

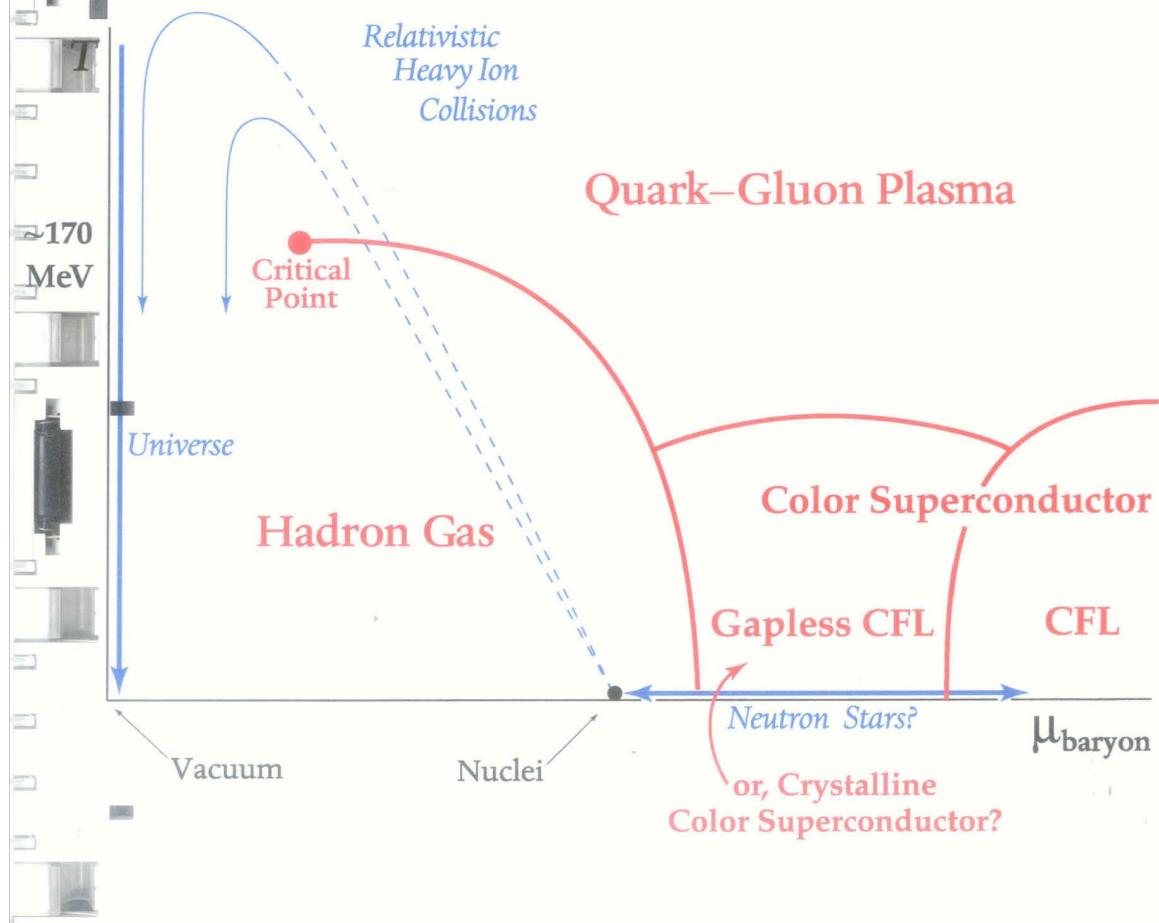
PROBING
THE
PHASES
OF
QCD MATTER

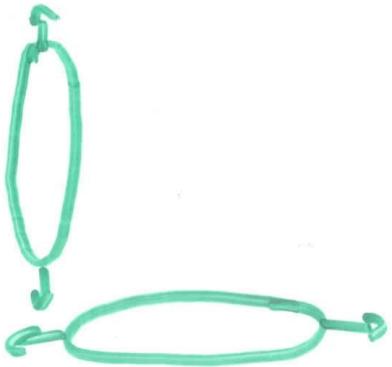
KRISHNA RAJAGOPAL

MIT & LBNL

Supercomputing RHIC Physics
TIFR, Mumbai, Dec 6, 2005

EXPLORING the PHASES of QCD





The interesting
"in-between regions"

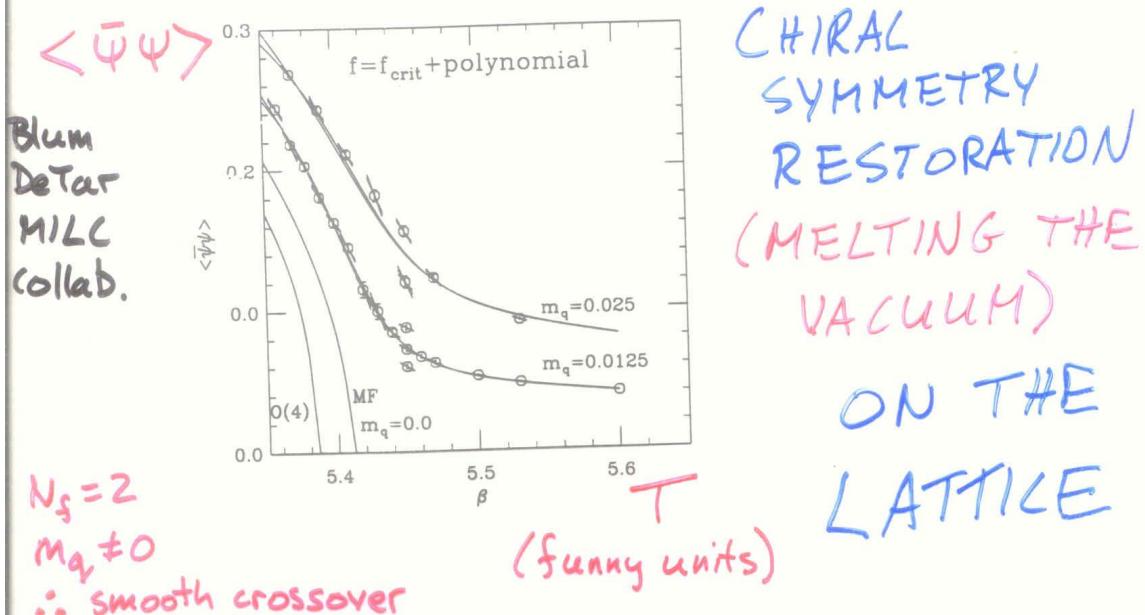
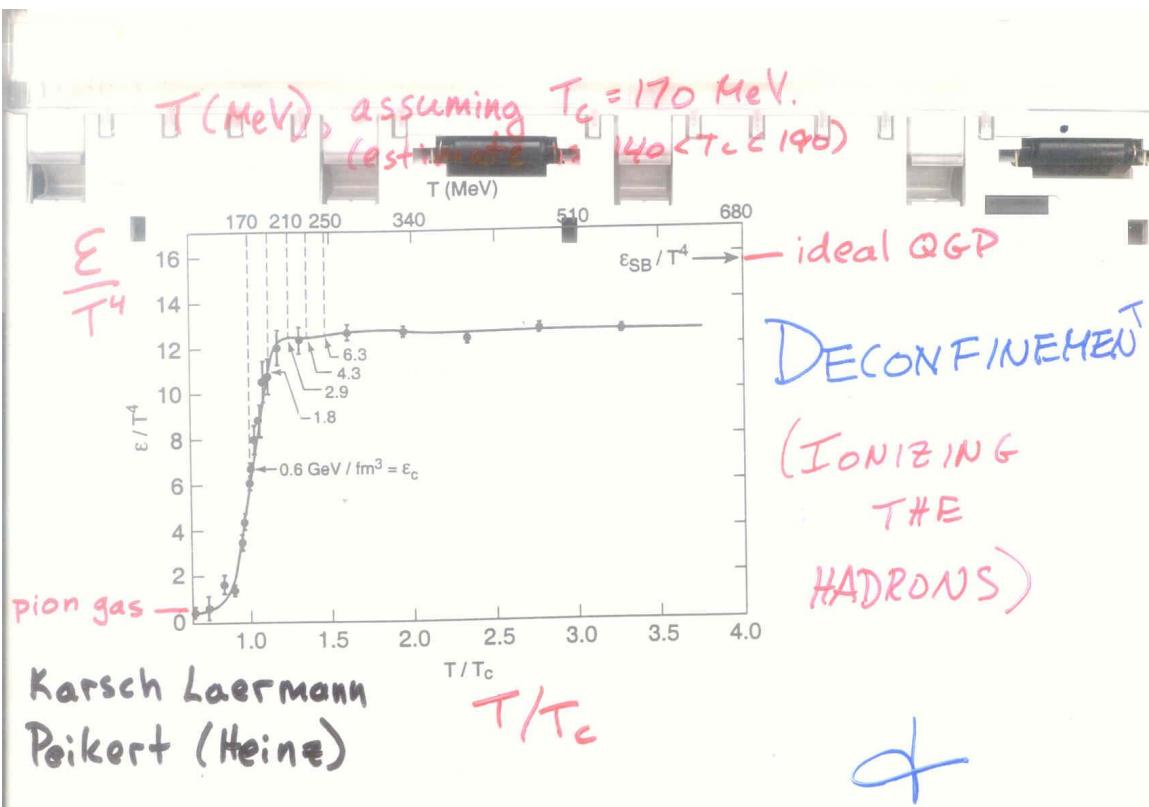


$T \neq 0 ; \mu = 0$

- vertical axis
 - we know a lot from lattice QCD. e.g. →
 - QCD describes a transition
- | | |
|--------------------------------------|--------------------------------------|
| FROM | TO |
| gas of hadrons | plasma of quarks
and gluons |
| with chiral symmetry
badly broken | with chiral sym.
almost restored. |
- $T_c \approx 175 \pm 15$ MeV
 - The transition is a smooth crossover, like ionization of a gas,
occurring in a narrow range of T .
IF $m_s \gtrsim \frac{1}{5} m_s^{\text{physical}}$, and so in Nature.

NB: In world with $m_u = m_d = m_s$,
crossover if $m_q \gtrsim \frac{1}{T_c} m_s^{\text{physical}}$

Bielefeld
Columbia



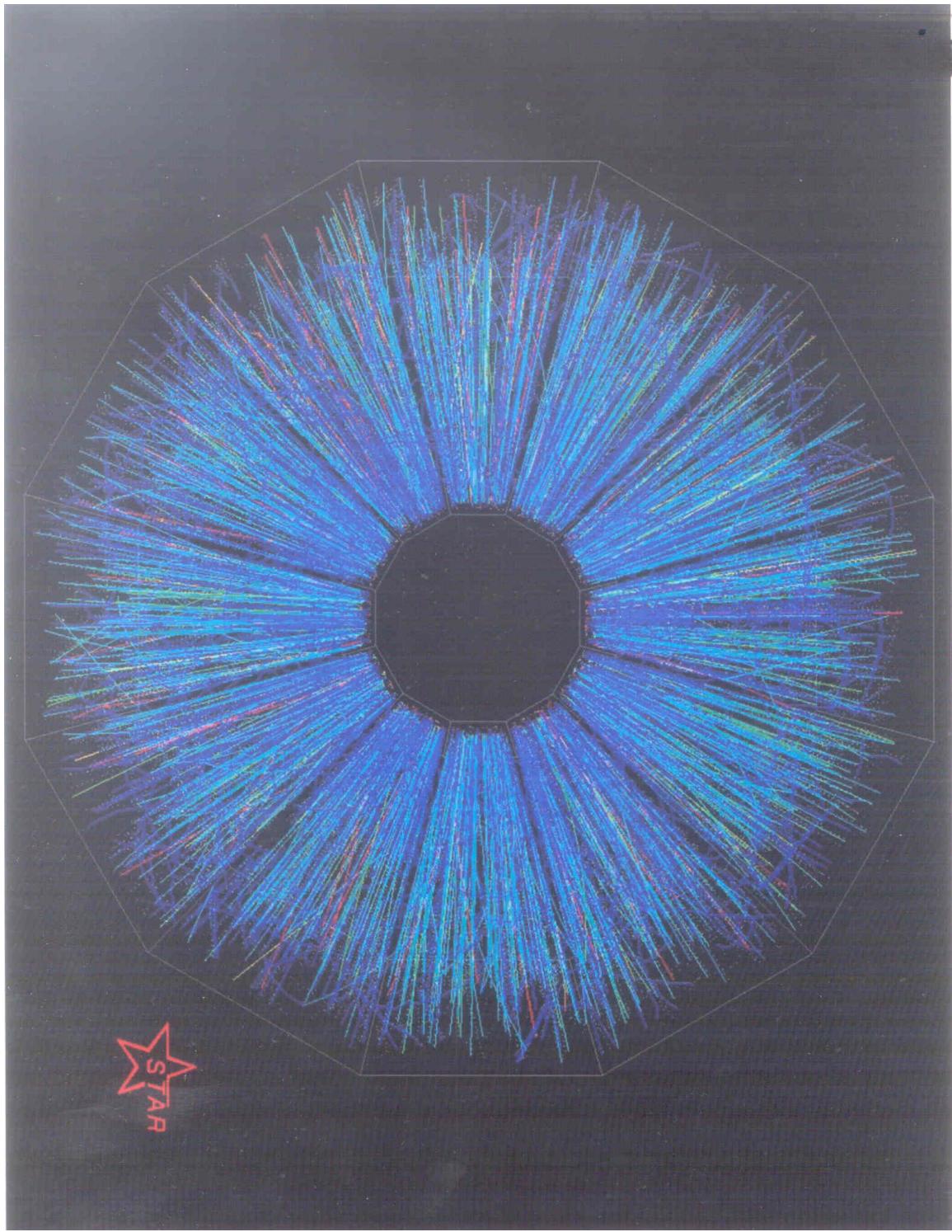
COSMOLOGICAL CONSEQUENCES?

Nobody has proposed an observable signature of the QCD transition in the early universe, if it is a crossover.

BUT: A sufficiently strongly first order transition messes up big bang nucleosynthesis, in a way inconsistent with data

SO: Cosmological "nonobservation" of a 1st order QCD transition is consistent with lattice QCD.

AND: If you want to probe the properties of hot quark matter, last seen microseconds after the big bang, you need experiments that recreate it.



HEAVY ION COLLISIONS: A BRIEF

INTRODUCTION

- A picture worth 1000 words →
- Sequence of events :
 - i) Collision leaves lots of gluons + quarks at mid-rapidity
 - ii) interaction → thermalization ??
 - must be tested experimentally
 - iii) if yes, hot fireball expands, cools, follows some track on phase diagram
 - iv) "Freezeout" (after which hadrons fly outwards in detector.) Much evidence from SPS + RHIC suggests final state at freezeout is expanding, \sim equilibrated, hadron gas.
- What does higher $\sqrt{s'}$ buy?
 - higher initial T , we hope
 - lower baryon#/entropy → lower μ
 - little change in freezeout T .

Chemical freeze-out in the T- μ_B plane

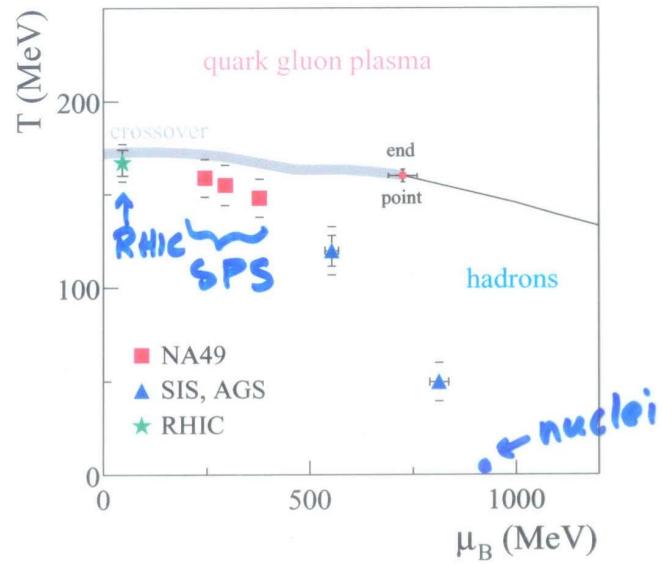
40 and 80 AGeV yields also fitted

	40 AGeV	80 AGeV	158 AGeV
T (MeV)	148 ± 2	155 ± 4	159 ± 2
μ_B (MeV)	377 ± 7	294 ± 15	244.5 ± 4.7
γ_S	0.75 ± 0.02	0.72 ± 0.03	0.82 ± 0.02
χ^2/NDF	14.8/4	10.4/4	23.5 / 11

$\sqrt{s} = 9$

fits by F. Becattini

- Freeze-out parameters on a (relatively) smooth curve
- Curve approaches phase boundary in the SPS energy range
- Even at RHIC, the parameters do not enter QGP-phase



Cross-over line from Z. Fodor, S.D. Katz hep-lat/0204029

EXPLORING QGP PROPERTIES

"Making QGP" is not a yes/no question:

No sharp boundary between hadrons, QGP.

