_q = \prod_i^n f_i(x_i,\mathrm{p}_i)$
- **sudden (instantaneous) hadronization**

$$f^{(H)}(p_T,\phi,x,t_c) = \left[f^{(q)}(p_T/n,\phi,x,t_c)\right]^n$$

→ *Binding potential turned on suddenly (no energy conservation)*

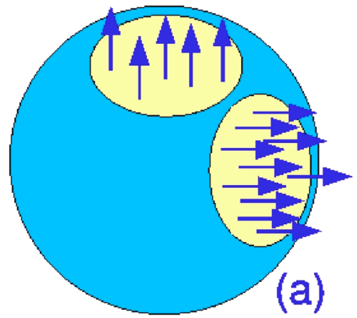
→ *For thermal distribution* $f^{(q)} \prec \exp[-(p_T/n - \mu)/T],\ \therefore \mu^{(H)} = n\mu^{(q)}$

Phase-space density vs emission volume

Define: $\bar{f} = \int d^3r\, f_q^2 \Big/ \int d^3r\, f_q\,, \qquad \Omega = \left(\int d^3r\, f_q\right)^2 \Big/ \int d^3r\, f_q^2$

Spectra determined by both average phase-space density and effective emission volume of partons

$$\frac{dN_H(p_T)}{d^3p} \prec \int d^3r [f_q(p_T/n,\phi)]^n$$
$$\approx \Omega(p_T/n,\phi)\, [\bar{f}(p_T/n,\phi)]^n$$

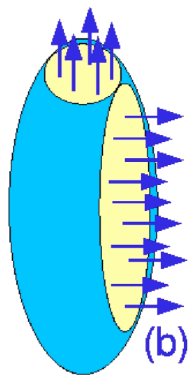


$v_2/n \rightarrow$ universal

If both mean $\bar{\text{f}}$ and Ω have small elliptic anisotropies

$$\bar{f}\,(p_T,\phi) = \bar{f}_0\,(p_T)[1+2\,\mathrm{v}_2^{(\bar{f})}\cos\,(2\phi)]$$

$$\Omega\,(p_T,\phi) = \Omega_0(p_T)[1+2\,\mathrm{v}_2^{(\Omega)}\cos\,(2\phi)]$$



quark-number scaling violated

The anisotropy for the spectra obtained from <cos(2φ)> :

$$\mathrm{v}_2^{(H)}(p_T) = \mathrm{v}_2^{(\Omega)}(p_T/n) + n\,\mathrm{v}_2^{(\bar{f})}(p_T/n)$$

Quark number scaling violated if effective emission volume depends on φ

Blast wave parametrization

- Hydro parametrization for the phase-space density at kinetic freeze-out

- Boost invariance with transverse anisotropic collective flow field

Schnedermann et al, PRC 48 ('93)
Huovinen et al, PLB 503 ('01)

Emission probability for n quarks in blast wave with azimuthally symmetric shell

$$\frac{dN}{d^3p} = \int d\phi_u d\eta \cosh\eta\, \rho_s(\phi_u) \exp[\{-\cosh\eta\, m_T \sqrt{1+u_T(\phi_u)} + u_T(\phi_u) p_T \cos(\phi-\phi_u) + n\mu(\phi_u)\}/T(\phi_u)\}$$

Anisotropies is added via: ρ_s, μ, T, u_T

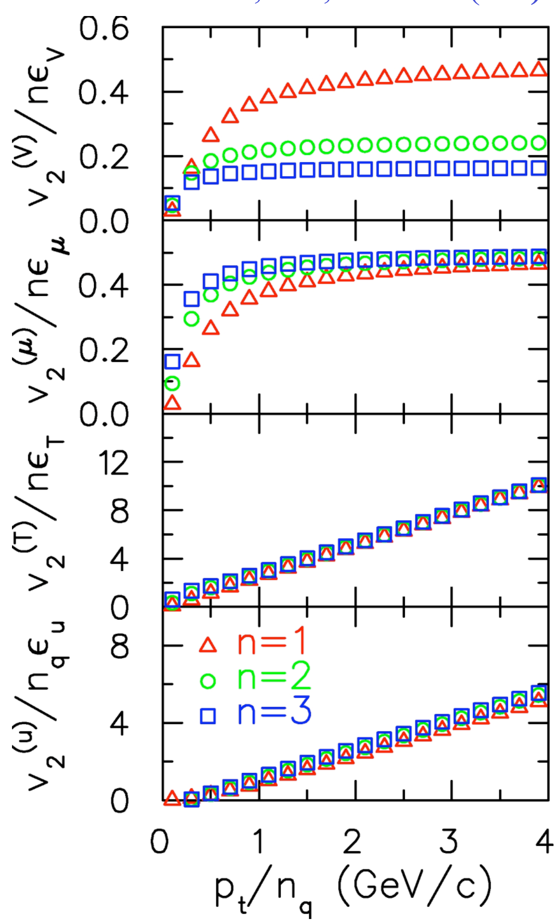
$$\rho_s(\phi_u) = (1+\varepsilon_{\rm v} \cos 2\phi_u)\rho_0$$