Goal of RHIC: Create matter⁽¹⁾ that
is above the crossover⁽²⁾ and
study its properties.⁽³⁾

(1): RHIC data (on V_A) tell us interactions sufficient to yield \sim equilibrated matter, expanding collectively as a fluid, by a time $\sim 0.6 - 1$ fm.

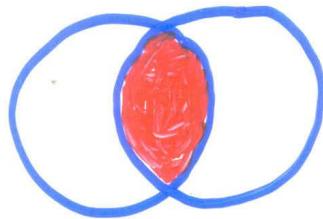
After that hydrodynamics (ideal hydro; zero mean free path; ideal liquid not ideal gas) describes "bulk" of particles ($P_T \lesssim 1 - 2$ GeV) well.

(2): RHIC data (dE_T/dy) tell us
 $E(1\text{ fm}) > 5 \text{ GeV/fm}^3 \Rightarrow$ ^{above}_{crossover}
So, on to (3).....^{NB}

TOWARD MEASURING SHEAR VISCOSITY

Elliptic flow indicates extent of
early equilibration.

Look at non-head-on collisions:

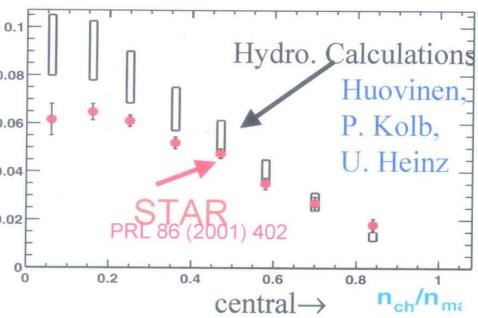


If just lots of p-p collisions followed by free streaming, then final state momenta uniformly distributed in azimuth angle ϕ .

If interaction \rightarrow equilibration \rightarrow pressure, pressure gradients \rightarrow collective flow.

If this happens early, before circularizes by free streaming, then nonzero $V_2 \sim \langle \cos 2\phi \rangle$.

v_2 predicted by hydrodynamics

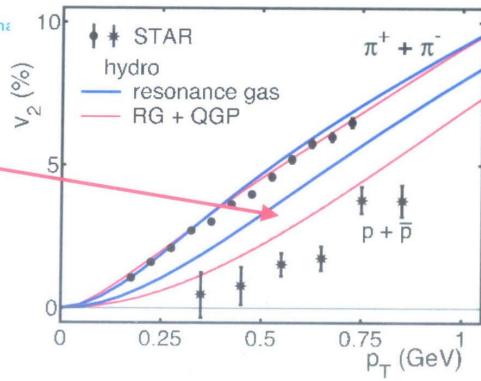


pressure buildup →
explosion
happens fast →
early equilibration !

Hydro can reproduce magnitude
of elliptic flow for π , p. BUT
must add QGP to hadronic EOS!!

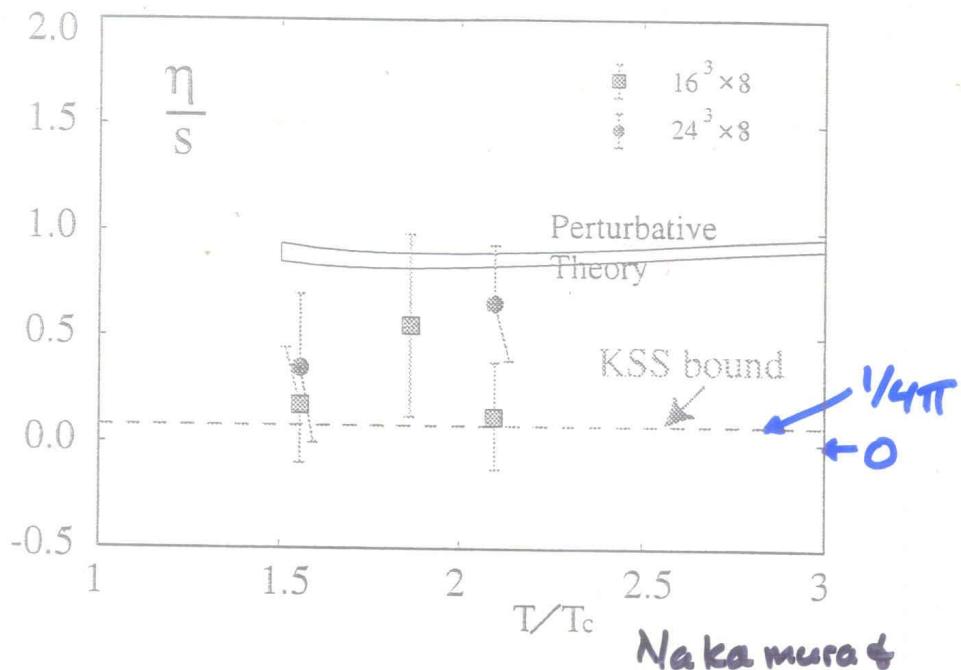
Similar conclusion reached by
CM Ko, et al., Kapusta, et al.,
Bleicher, et al., among others...

full by B. Jocak



- Ideal hydrodynamics based on assumption of local eqbm.
- Hydro never agreed with data before RHIC. (At SPS, $v_2^{\text{data}} \sim \frac{v_2^{\text{hydro}}}{2}$)
- At RHIC, hydro does good job of describing v_2 , spectra for $P_T < 2 \text{ GeV}$
- MEANS: "hydro works" by $t \sim 0.6 - 1 \text{ fm}$
Heinz Kolb
- Challenge to theory: how can Mrowczynski
 ~ equilibration occur so quickly?
 Rehan Sidi
 Strong interactions? Strong color Rometschke
 fields \rightarrow plasma instabilities? Strickland
 Arnold Moore
 Yaffe, ...
- MEANS: "small" shear viscosity η .
 Teaney: $\eta/s < \Theta(1)$
 cf water: $\eta/s > 10$
 CHALLENGES: Real extraction of η
 requires hydro calculations with $\eta \neq 0$.
 Muronga; Heinz Song Chaudhuri

η/s FROM LATTICE QCD ?



Nakamura &
Sakai

Not an ab initio lattice calculation;
doing that for transport coefficients
is hard. (See, e.g., Petreczky + Teaney)

Parametrize Minkowski space spectral
function with few-parameter
ansatz, fit those parameters to
lattice calculation of Euclidean
correlation function.

See also Gorkai + Greiter. They extract
electrical conductivity this way,
then use kinetic theory to $\rightarrow \eta/s \approx 0.2$

RHIC experiments seem to be
studying the properties of a
QUARK - GLUON LIQUID

In a gas, or a weakly-coupled plasma,

mean free paths \gg spacing between ($\sim \frac{1}{T}$
particles
for us)

→ quasiparticles with width \ll mass.

→ $\eta/s \gg 1$

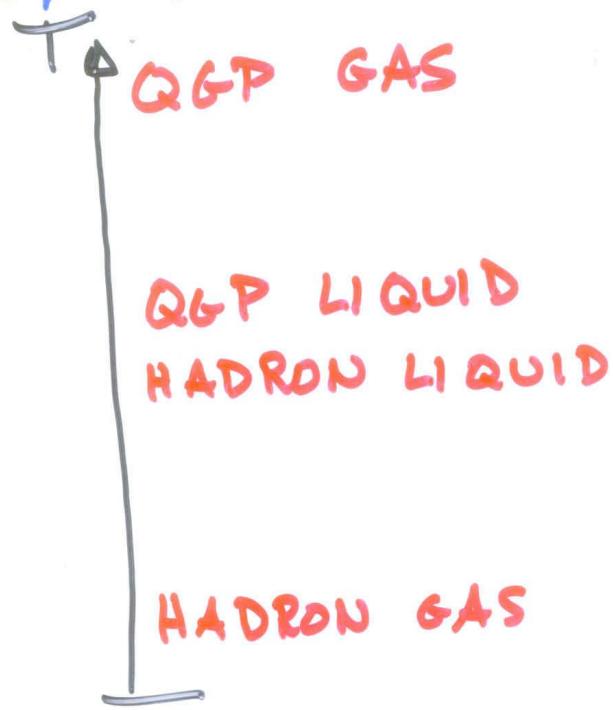
In a liquid, m.f.p. \lesssim spacing,
no well-defined quasiparticles.

NB: for T large enough, PQCD
works, $mfp \sim \frac{1}{\alpha^2 T} \rightarrow \frac{\eta}{s} \sim \frac{1}{\alpha^2} \rightarrow$
QGP not a liquid anymore.

Should we be surprised if/that
the QGP turns out to be liquid-like?

- 1) No. At $T \sim$ few T_c , coupling not small
- 2) But.... Lattice shows ϵ/T^4 reaches 80% of its value in an ideal-gas-QGP (ie noninteracting) already just above T_c . Doesn't this imply interactions are "just" a 20% correction???
3) $N=4$ SUSY QCD can teach us a lesson:
 - $\epsilon/T^4 = 75\%$ of its value in a Gubser-Klebanov-Tseytlin noninteracting SUSY-QGP
 - interactions very strong.
Policastro $\eta/s = \frac{1}{4\pi} \rightarrow$ m.f.p. \sim spacing
Son Sterinets η/s \sim per entropy than water
 - ideal hydro!
 - Teaney uses v_2 data to suggest η/s of real world QGP \sim as small.

So, a posteriori, (i.e after the data) it is not surprising to find a QGP liquid. In fact, given that the transition is a crossover, it probably has to be this way:



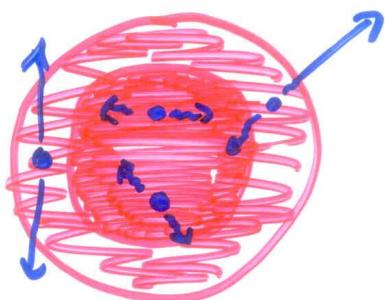
Also a posteriori (in this case, after the string theorists) we realize that $\frac{\Sigma}{T^4} = 90\%$ of noninteracting is closer to 75% (strong coupling) than to 1.

TOWARD MEASURING OPACITY, AND

PERHAPS η_{sound} , AND BOUNDING ϵ

"Jet quenching": RHIC data suggests that the rare high- P_T particles produced in initial hard scatterings are efficiently stopped \Rightarrow matter is "opaque".

Picture suggested is:

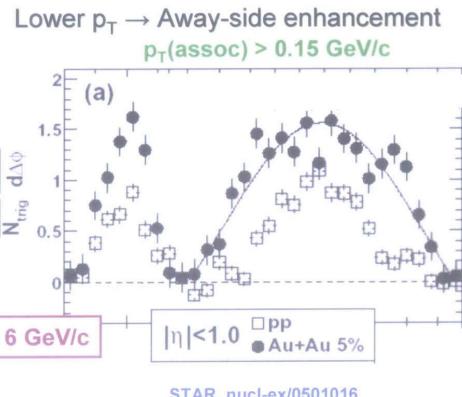
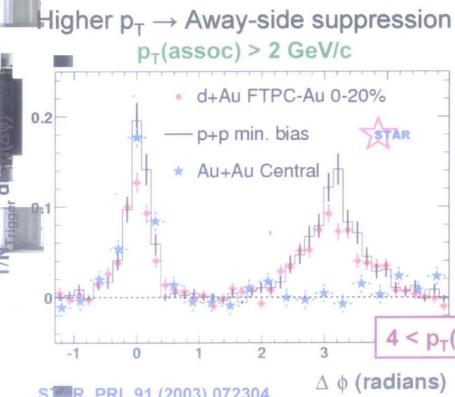
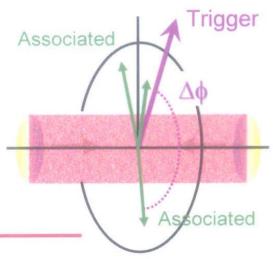


Ingoing, and interior, jets quenched.
Should see some back to back jets
at any P_T , and more and more
at higher P_T .

Evolution of $\Delta\phi$ correlations at RHIC

correlations

- "Trigger-associated" technique valuable for tagging jets in high-multiplicity environment (vs. jet-cone algorithms)
- Probes the jet's interaction with the QCD medium
- Provides stringent test of energy-loss models



QM 2005 Budapest

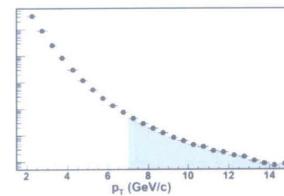
Dan Magestro, STAR

3

Emergence of dijets w/ increasing $p_T(\text{assoc})$

$\Delta\phi$ correlations (not background subtracted)

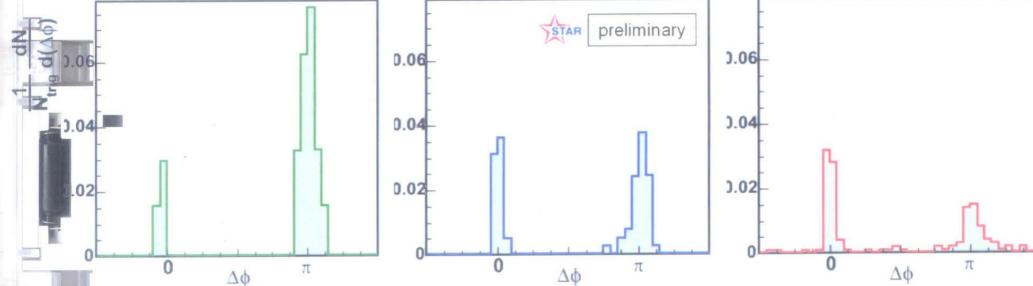
$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 7 \text{ GeV}/c$



d+Au

Au+Au, 20-40%

Au+Au, 0-5%



- Narrow peak emerges cleanly above vanishing background

v_{sound} ?

Some reports of a "Mach cone" on the away side, where the supersonic jet was heading before it was quenched.

If this persists as the data is further analyzed (via 3-particle correlations) then measure opening angle of the sonic boom $\rightarrow v_{\text{sound}}$.

CAN WE MEASURE (OR BOUND) ν AND DEMONSTRATE DECONFINEMENT?

$$\nu \sim \frac{\epsilon}{T^4} \sim \frac{s}{T^3} \sim \frac{s^4}{\epsilon^3}$$

We have a lower bound on $\epsilon(1\text{ fm})$.

Can we get upper bound on $T(1\text{ fm})$?

Challenge to exp. + th. (γ 's? J/ψ 's?)

We can estimate $s(1\text{ fm})$ from final state entropy, assuming equilibration before 1 fm. $s(1\text{ fm}) = 33 \pm 3 \text{ fm}^{-3}$ Muller KR
(Error theory-dominated, improvable)

Now that jets seen to be punching through,
can we use the whole suite of jet quenching observables to put an upper bound on $\epsilon(1\text{ fm})$?

Challenge to theory.

Motivation: if you could show $\epsilon(1\text{ fm}) < 7 \frac{\text{GeV}}{\text{fm}^3}$
you would have shown $\nu > 26 \pm 8$.

$T \neq 0 ; \mu \neq 0 ; \mu/T \text{ NOT LARGE}$

$\mu \neq 0 \rightarrow$ complex Euclidean action.

→ sign problem that makes difficulty
of standard Monte Carlo $\sim e^V$.

Nevertheless, we are learning about
this regime from lattice calculations
that rely on smallness of μ/T .

These methods may be used to
locate the...

CRITICAL POINT

A 2nd order point in the phase
diagram where a line of 1st
order transitions end. (Location
is sensitive to quark masses.
Moves leftward as masses \downarrow .)

LOCATING THE CRITICAL POINT

Range of estimates: $\lceil \text{NB}: \mu_B = 3\mu \rfloor$

$$\frac{\mu_B^{\text{Endpoint}}}{T_c(\mu=0)} \sim 1, \sim 2, \sim 3$$

Gavai Fodor Ejiri et al
Gupta Katz

Error estimates uncertain and clearly still large. Not at all like calculations of T_c . Yet.

Race between lattice QCD and experiment to locate the critical point....

HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

Increasing $\sqrt{s} \rightarrow$ decreasing μ_B .

ENERGY:	AGS	SPS	RHIC
$\mu_B^{\text{freezeout}}$:	550 MeV	30 MeV	

Vary \sqrt{s} , and hence μ_B , and look for enhancement (rise + then fall) of event-by-event fluctuations of:

i) mean P_T of low P_T pions Stephanov KR
Shuryak

ii) observables that are proxies for baryon number, like # of protons - \bar{P} .

Hatta Ikeda, Hatta Stephanov

Seen on lattice by Bielefeld-Swanson. \rightarrow

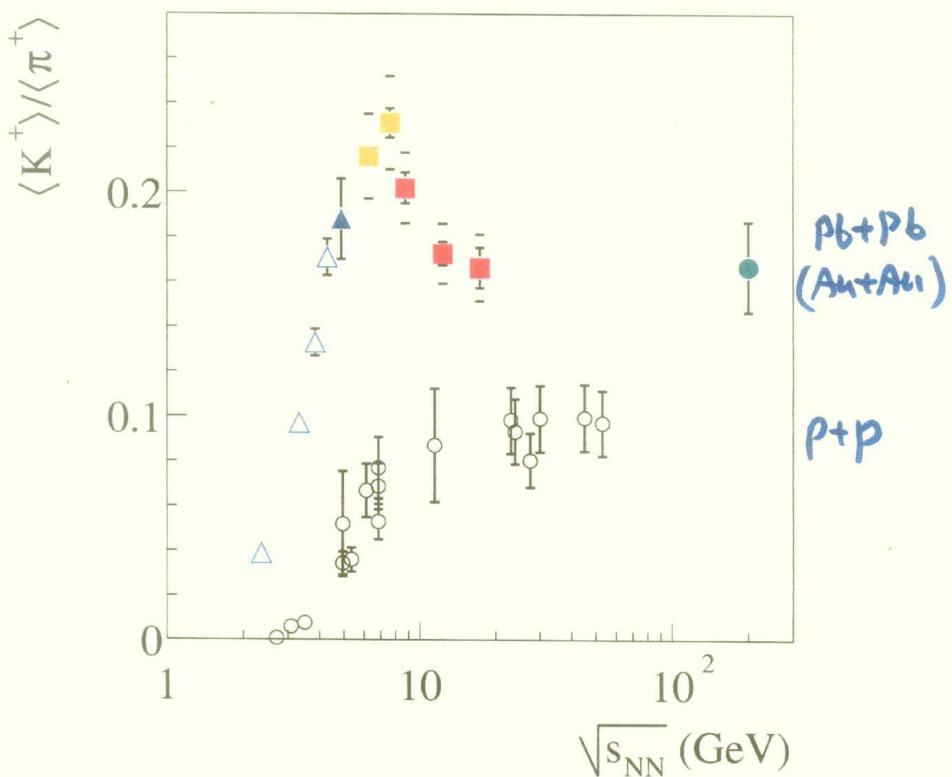
iii) particle ratios ???

- will better survive late time hadron gas than P_T -fluctuation

And....

Here is another quantity - not an e-by-e fluctuation - that varies nonmonotonically with \sqrt{s} ...

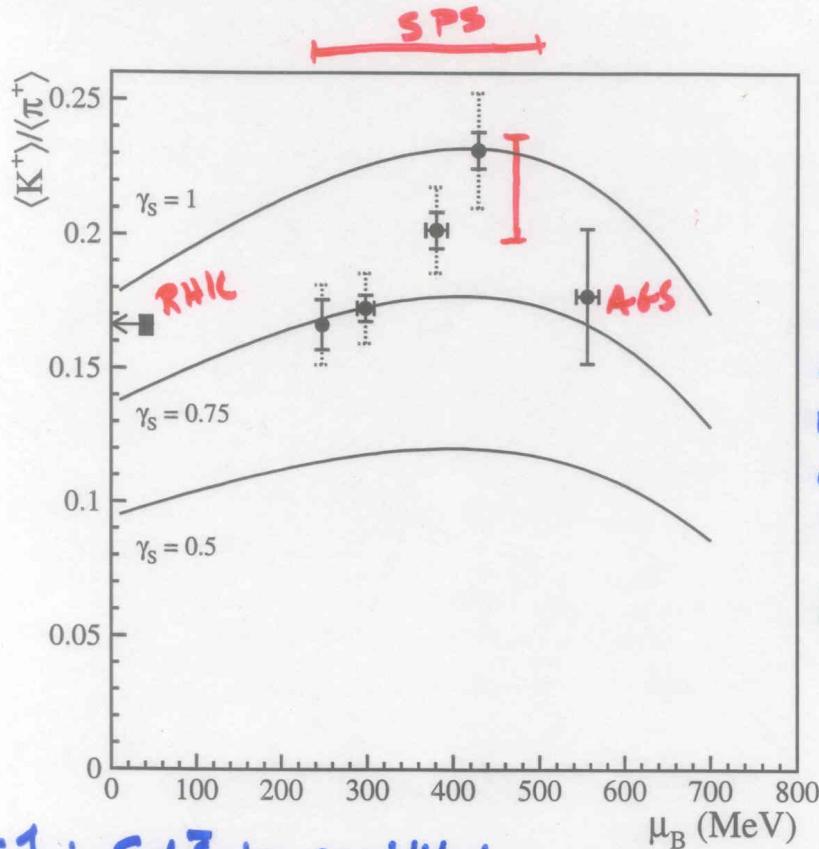
Hadron Multiplicities (\bar{s} -QUARK CARRIER)



THE HORN

Talk by M. Gospodicki, QM04, reporting
NA49 results.
(Horn also seen in $\frac{K^+ + \Lambda}{\pi^-}$.)

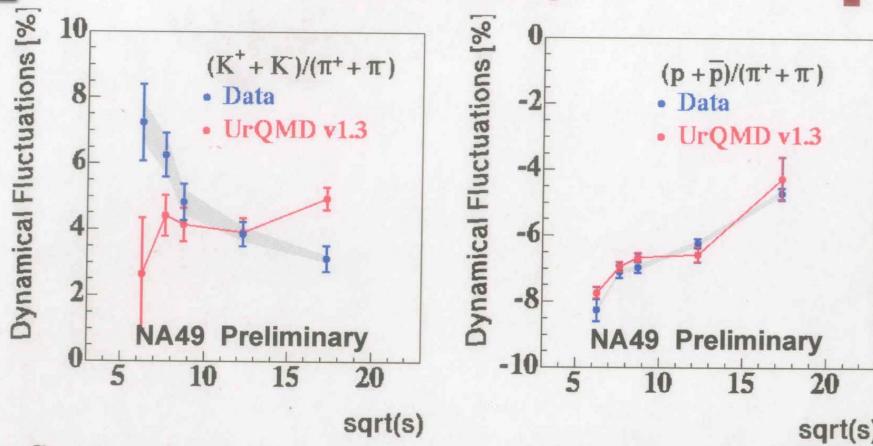
FIG. 13: Measured $\langle K^+ \rangle / \langle \pi^+ \rangle$ ratio as a function of the fitted baryon-chemical potential. The full square dot is a preliminary full phase space measurement in Au-Au collisions at $\sqrt{s}_{NN} = 200$ GeV [37] and the error is only statistical; the arrow on the left signifies that its associated baryon chemical potential is lower than that estimated at $\sqrt{s}_{NN} = 130$ GeV [11] used here. For the SPS energy points the statistical errors are indicated with solid lines, while the contribution of the common systematic error is shown as a dotted line. Also shown are the theoretical values for a hadron gas along the fitted chemical freeze-out curve shown in fig. 11, for different values of γ_S .



$\gamma_S = 1$: $S + \bar{3}$ in equilibrium

$\gamma_S = 0.75$: $S + \bar{3}$ is 75% of eqbm value

To explain "horn", need $\gamma_S = 1$ at horn and
 $\gamma_S \sim .7 - .8$ on either side of horn.



- K/π fluctuations increase towards lower beam energy
 - Significant enhancement over hadronic cascade model
- p/π fluctuations are negative
 - indicates a strong contribution from resonance decays

Intriguing...

Large e-by-e fluctuations at
 $M_B \sim 400 - 450$ MeV, in K/π .

Are the fluctuations dominated by
low $p_T \pi + K$?

Why no P/π fluctuations?

Majumder & Koch suggest this
points towards first order, not
critical point.

Are there p_T -fluctuations?

Cf: Fodor Katz $\rightarrow M_B^{\text{Endpoint}} \sim 360$ MeV $\rightarrow \sqrt{s} \sim 86$ GeV

Gavai Gupta $\rightarrow M_B^{\text{Endpoint}} \sim 180$ MeV $\rightarrow \sqrt{s} \sim 25$ GeV

Motivation for an energy scan at
low energies at RHIC.

Goal: Mark a ● on phase diagram.

HIGH DENSITY + LOW TEMPERATURE

Whereas at high T entropy wins
→ quark-gluon plasma with symmetries
of QCD Lagrangian manifest

At large μ with small T we find
quark matter with new patterns
of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity
:
- At large enough μ (to be defined
below) we have answers.
- At large but not so large μ , we have
a puzzle, and hints.
- How can we use astrophysical
observations of compact stars
to provide answers?

WHY COLOR SUPERCONDUCTIVITY?

Large $\mu \rightarrow$ quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom \rightarrow weak interactions between quarks at Fermi surface.

BUT any attractive interaction, no matter how weak, \rightarrow COOPER PAIRS ; $\langle q, q \rangle$

One gluon exchange (& instanton interaction) attractive in color 3.

(no need to resort to phonons; \therefore superconductivity more robust in QCD than in metals. Higher T_c/E_F .)

$\langle q, q \rangle$, ie Cooper pairs of quarks,
 \Rightarrow electric & color currents superconduct
- mass for photon & (some) gluons (?)
- Meissner effects. (magnetic & color magnetic fields excluded.)

Barrois; Boilin & Love

GAP AND T_c

Much work (that I will not review)

suggests that @ $\mu_q \sim 500$ MeV $\Gamma \sim 10 \times$ nuclear density

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ This is high T_c s.c.!

Two classes of methods \sim agree :

i) models normalized to $\mu=0$ physics

(Alford, KR, Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetje,)

ii) weak-coupling QCD calculations, valid

for $\mu \rightarrow \infty; g \rightarrow 0$. (Quantitatively, valid

for $g \lesssim 1$ which means $\mu \gtrsim 10^8$ MeV KR, Shuster)

$$\frac{\Delta}{\mu} \sim 256 \pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2} \frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

Schuster, Wilczek; Pisarski, Rischke; Wong, Miransky, Son
Shatkovy, Wijewardhana; Evans, Hsu, Schwetje;

Brown, Liu, Ren; Beane, Bedaque, Savage; KR, Shuster; Rischke, Wong;

$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering
via exchange of unscreened magnetic gluons:

$$\rightarrow x = \frac{\Delta}{\mu} \rightarrow 1 = g^2 \underbrace{\ln \frac{\Delta}{\mu}}_{BCS} \underbrace{\ln \frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

A LATTICE NJL CALCULATION

Hands & Walters, 2004

Prior to this work, "models normalized to $\mu=0$ physics" typically meant:

- Choose an NJL model, fitting its parameters to $\mu=0$ physics
- Make a mean field approximation or some other uncontrolled truncation
- Calculate Δ at $\mu \neq 0$

Hands & Walters do a lattice calculation of an NJL model at $\mu \neq 0$.

They find a diquark condensate and a gap in the quasiparticle dispersion relation: $\Delta \approx 60$ MeV

To me, this is now the most reliable estimate of Δ at realistic density.
(consistent with other estimates. Still in a way)

CFL

In cold quark matter, quarks near their Fermi surfaces pair
→ color superconductivity

Pattern of pairing:

$$\langle \Psi_a^\alpha (\gamma_5 \Psi_b^\beta) \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1} \epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2} \epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3} \epsilon_{ab3}$$

color

flavor Lorentz scalar

- antisymmetry in color + Dirac indices energetically favored; flavor antisym. forced by Pauli
- If density great enough that M_s can be neglected, $\Delta_1 = \Delta_2 \approx \Delta_3$
- All q quarks pair, maximizing condensation energy; leaves largest symmetry unbroken
- Demonstrated rigorously at asymptotic density.
- Unbroken symmetries all are color+flavor

	ru	gd	bs	rd	gu	bu	rs	gs	bd
ru		$-\Delta_3$	$-\Delta_2$						
gd	$-\Delta_3$		$-\Delta_1$						
bs	$-\Delta_2$	$-\Delta_1$							
rd					Δ_3				
gu				Δ_3					
bu							Δ_2		
rs						Δ_2			
gs							Δ_1		
bd								Δ_1	

Define

$$\tilde{Q} = \begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix} \text{ for } d + \begin{pmatrix} -2/3 \\ 1/3 \\ 1/3 \end{pmatrix} \text{ for } g$$

and check $\tilde{Q} = 0$ for every pair in the condensate.

⇒ One linear combination of photon & gluon does not get "Meissnered".

$$U(1)_{EM} \times SU(3)_{color} \rightarrow U(1)_{\tilde{Q}}$$

COLOR-FLAVOR LOCKED QUARK MATTER

- occurs for $\mu \rightarrow \infty$, and at any μ if $m_s = m_u = m_d$
- all 9 quarks pair and are gapped
- superfluid
- chiral symmetry spontaneously broken, by a new mechanism (CFL)
⇒ "pions" and "kaons" lightest excitations
 - massless if $m_s = m_u = m_d = 0$
 - \sim few MeV mass ($\ll \Delta$) for real $m_{s,u,d}$.
 - "K₀" may condense
- Unbroken gauged U(1) \rightarrow massless photon
- As long as $T <$ meson masses \sim few MeV:
 - Transparent insulator (neutral without electrons)
- index of refraction and reflection/refractive coefficients known
 - Very small specific heat, neutrino emissivity, viscosity. Good thermal conductor
- All these properties, and more, rigorously calculable in $\mu \rightarrow \infty, g \rightarrow 0$ limit. Chiral symmetry breaking and all its consequences understood at high density. ✓ density depend.
- Occurs in nature wherever $\mu > m_s^2 / 2\Delta$.
- What are the properties of quark matter at lower density ??? ..

WHAT CAN BE CALCULATED?

from QCD from first principles?

- At asymptotic densities, answer is "everything"; more than in any other circumstance in QCD.
 - in the CFL phase, there are no unresolved nonperturbative ambiguities: no gapless fermions; no massless gluons. No IR difficulties.
 - calculation of Δ is nonperturbative, but controlled by smallness of g .
 - analogues of confinement and chiral symmetry breaking are calculable at weak coupling.

- At potentially accessible densities, g not small. Means Δ cannot be calculated precisely (barring a major lattice QCD breakthrough.)

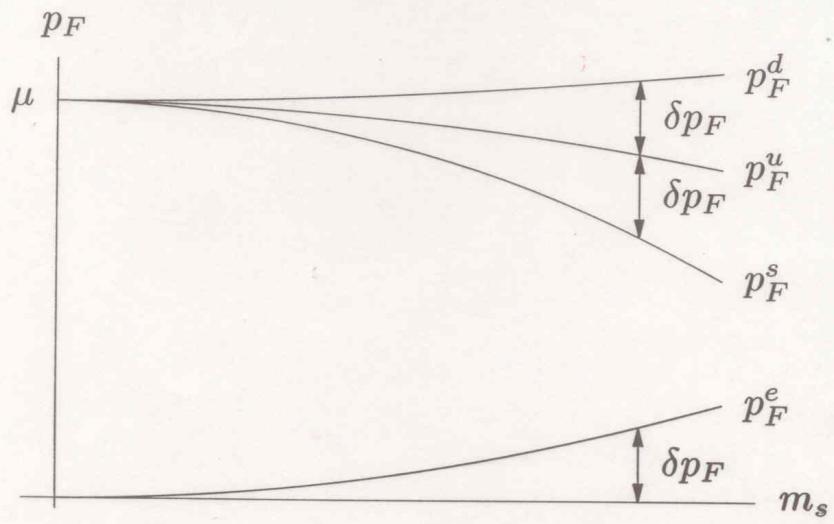
BUT: if you take Δ as given (ie treat as a parameter whose value known at order of magnitude level) then many physical properties calculable in terms of Δ .