$$\exp[\mu(\phi_u)/T(\phi_u)] = (1+\varepsilon_\mu \cos 2\phi_u)\exp(\mu/T)$$

$$T(\phi_u) = (1+\varepsilon_{\rm T} \cos 2\phi_u)T$$

$$u_T(\phi_u) = (1+\varepsilon_{\rm u} \cos 2\phi_u)u_T$$

S. Pratt, S.P, PRC 71 ('05)

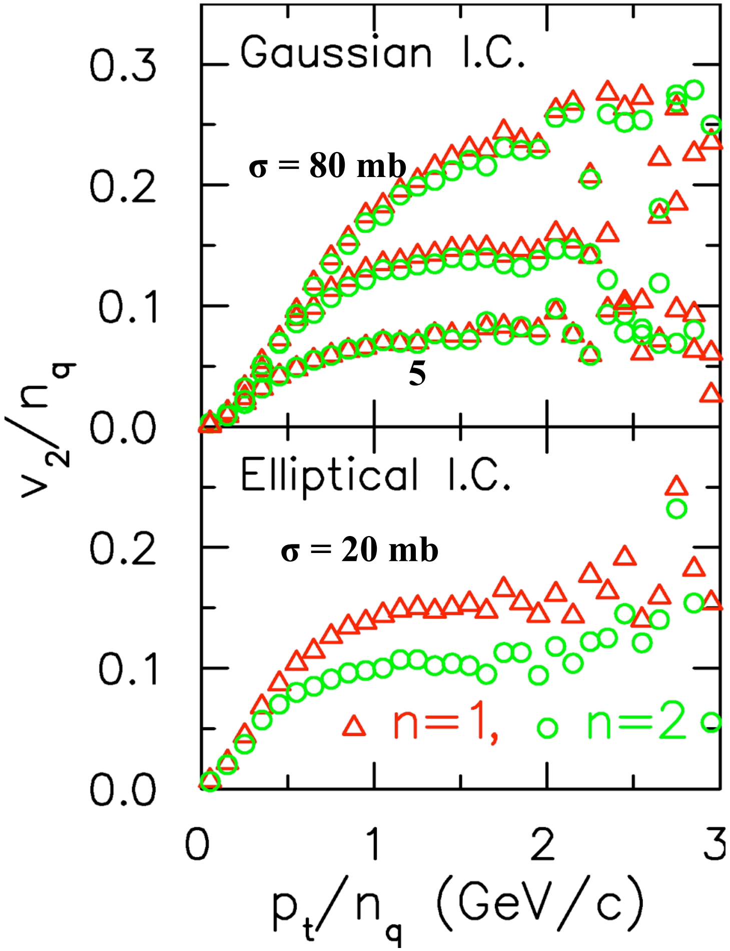


Microscopic simulation

GROMIT: Cheng, Pratt, Csizmadia, …. PRC 65 ('02) 024901

- Boltzmann description provide complicated final phase-space density
- Constituent quarks initially thermalized
- Quarks collide with simple s-wave cross section of σ = 5, 20, 80 mb
- Initial transverse density distributions: gaussian and elliptic