Eg: specific heat, thermal conductivity, index of refraction, neutrino opacity, neutrino emissivity, shear viscosity, bulk viscosity,

Many of these described within an effective field theory for the Goldstone bosons, whose parameters are determined by Δ .

INTERMEDIATE DENSITY QUARK MATTER

- M_s important
- For orientation, consider noninteracting quarks, $M_u = M_d = 0$, $M_s \neq 0$, impose electrical neutrality and weak eqbm:



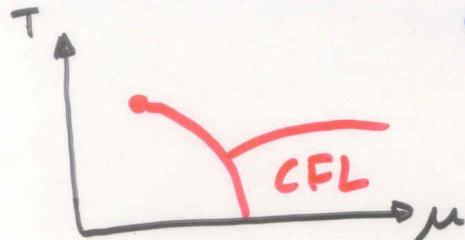
- In noninteracting quark matter, $\delta p_F \approx \frac{m_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\frac{m_s^2}{4\mu} > \Delta ??$
- Also, when CFL "breaks", no residual $\langle \bar{u}d \rangle$ pairing either. Alford, KR

WHAT REPLACES CFL, AT LOWER μ ?

We don't yet know.....

We do know:

- CFL pairing is unstable once $\mu < \frac{M_s^2}{2\Delta}$
(Alford Kouvaris KR)
and stable for larger μ .
- \therefore If Δ large enough & M_s not too large, CFL quark matter is stable all the way down to transition to nuclear matter.



$$\text{eg: } M_s = 300 \text{ MeV}$$

$$\Delta > 125 \text{ MeV}$$

OR

$$M_s = 200 \text{ MeV}$$

$$\Delta > 55 \text{ MeV}$$

QUESTIONS:

What if less symmetrically paired quark matter intervenes? Ie, what are properties of quark matter with $\mu < M_s^2/2\Delta$?

What are astrophysical consequences if neutron stars have CFL cores?

LESS SYMMETRICALLY PAIRED Q:M.

Gapless CFL Phase?

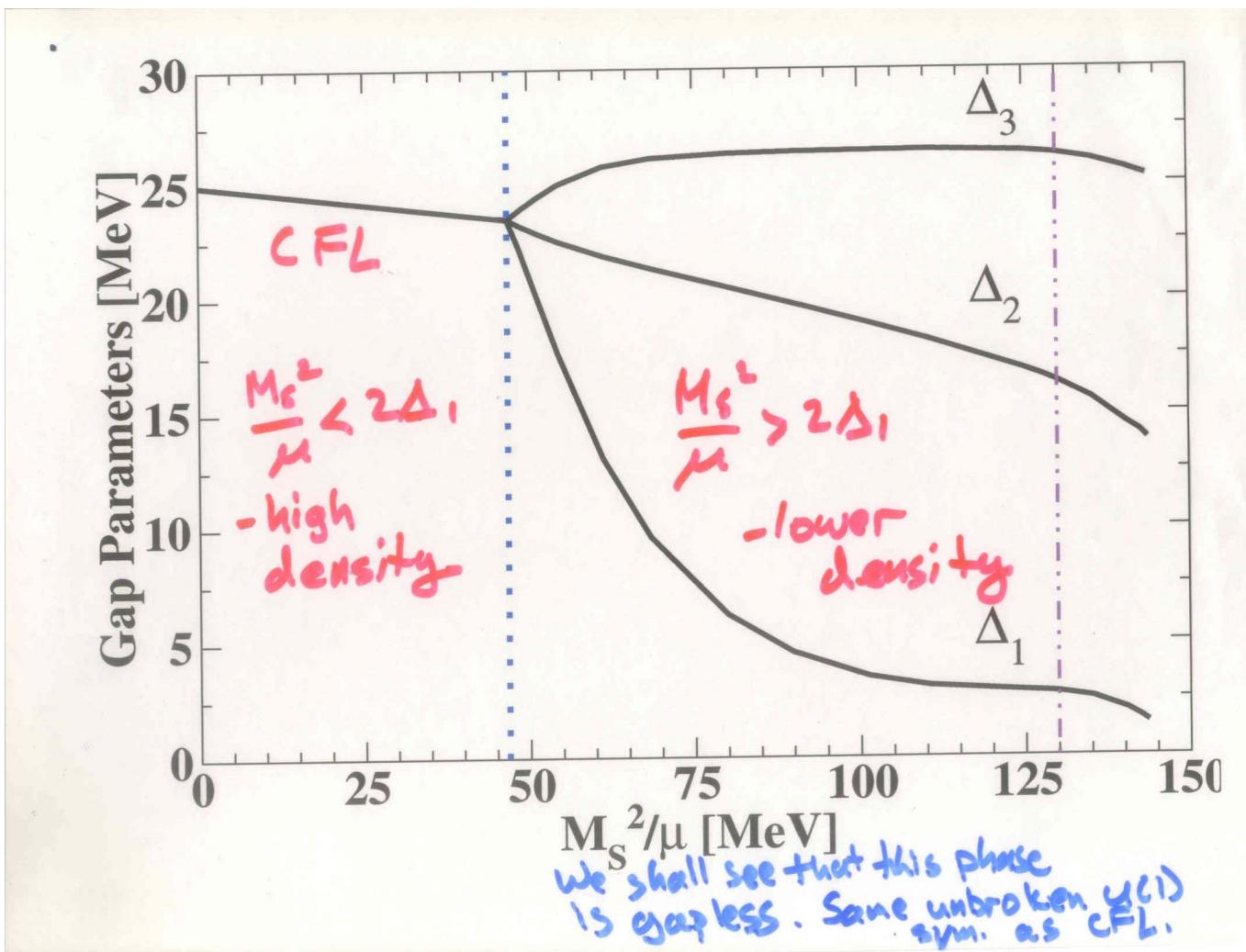
- 2nd highest density phase within a spatially uniform ansatz
- nice distinctive astrophysical signature (Alford Jotwani Kouvaris Kundu KR)
- unstable to currents \rightarrow inhomogeneity (Huang Shovkry; Casalbuoni et al.; Giannakis Ren; Fukushima ;...)

Crystalline Color Superconductivity?

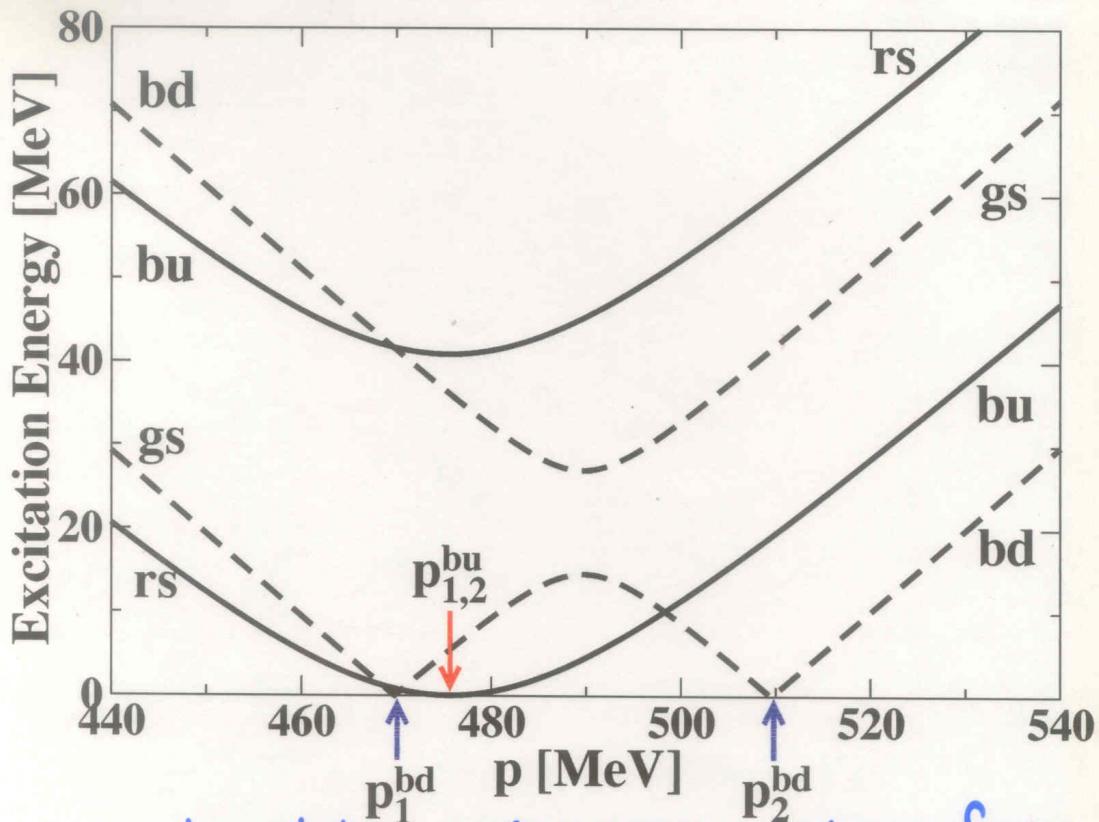
- may be the answer, but:
- until recently, analyzed only in 2-flavor setting, without imposing neutrality
- potential for astrophysical signatures, (Alford Bowers KR) but not yet analyzed sufficiently to say how distinctive

THE GAPLESS CFL PHASE

- All 9 quarks still pair, with same partners as before. BUT:
 - $\Delta_1 < \Delta_2 < \Delta_3$
 - there are shells in momentum space containing unpaired quarks.
Eg: for some momenta, find
bd quarks with no gs
quarks with which to pair.
 - bd-gs pairing disrupted;
Fermi surfaces split.
 - Nonzero density of electrons
needed to maintain neutrality
 - $CFL \rightarrow gCFL$ transition is
an insulator \rightarrow conductor
transition, 2nd order at $T=0$
and crossover at $T>0$.



gCFL DISPERSION RELATIONS



- Conventional linear dispersion relations for gapless fermions at two momenta (with unpaired quarks between those momenta)
- unconventional "grazing" dispersion relation, practically quadratic, at a third momentum
 - characteristic of and unique to gCFL
 - due to the way electric neutrality is maintained, not due to any fine tuning
 - and, has consequences....

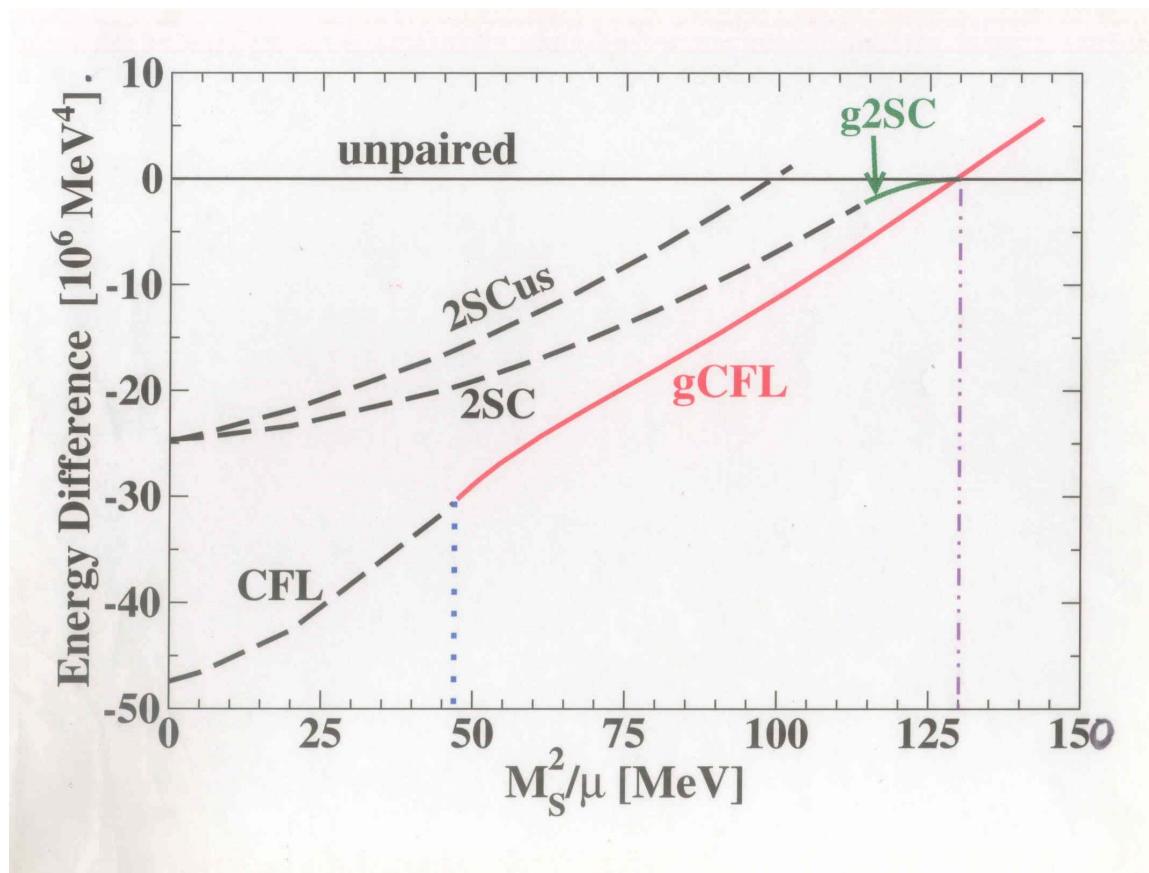
gCFL INSTABILITY

Before astrophysical observations have had a chance to rise to the challenge of ruling out the presence of gCFL quark matter, theorists have done so first:

The gCFL phase has a "magnetic instability." (Huang Shovkovy; Casalbuoni et al
Giannakis Ren; Fukushima)
⇒ It can lower its energy by turning on currents.

BUT: ground state of a system has ^{net} no current. (Bloch's theorem)
SO: the instability tells us there must be some lower energy phase, but does not tell us what.

- Not ZSC or variants. Not a mixed phase
- Currents hint we should revisit....



↓ ↓ ↓ ↓ ↓
INSTABILITY

Challenge : Find a phase
with lower energy
than g CFL.

ENUMERATION OF (SOME) POSSIBILITIES

- For large enough Δ_{CFL} , phase diagram is just



Furthermore, augmenting CFL phase by rotating CFL condensate in K^0 -direction delays the onset of gaplessness (and the attendant puzzle) by a factor of $4/3$ Kryjeuski Schäfer; Kryjeuski Yamada

1.21 Forbes

- for smaller Δ_{CFL} (preceding plot was for $\Delta_{\text{CFL}} = 25 \text{ MeV}$) gCFL beats 2SC, mixed phases, ..., but is unstable. To what?

- 2SC phase? ($\Delta_3 \neq 0$; $\Delta_1 = \Delta_2 = 0$)

- 4 of 9 quarks pair. Others may pair with small (\sim keV)
spin-one gaps \rightarrow interesting consequences (Schmitt Shovkovy; Aguilera et al; ...)

- Can arise on phase diagram if Δ_{CFL} not too large and not too small. (Rüster et al)
- generally get gCFL region also, so this is not a full answer

- Phase Coexistence?

- always possible if you are willing to have coexisting charged and colored phases. (Bedaque Coldas Rupak) (applied here; Forbesch)
- Coexistence of charged but colorless phases can resolve "g2SC puzzle" in 2-flavor QCD (Reddy Rupak) but not gCFL puzzle in 3-flavor QCD (Alford Kouvaris KR)

NEWS

Nice, Slepnev, Stairs, Löhmer, Jessner, Kramer, Cordes
astro-ph/0508050 (appeared this week)

A pulsar (named PSR J0751+1807)
with mass:

$$M = 2.1 \pm 0.2 \text{ solar masses}$$

$$1.6 < M < 2.5 \text{ at 95\% confidence}$$

- A 3.5 ms pulsar in a 6.3 hr orbit around a 0.19 solar mass white dwarf.
- Over 10 years of observation, the 6.3 hr orbit has slowed by $19.6 \pm 2.5 \mu\text{s}$!
due to gravitational wave emission.
- Shapiro delay measured. No accretion, mass transfer, or X-ray emission.
- i.e. this is gold-plated, as clean as the best previous mass measurements. And, the error bar will shrink like $(1/\text{duration of observation})^{2.5}$
- cf: $M_{NS} \lesssim 2.3 M_\odot$ (stiff nuclear E.O.S.)
- cf: M_{NS} with quark core $< (1.9 - 2.0) M_\odot$

Alford Baby,
Paris Rold,

$N_f = 3$: COLOR-FLAVOR LOCKING

Condensate pairs quarks of all colors + flavors:

Alfred
KR
Wilczek

$$\langle \bar{u}d - d\bar{u} - \bar{u}d + d\bar{u} + \bar{d}s - \bar{s}d - \bar{s}d + s\bar{d} + \bar{s}u - \bar{u}s - \bar{s}u + u\bar{s},$$
$$\rangle \neq 0$$

Locks $SU(3)_{\text{color}}$ to $SU(3)_L$.

i.e. $SU(3)_{\text{color}+L}$ is a symmetry.

Similarly, condensate of R-quarks locks $SU(3)_{\text{color}}$ to $SU(3)_R$.

Result:

$SU(3)_{\text{color}+L+R}$ unbroken. \leftrightarrow Use this chiral symmetry broken. "EM" + "isospin" to

$U(1)_Q$ unbroken. \leftrightarrow classify excitations

All other gauge symmetries broken.
 $U(1)_B$ broken. \therefore superfluid.

A TRANSPARENT INSULATOR

What can the \tilde{Q} -photon scatter off? No electrons.... And, ...

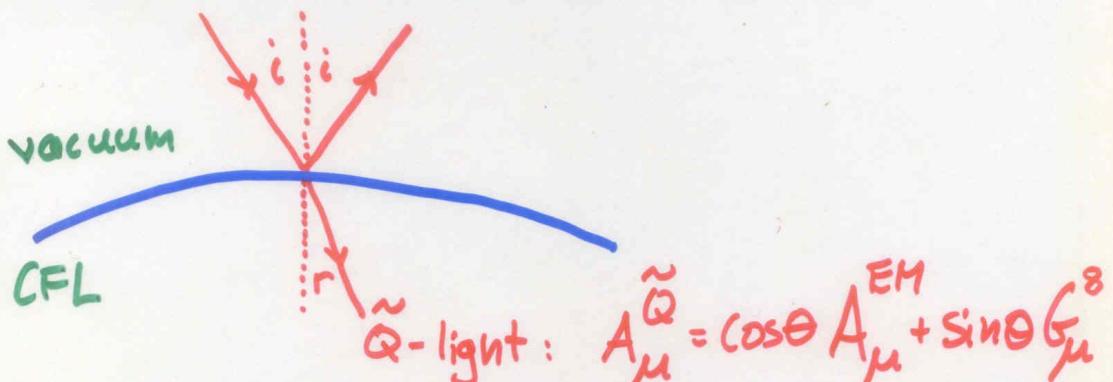
- the CFL condensate itself is \tilde{Q} -neutral.
- once you include non-zero quark masses, all excitations with $\tilde{Q} \neq 0$ are massive.
- \therefore for $T \ll M_{\text{lightest excitation}}$,
with $\tilde{Q} \neq 0$
 \uparrow
likely a kaon

the CFL phase is transparent to the \tilde{Q} -photon. It is a \tilde{Q} -insulator, with some index of refraction $n_{\text{CFL}} \neq 1$.

ILLUMINATING CFL QUARK MATTER

Manuel, KR

Suppose (just for fun) you had a quark star in CFL phase, and shone light on it:



$$n_{\text{CFL}} = 1 + \frac{4\alpha}{9\pi} \frac{\mu^2}{\delta^2} \cos^2\theta \quad (\text{Litim, Manuel})$$

Find: $\frac{\sin i}{\sin r} = n_{\text{CFL}}$

Explicit expressions in terms of n, θ
for reflection & refraction coefficients
for light of either possible
polarization.

It's fun to think of 10 km
lenses in space, but more likely
applicable version of this is
in the static limit:

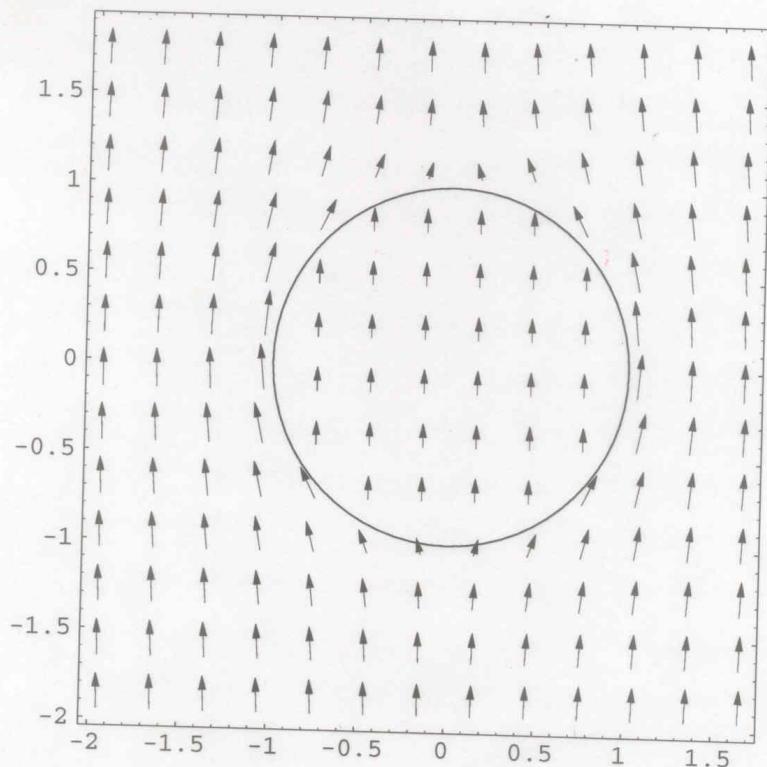
Suppose core of a neutron
star is CFL. How does it respond
to the large static \vec{B} it finds
itself in?

ANSWER: (Alford, Berges, KR)
† (found via magnetic b.c.'s ...)

Partial Meissner effect...

Magnetic field solution (sharp boundary)

Stitching together the inside and outside solutions, we find^{*} the solution. For $\cos \alpha_0 = 0.5$



In the real world α_0 is small, so the field is mostly converted into \tilde{Q} flux by the supercurrents and monopoles, and penetrates the interior. Only a weak field is excluded.

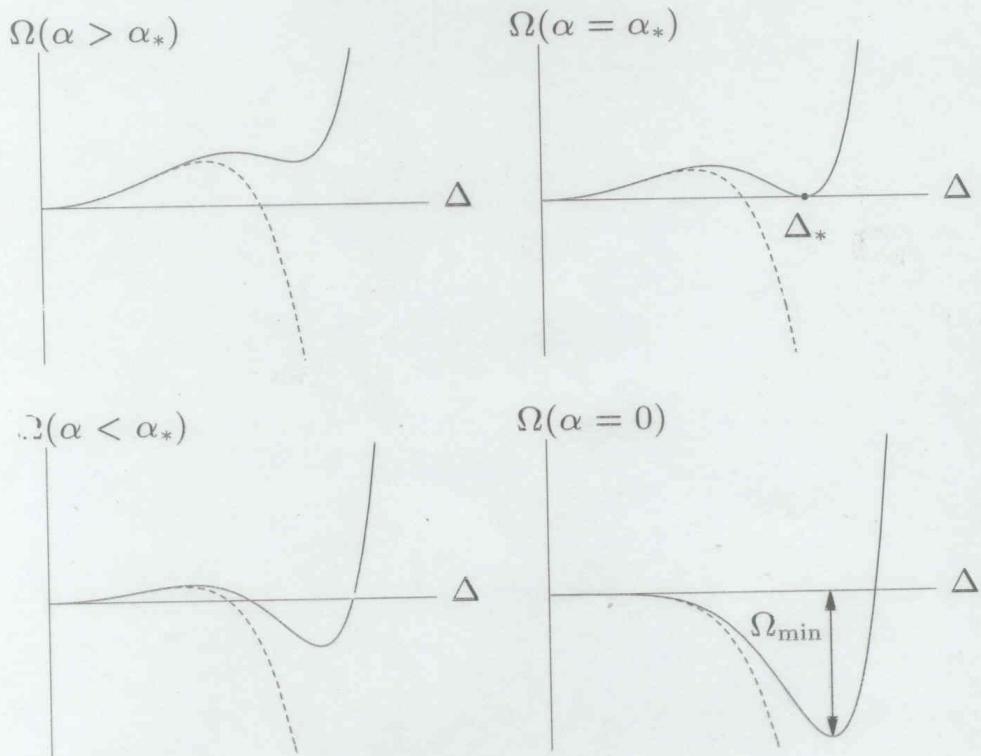
Crystal structures

Candidate crystal structures with P plane waves, specified by their symmetry group \mathcal{G} and Föppl configuration. Bars denote dimensionless equivalents: $\bar{\beta} = \beta \delta\mu^2$, $\bar{\gamma} = \gamma \delta\mu^4$, $\bar{\Omega} = \Omega / (\delta\mu_2^2 N_0)$ with $N_0 = 2\bar{\mu}^2/\pi^2$. $\bar{\Omega}_{\min}$ is the (dimensionless) minimum free energy at $\delta\mu = \delta\mu_2$. The phase transition (first order for $\bar{\beta} < 0$ and $\bar{\gamma} > 0$, second order for $\bar{\beta} > 0$ and $\bar{\gamma} > 0$) occurs at $\delta\mu_*$.

Structure	P	\mathcal{G} (Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{\min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty v}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	—	—
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramidal	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	—	—
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	—	—
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramidal	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	—	—
cube	8	$O_h(44)$	-110.757	-459.242	—	—
square antiprism	8	$D_{4d}(4\bar{4})$	-57.363	-6.866	—	—
hexagonal bipyramidal	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-6}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

Unstable structures?

- Ginzburg-Landau instability guarantees a strong first-order transition at some $\delta\mu = \delta\mu_* \gg \delta\mu_2$
- Δ_* , Ω_{\min} are large, but cannot be predicted by the Ginzburg-Landau method
- Larger instability \Rightarrow more robust ground state (cube has the most unstable Ginzburg-Landau free energy)

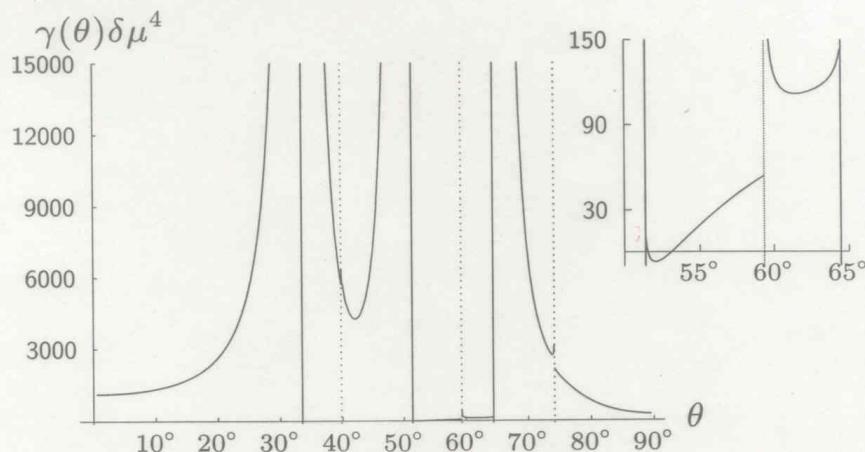


Continuous variations

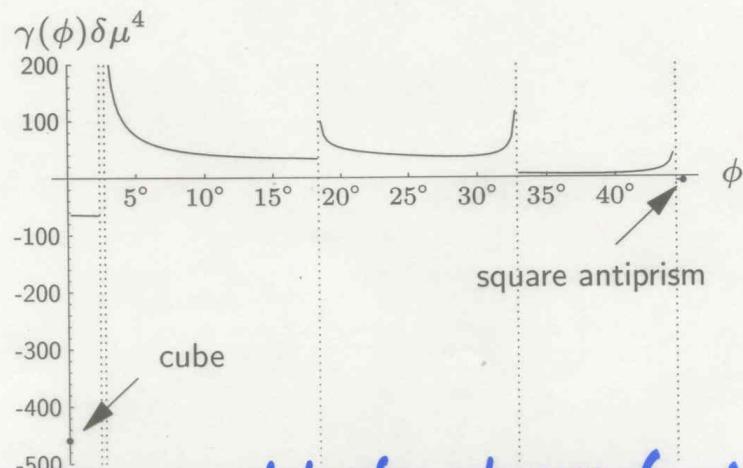
$$\Omega = \alpha \Delta^2 + \beta \Delta^4 + \gamma \Delta^6 + \dots$$

γ FOR DIFFERENT CRYSTALS WITH 8 WAVES

- Varying the "height" of a square antiprism

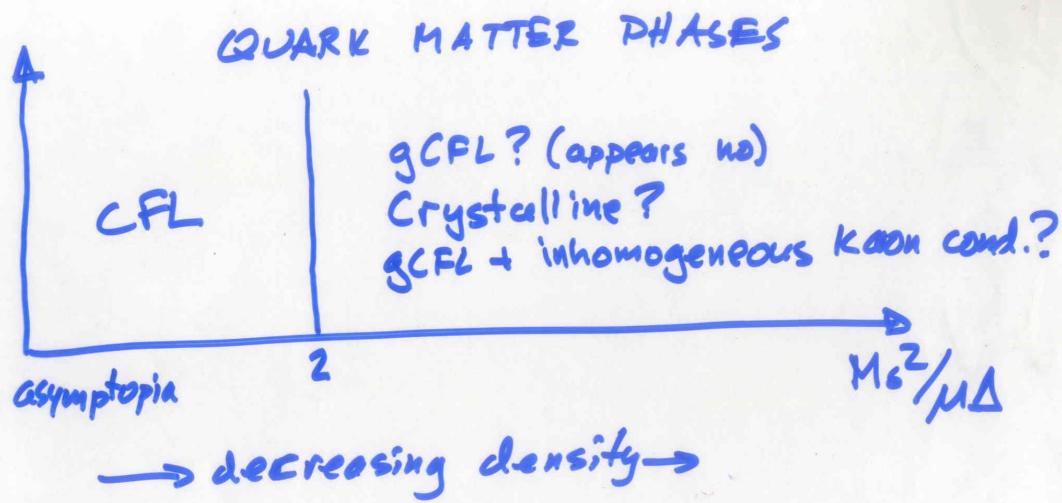


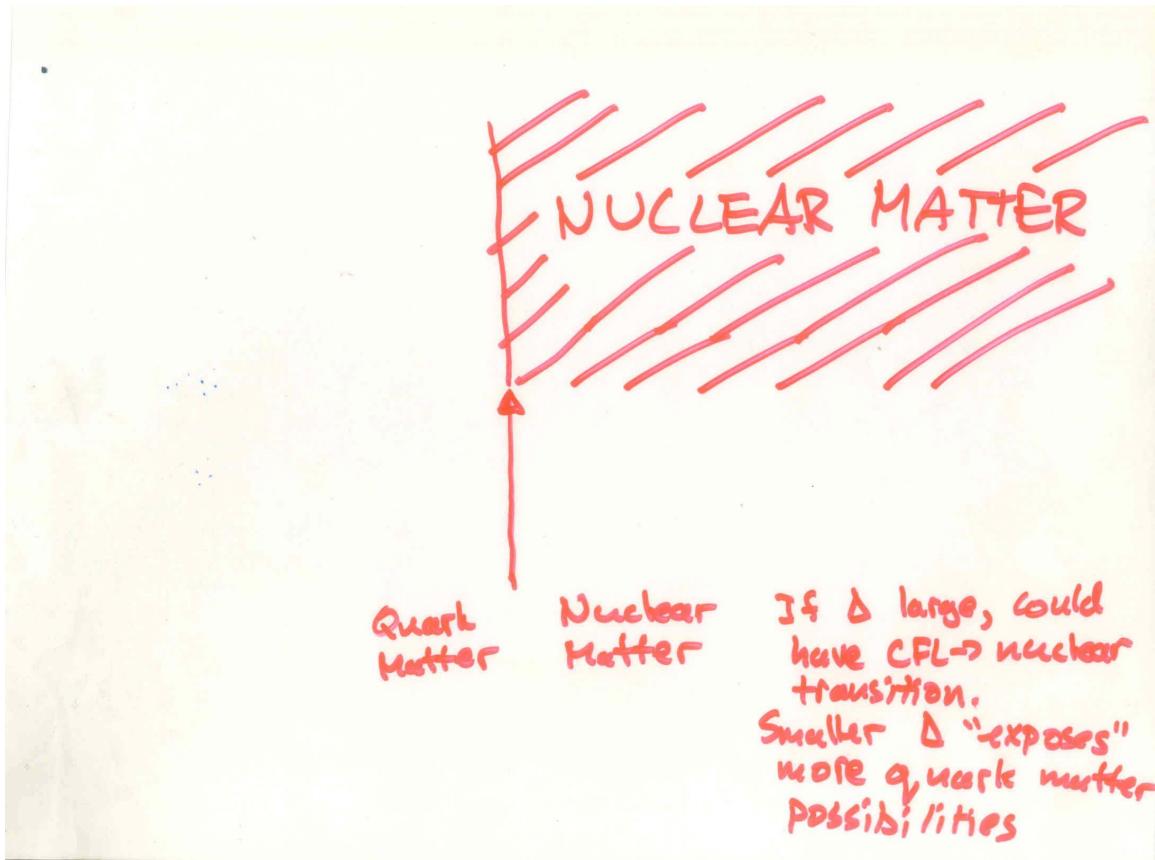
- Varying the "twist" of a square prism



of all 23 crystal structures (and their continuous variations) we investigated,
- Typeset by FoilTeX - CUBE has most negative β and γ