Quark number scaling violated for elliptic quark density profile

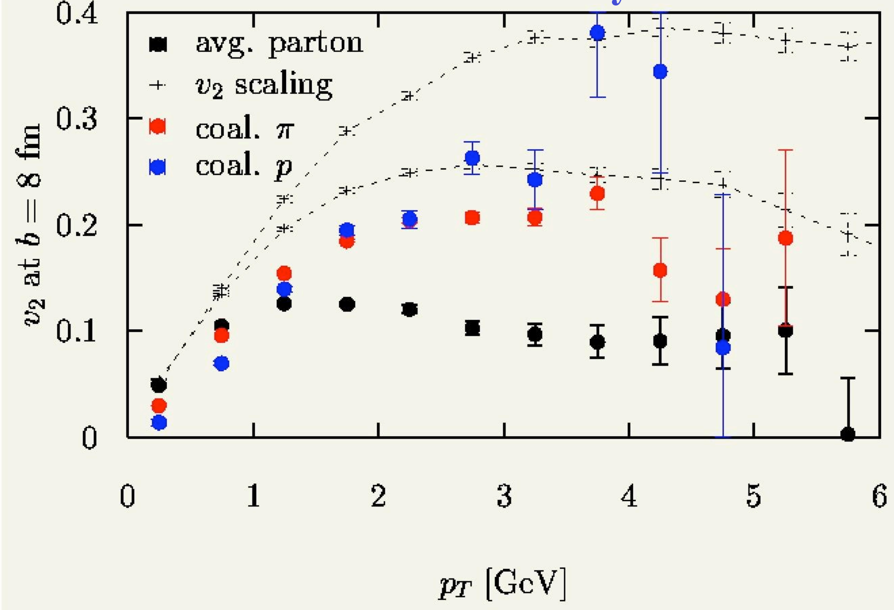


Quark scaling violation in MPC

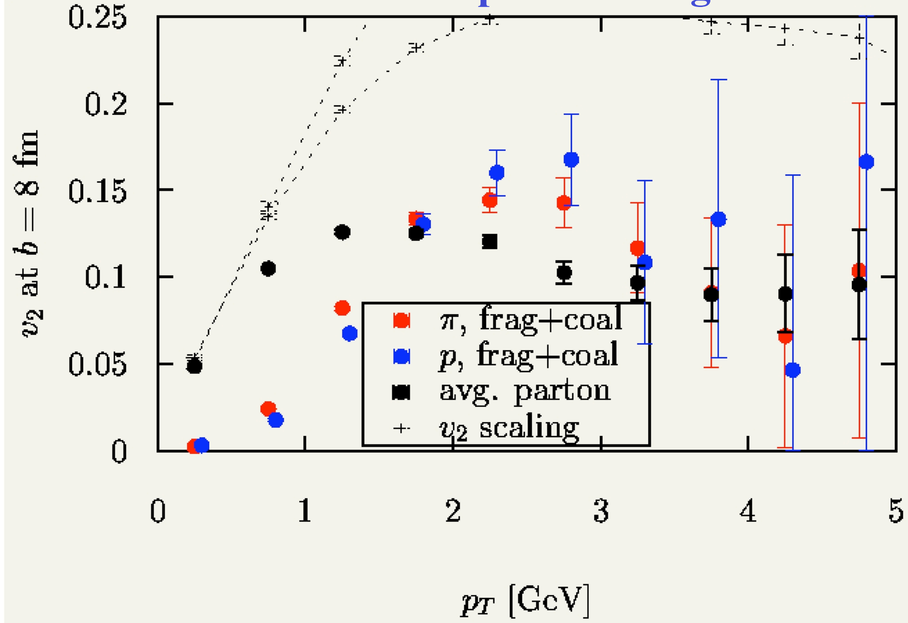
Au+Au @ 200A GeV, b= 8 fm, σ_{gg} = 10 mb

Molnar, J.Phys G30 ('04)

Coalescence only



Coalescence + independent fragmentation



v_2 for π(p) reduced by 20(40)% relative to v_2 scaling law

Fragmentation quench spectra and smears out anisotropy

→ v_2 scaling further violated

➢ parton $v_2(p_T)$ extracted from hadron v_2 will be underpredicted

Summary

- ❖ Hadronization of dense phase of partons can be described by recombination
- ❖ For (semi)central Au+Au collision at RHIC, hadron production is dominated via recombination at intermediate $1.5 < p_T < 4\text{-}6$ GeV
- ❖ Simple recombination formalism explains, large baryon/meson ratio, elliptic flow systematic for various hadron species & elliptic flow scaling with quark number
- ❖ Quark number scaling can be significantly affected by anisotropy in emission volume, emission profile, space-time dynamics (x-p correlations) ……

Recombination challenges:

- ➢ **Without jet quenching recombination may not be seen**
- ➢ **Simple formula for rare process, current formalism not suited at low p_T**
- ➢ **No energy conservation, decreasing entropy ?**
- ➢ **Effects of hadron scattering, jet-like correlations…**
- ❑ **Space-time evolution not well understood in heavy ion collisions**
  $\longrightarrow$ **requires study in a consistent dynamical calculation** ☻

Charm elliptic flow

Tq=170 MeV, v_T = 0.5-0.65