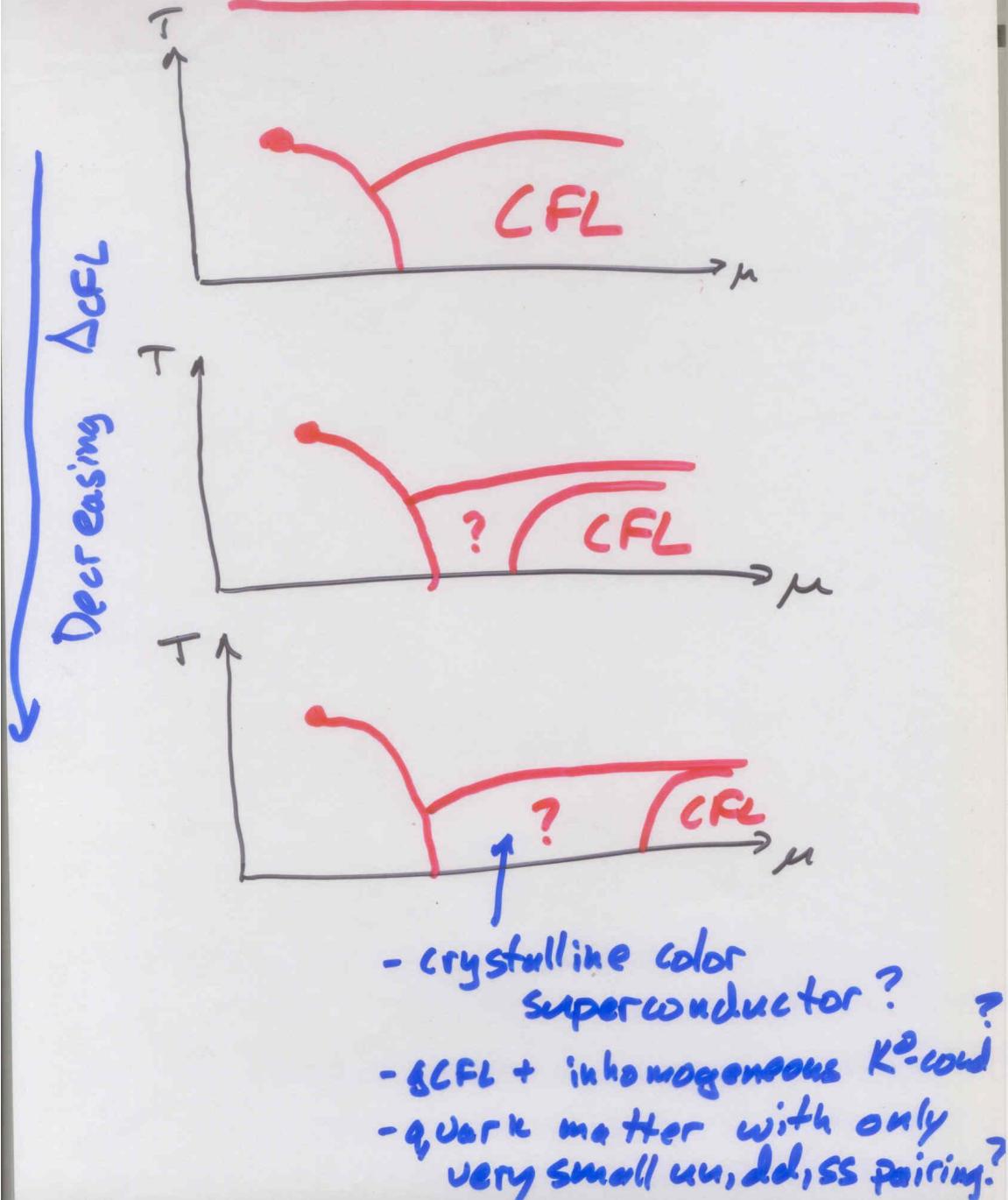
PUTTING THE PHASES TOGETHER



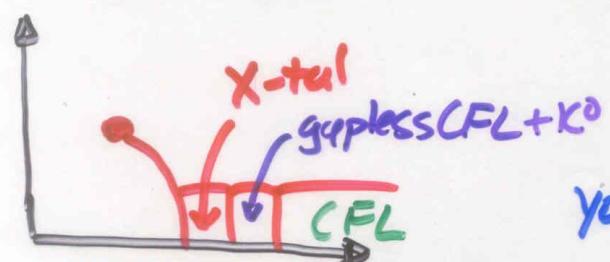
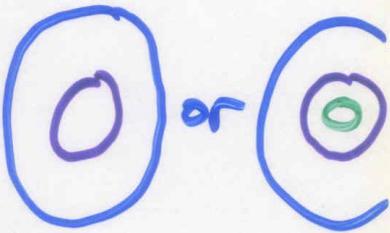
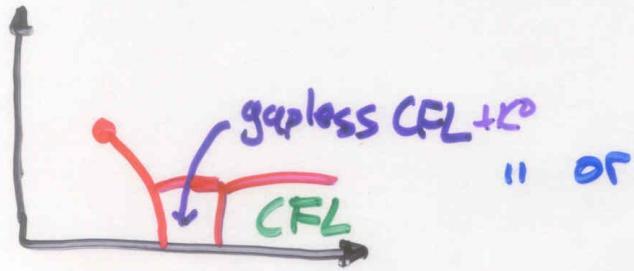
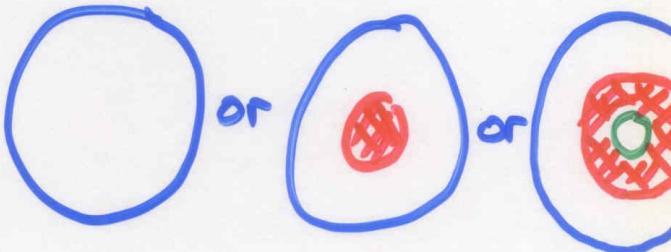
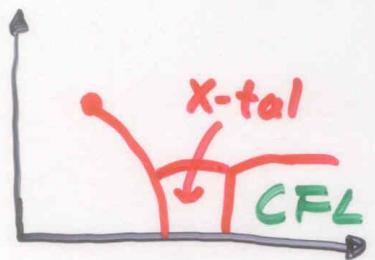
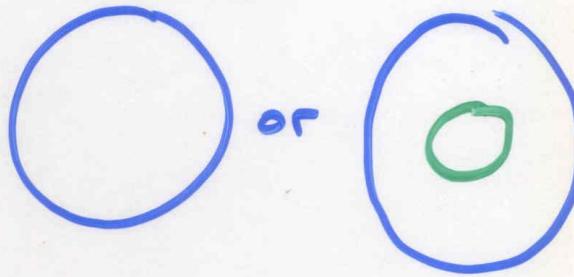
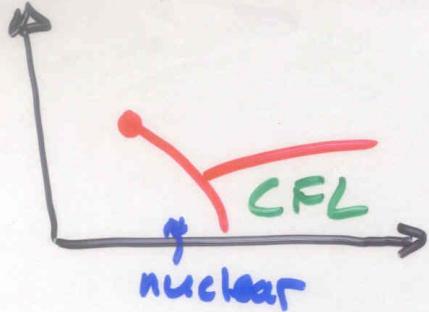


$r=0$ NEUTRON STAR PROFILE
to the surface

POSSIBLE PHASE DIAGRAMS

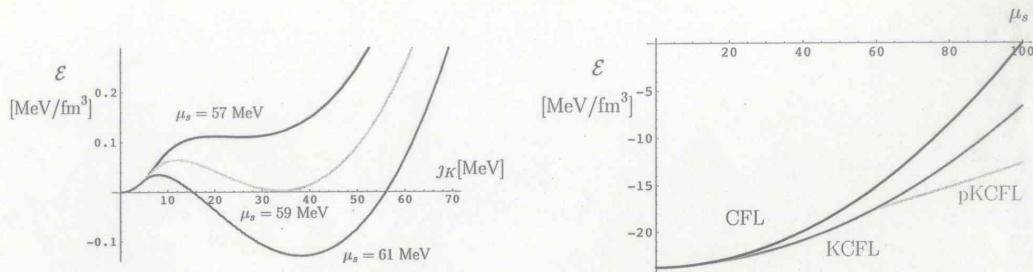


TOO MANY POSSIBILITIES



you get the idea

Energy Functional



$$\left. \frac{3\mu_s - 4\Delta}{\Delta} \right|_{crit} = ah_{crit} \quad h_{crit} = -0.067 \quad a = \frac{2}{15^2 c_\pi^2 v_\pi^4}$$

[Figures include baryon current $j_B = \alpha_B / \alpha_K j_K$]

WHAT IS QCD?

A theory of quarks and gluons....

WHAT DOES QCD DESCRIBE?

Colorless, heavy, hadrons...

Hadrons are the (rather complicated
quasi-particles of the QCD
vacuum.

The vacuum, whose excitations
are the hadrons, is therefore
quite a nontrivial [confinement;
chiral symmetry breaking; strong
coupling; ...] phase of the theory

BUT: QCD is asymptotically
free

DO OTHER (SIMPLER?) PHASES EXIST?

Do other phases exist whose quasiparticles look more like the quarks and gluons of the QCD lagrangian? And look more like phases familiar from QED?

Asymptotic freedom: quarks and gluons weakly interacting

- i) when close together
- ii) when interact at large momentum.

Suggests look at high density or high temperature.

NB: condensed matter physics teaches us that phases may be far from simple even for α as small as $\frac{1}{137}$

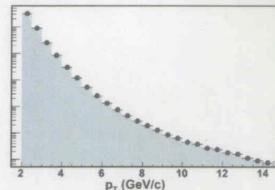


Emergence of dijets w/ increasing $p_T(\text{assoc})$

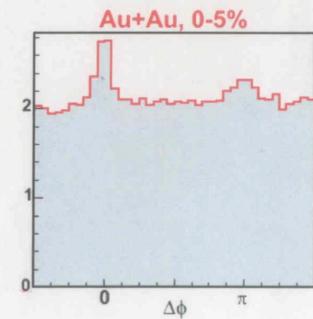
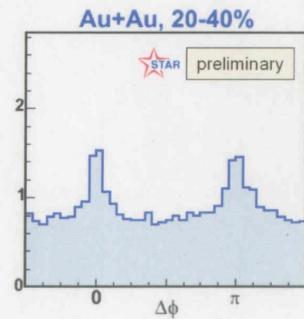
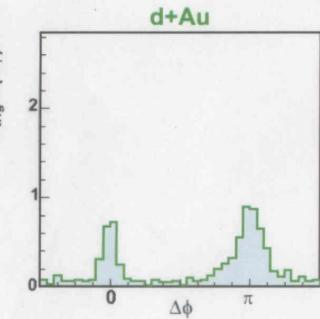


- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 2 \text{ GeV}/c$



$\frac{1}{N_{\text{trig}}} \frac{dN}{d\Delta\phi}$



- Narrow peak emerges cleanly above vanishing background

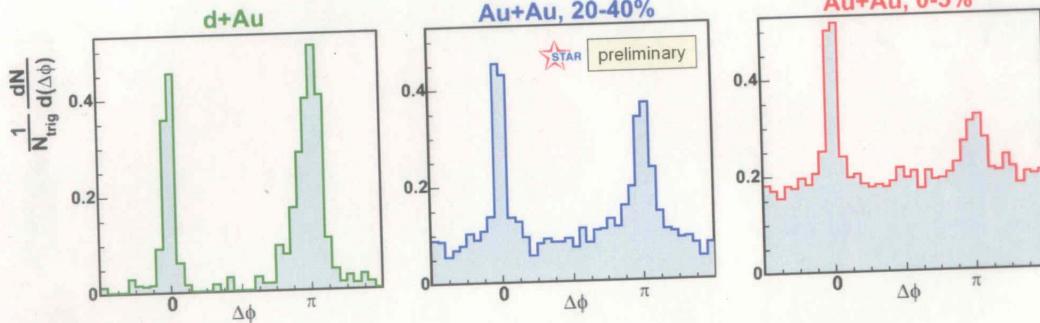
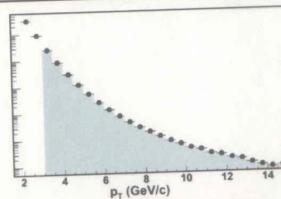


Emergence of dijets w/ increasing $p_T(\text{assoc})$



- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 3 \text{ GeV}/c$



- Narrow peak emerges cleanly above vanishing background

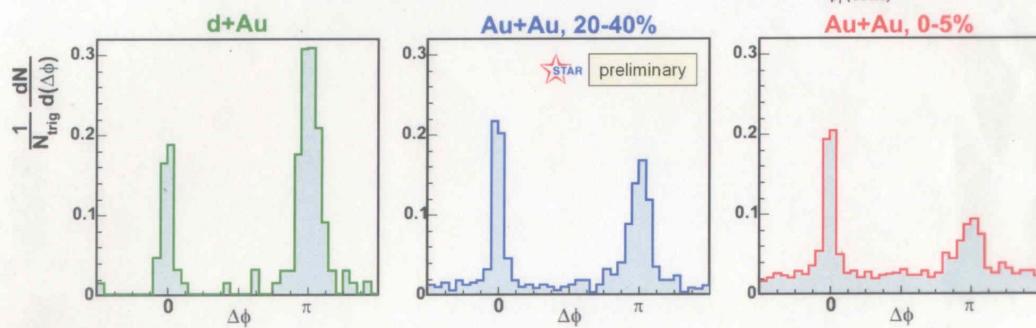
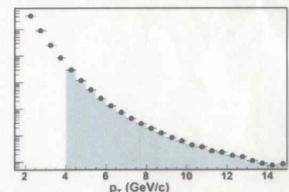


Emergence of dijets w/ increasing $p_T(\text{assoc})$



- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 4 \text{ GeV}/c$



- Narrow peak emerges cleanly above vanishing background

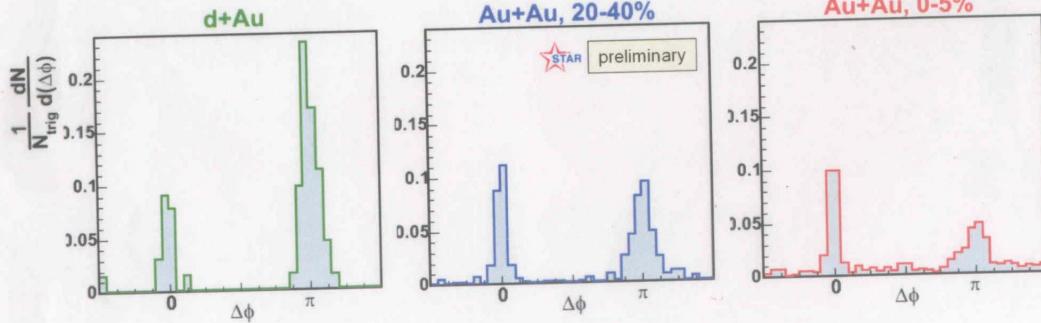
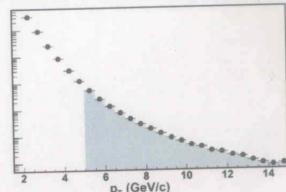


Emergence of dijets w/ increasing $p_T(\text{assoc})$



- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 5 \text{ GeV}/c$



- Narrow peak emerges cleanly above vanishing background

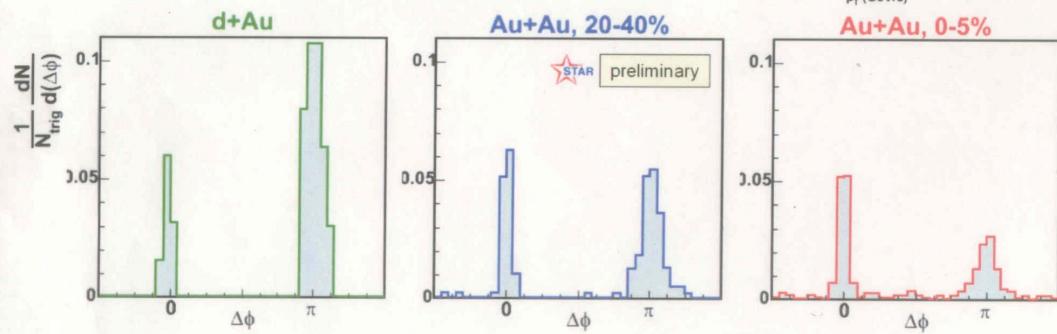
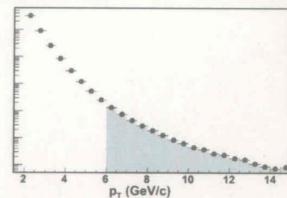


Emergence of dijets w/ increasing $p_T(\text{assoc})$



- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 6 \text{ GeV}/c$



- Narrow peak emerges cleanly above vanishing background

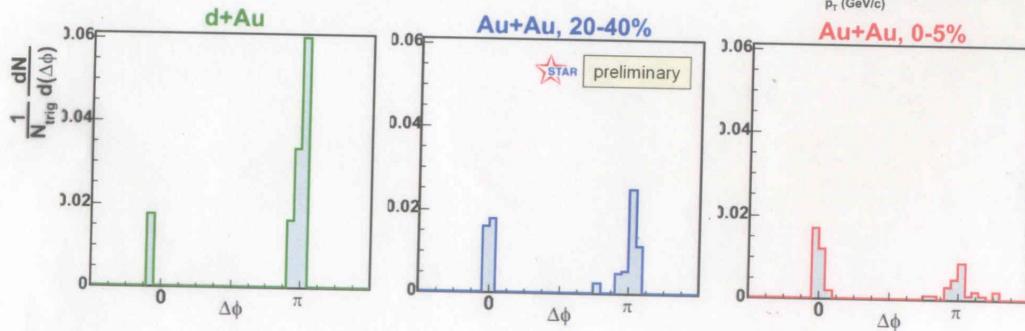
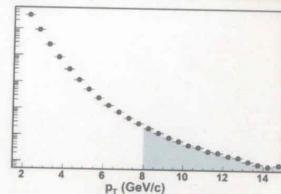


Emergence of dijets w/ increasing $p_T(\text{assoc})$



- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 8 \text{ GeV}/c$

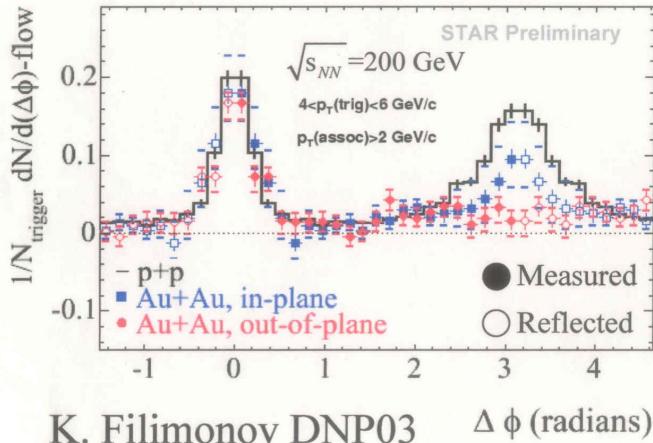


- Narrow peak emerges cleanly above vanishing background



Path Length Dependence

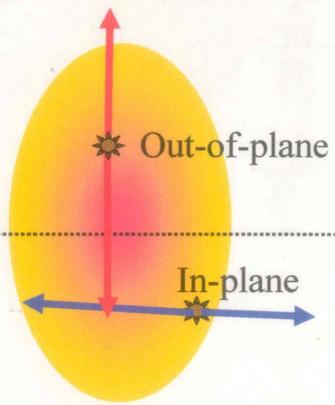
di-hadron, 20-60% Central



K. Filimonov DNP03

Suppression larger out-of-plane

Background Subtracted
See J. Bielcikova *et al.*,
(nucl-ex/0311007) for
background derivation



$$\Delta E_{GLV} \sim L^n(\phi)$$



- Maybe gCFL is not really gapless?
 - spin-one condensates could gap the gapless modes (Hong)
 - resulting gaps small, meaning that gCFL instability returns at a small but non zero T. (Alford Wang)
- P-Wave K^0 -condensate (Schäfer; Kryjevski)
- Crystalline color superconductor (rest of this talk)
 - can this have lower free energy than gCFL, first of all, and lower than p-wave K^0 -condensate and any other contenders?
 - possible, I will argue, but not demonstrated.

CRYSTALLINE COLOR SUPERCONDUCTIVITY

Alsord Bowers KR; Bowers Kunder KR Shuster; Leibovich KR Shuster;
Casalbuoni Guetta Mancarelli Verdulli; Giannakis Liu Ren; Bowers KR;

As $\mu \downarrow$, if CFL "breaks" before you get to hadronic matter, quark matter at intermediate density may have:

Pairing between quarks with different p_F

GOAL: both quarks in a pair on respective Fermi surfaces

IDEA: Cooper pairs with momentum!

$$(\vec{p} + \vec{q}, -\vec{p} + \vec{q}) \text{ for any } \vec{p}.$$

Each pair has total momentum $2\vec{q}$

- $|q| \approx 1.2 p_F$ determined energetically

- "pattern" of $\{\hat{q}_i\}$ " Bowers KR

$$\langle \psi \psi \rangle \sim \delta \sum e^{i \vec{q}_i \cdot \vec{x}}$$

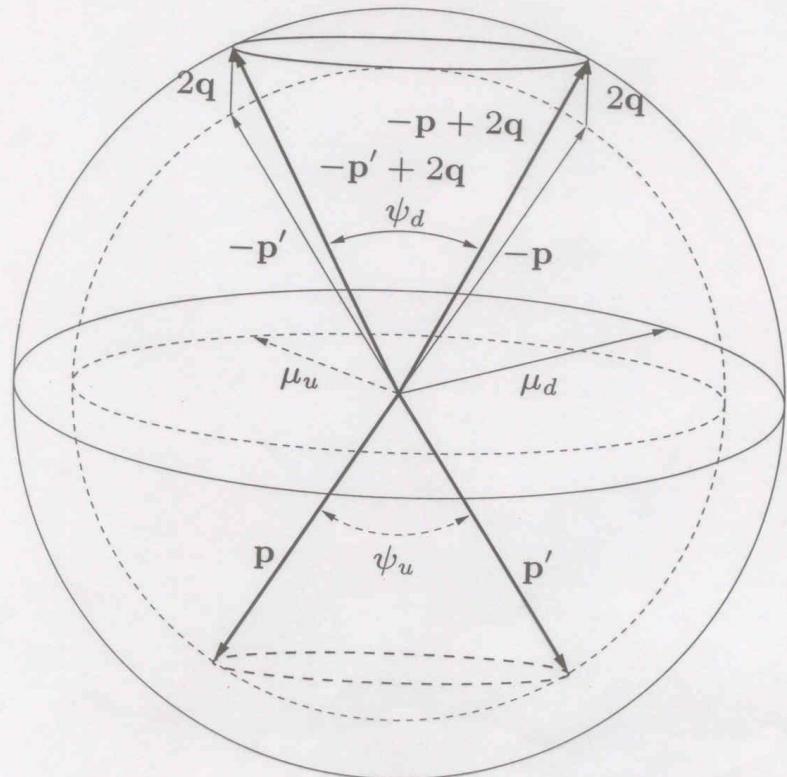
- spontaneous breaking of rotational and translational symmetry.

LOFF: Larkin Ovchinnikov Fulde Ferrell (1964) considered this state for $\langle e_\uparrow e_\downarrow \rangle$ pairing with Zeeman splitting. State not seen in condensed matter. Problem is that $\vec{B} \rightarrow$ orbital effects, not just Zeeman. QCD, with its "flavor Zeeman splitting" turns out to be the natural context for LOFF's idea!

Basic LOFF idea

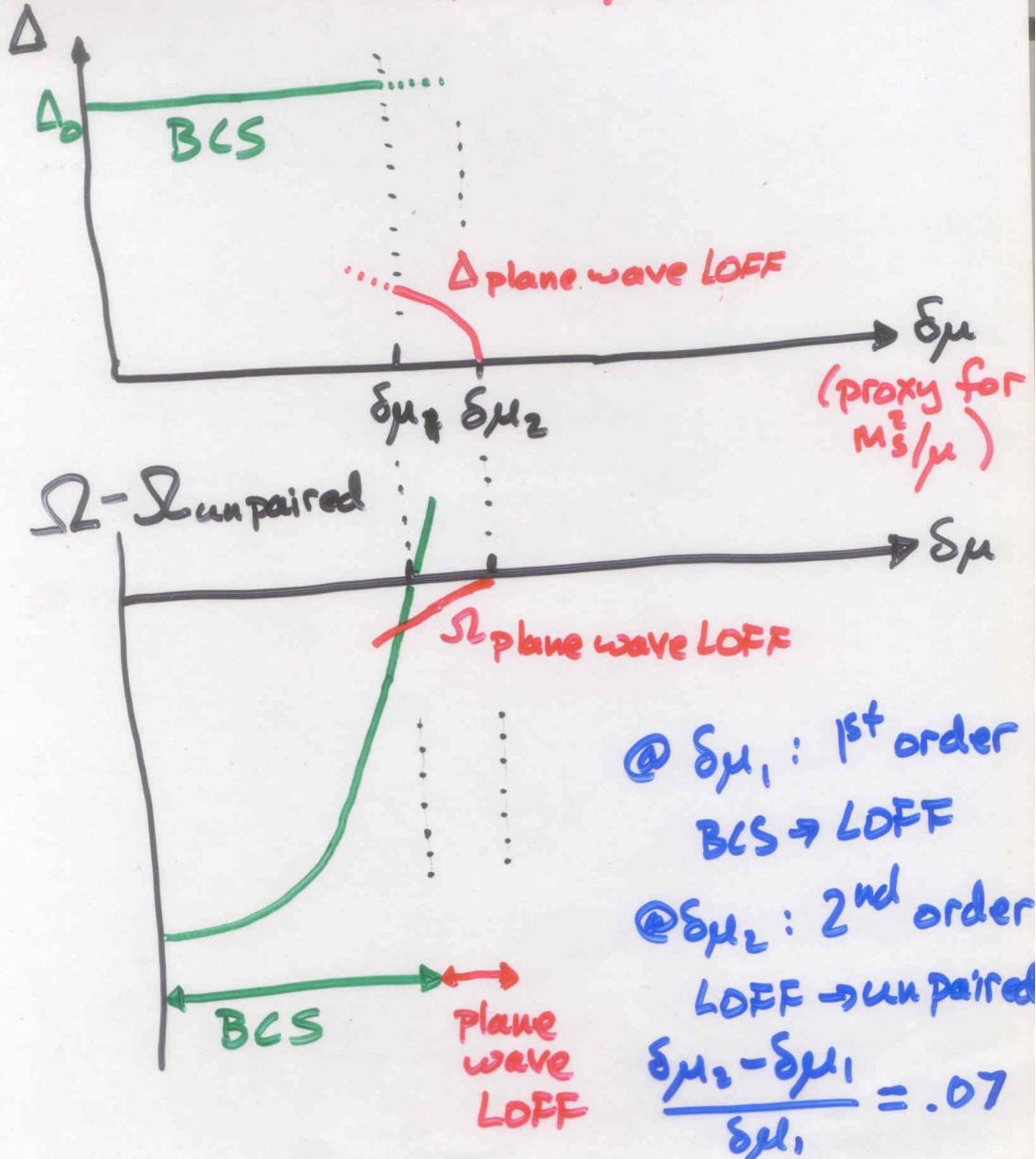
Try Cooper pairs $(\mathbf{p}, -\mathbf{p} + 2\mathbf{q})$

- total momentum $2\mathbf{q}$ for each and every pair
- each quark at its Fermi surface, even with $p_F^u \neq p_F^d$
- $\hat{\mathbf{q}}$ chosen spontaneously, $|\mathbf{q}|$ determined variationally
(result is $|\mathbf{q}| = q_0 \approx 1.20\delta\mu$)
- condensate forms a ring on each Fermi surface, with opening angle $\psi_u \approx \psi_d \approx 2\cos^{-1}(\delta\mu/q_0) \approx 67.1^\circ$



SINGLE PLANE WAVE

$$\langle \Psi(x) \Psi(x) \rangle \sim \Delta e^{i 2 \vec{q} \cdot \vec{x}}$$



MULTIPLE PLANE WAVES

If system unstable to formation of 1 plane wave, this allows quarks lying on one ring on each F.S. to pair. Much of F.S. remains unpaired....

Why not multiple \vec{q} 's? i.e. multiple rings?

Want to compare many different possible $\{\vec{q}_i\}$;

$$\langle \Psi(x) \Psi(x) \rangle = \sum_{\{\vec{q}_i\}} \Delta e^{i 2\vec{q}_i \cdot \vec{x}}$$

and for each $\{\vec{q}_i\}$ calculate Δ and Ω $\{\vec{q}_i\}$, ie crystal structure, with lowest Ω wins.

GINZBURG - LANDAU

For $\Delta \ll \Delta_0$, ie for $\delta\mu \gg \delta\mu_2$,
the free energy Ω can be evaluated
order-by-order in Δ , for many
crystal structures.

Order Δ^2 : $|\vec{q}_{i1}| = 1.2 \delta\mu$ for all q_i 's

→ each q_i gives pairing on a ring
with opening angle 67° .

- the more q_i 's, the better.

Order Δ^4 and Δ^6 : "interaction between rings"

- intersecting rings costs a lot
⇒ at most 9 plane waves
- "regularity" (lots of different
ways of making closed
4-, 6-, ... sided figures from q_i 's)
strongly favored.
- indicates that best choice is.....

FCC Crystal

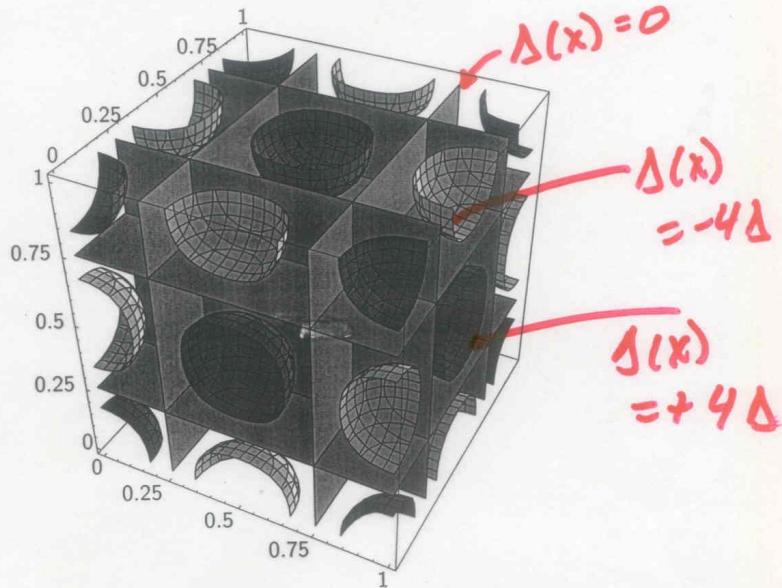
Favored according to Ginzburg-Landau analysis, that is not yet quantitatively reliable. Bowers & R

- The cube structure is the favored ground state: eight wave vectors pointing towards the corners of a cube, forming the eight shortest vectors in the reciprocal lattice of a face-centered-cubic crystal. The gap function is

$$\Delta(x) = 2\Delta \left[\cos \frac{2\pi}{a}(x+y+z) + \cos \frac{2\pi}{a}(x-y+z) \right. \\ \left. + \cos \frac{2\pi}{a}(x+y-z) + \cos \frac{2\pi}{a}(-x+y+z) \right]$$

$\Delta \sim \Delta_{CFL}$

A unit cell:



with contours $\Delta(x) = +4\Delta$ (black), 0 (gray), -4Δ (white). Lattice constant is $a = \sqrt{3}\pi/|\mathbf{q}| \simeq 6.012/\Delta_0$.

Sum of 8 currents; zero net current.

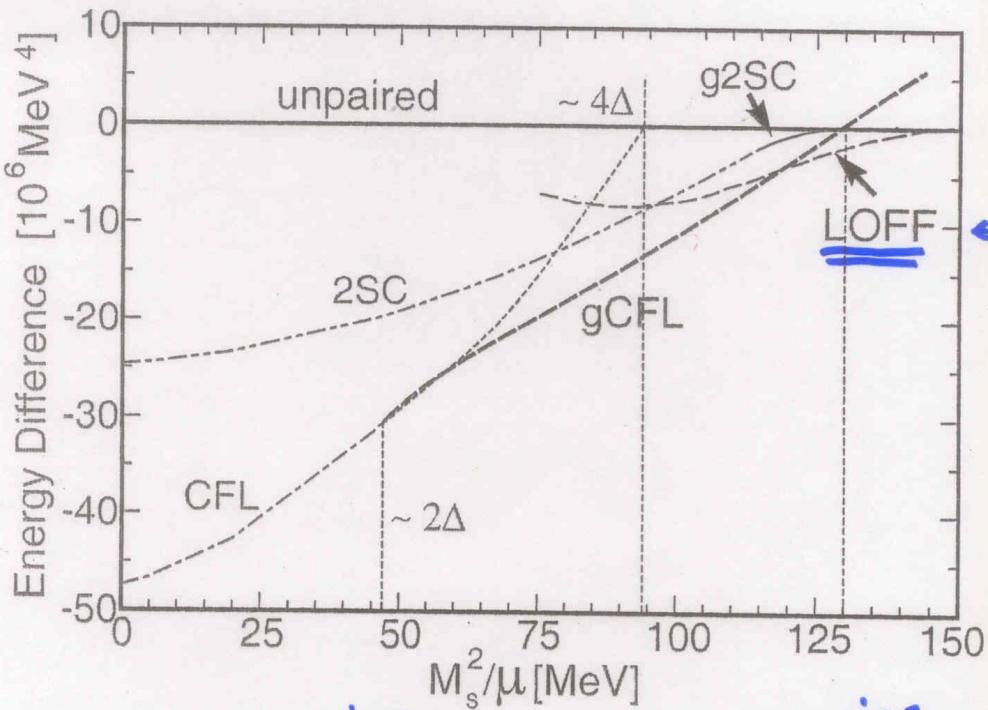
CONCLUSIONS

- FCC cube is favored structure.
Ginzburg-Landau analysis has taught us what features of a crystal structure are favored, and thus why FCC best.
- BUT: $\Omega = \alpha\Delta^2 + \beta\Delta^4 + \gamma\Delta^6 + \dots$ with β, γ large and negative, and $\alpha \sim (\delta\mu_1 - \delta\mu_2)$
 \Rightarrow Strong 1st order crystalline \rightarrow unpaired transition at a
 $\delta\mu_1 \gg \delta\mu_2$
 - crystalline "window" in phase diagram not small
 - Δ not small
 - \Rightarrow Ginzburg-Landau cannot provide quantitative calculation of Δ, Ω .
 - Make FCC ansatz, calculate λ, Δ variationally. (In progress)

cf G-L analysis of liquid-solid transition

THREE-FLAVOR "CRYSTALLINE" PHASE

Casalbuoni Gatto Ippolito Nardulli Ruggieri



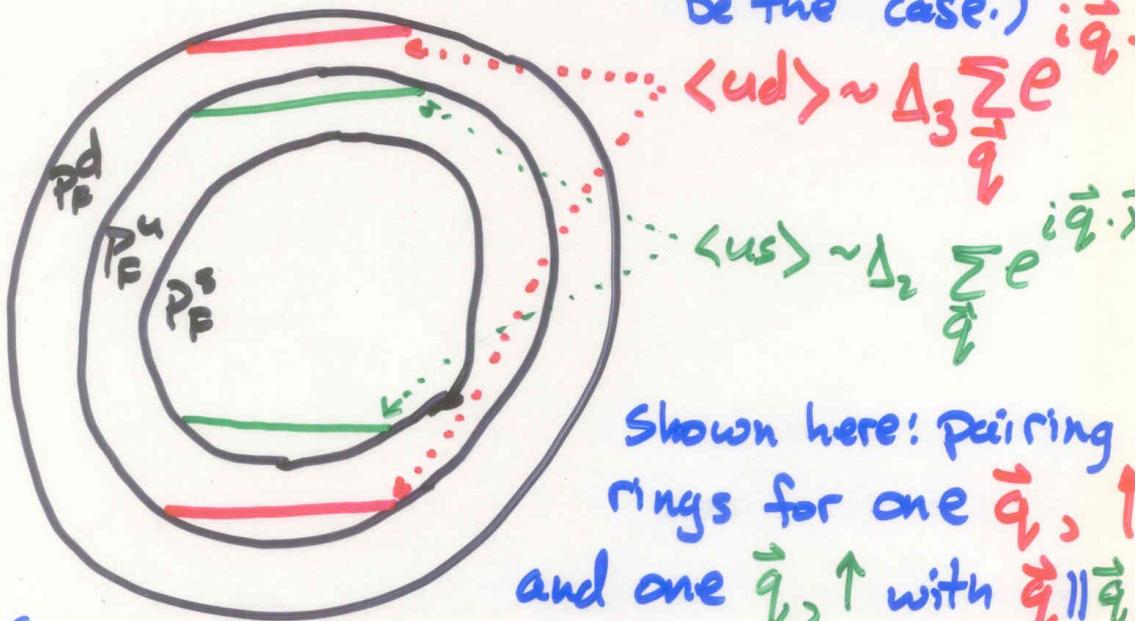
$$\langle ud \rangle \sim \Delta_3 e^{+iqz}; \quad \langle us \rangle \sim \Delta_2 e^{-iqz}$$

Greatly simplified ansatz for "crystal" structures and yet we see this phase does beat gCFL over a part of the gCFL window.

A strong hint, since whatever the favored three-flavor crystal structure turns out to be, it must have lower energy still.

THREE - FLAVOR CRYSTALLINE PHASES

If $\Delta \ll M_s^2/4\mu > \Delta_{CFL}$ then color + electric neutrality occur for $\mu_3 = \mu_8 = 0$; $\mu_B = M_s^2/4\mu$. We assume this, and assume $\Delta_1 = 0$, $\Delta_2 = \Delta_3$. (All reasonable for $\Delta \gg 0$, which may not really be the case.)



$$\langle ud \rangle \sim \Delta_3 \sum_{\vec{q}} e^{i\vec{q} \cdot \vec{r}}$$

$$\langle us \rangle \sim \Delta_2 \sum_{\vec{q}} e^{i\vec{q} \cdot \vec{r}}$$

Shown here: pairing rings for one \bar{q} , 1 and one \bar{q}_u , \uparrow with $\bar{q} \parallel \bar{q}_u$

Calculate α 's, β 's + γ 's :

$$\begin{aligned} \Omega(\Delta_2, \Delta_3) = & \alpha(\Delta_2^2 + \Delta_3^2) + \beta(\Delta_2^4 + \Delta_3^4) + \gamma(\Delta_2^6 + \Delta_3^6) \\ & + \beta_{23} \Delta_2^2 \Delta_3^2 + \gamma_{23} (\Delta_2^2 \Delta_3^4 + \Delta_2^4 \Delta_3^2) \end{aligned}$$

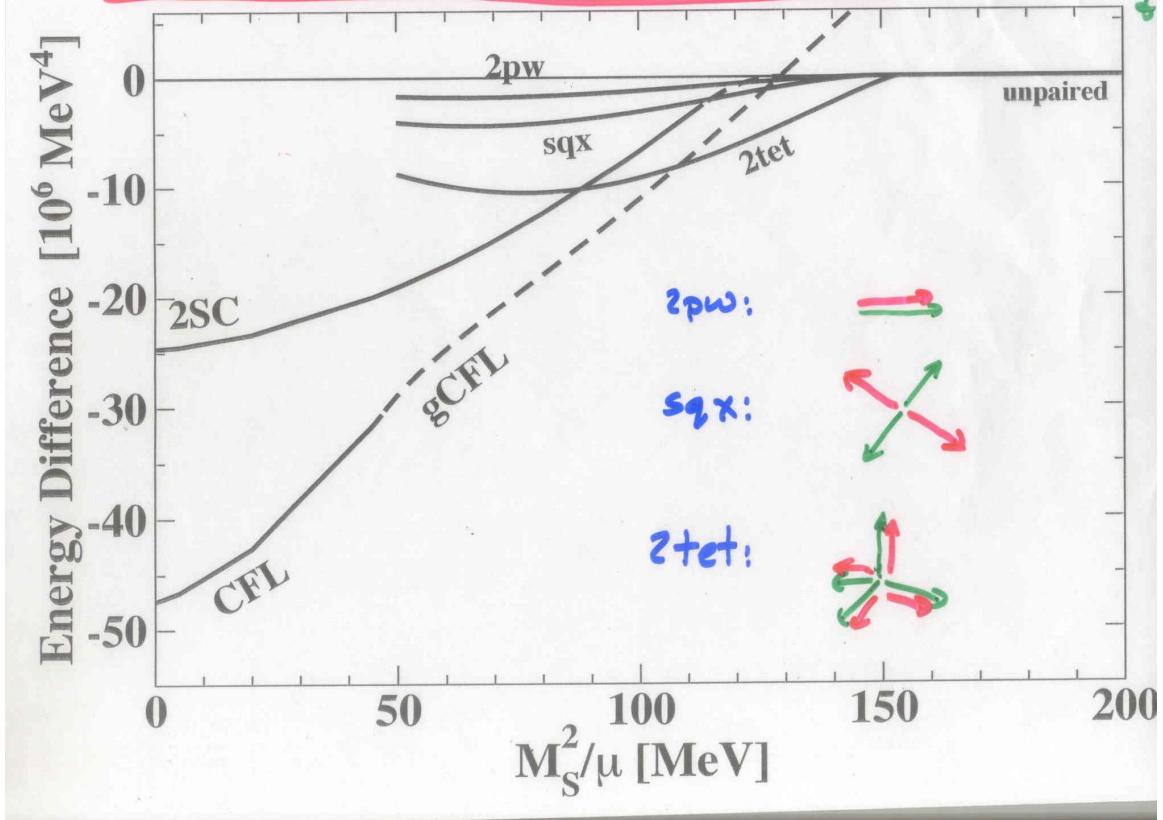
Results preliminary. So far:

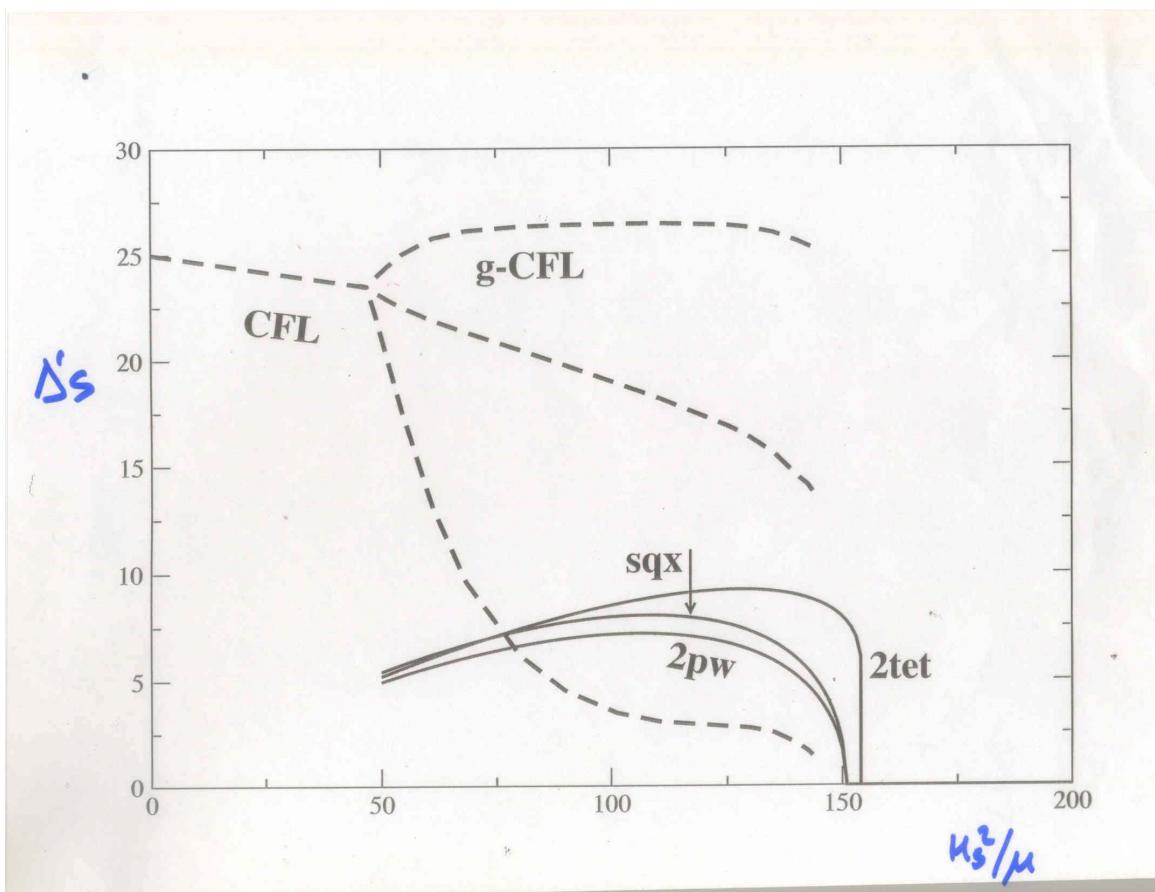
- $\beta_{23} + \gamma_{23} \rightarrow \infty$ if any ↑
becomes anti parallel to any ↑.

(Pairing rings on \mathbb{U} -Fermi sphere
"interfere".) (Disagrees w/ Casalbuoni et al.)

- Several structures with
 $2\beta + \beta_{23} > 0$ $\gamma + \gamma_{23} > 0 \rightarrow$ 2nd order trans.
→ two examples on fig.
- So far one with
 $2\beta + \beta_{23} < 0$ $\gamma + \gamma_{23} > 0 \rightarrow$ 1st order trans.
→ fig.
- So far, many with $2\beta + \beta_{23} < 0$,
 γ_{23} not yet calculated.
- Need to wait to see whether
clear qualitative understanding emerges,
as in 2-flavor case.

THREE - FLAVOR CRYSTALLINE PHASES, WITH
MORE ROBUST CRYSTAL STRUCTURES Rishi Shar





OUTLOOK

We have not yet calculated δ_{23} for enough cases to identify most favorable structures.

Nobody has shown that a crystalline phase can have lower energy than gCFL throughout gCFL window.

But, the gCFL instability points in that direction, and preliminary GL analyses are encouraging.

Ultimately, will need to compare three-flavor crystalline phase with Schäfer + Kryjevski's phase that has $gCFL + K^0$ condensate + meson supercurrent.

OUTLOOK AND IMPLICATIONS

CRYSTALLINE SUPERFLUIDITY

- A two species version of crystalline superfluid may be created in gases of ultra cold fermionic atoms
Combescot; Son Stephanov
 - trap 2 hyperfine states of atom;
 - arrange strong attractive interaction between 2 "species". (Done via a Feshbach resonance.)
 - load the trap with different number densities for 2 "species".

VORTEX PINNING & PULSTAR GLITCHES

- Rotate the crystal; what happens?
Alford Bowers KR
Vortices? Vortices pinned at intersections of crystal's nodal planes?
- If there are pinned vortices, the presence of a layer of crystalline color superconducting quark matter within neutron stars could make this layer a locus for Pulsar Glitches.

IMPLICATIONS FOR COMPACT STARS

or, flipping that around, ... How can we use observations of compact stars to determine the high density region of the phase diagram?

- If core of "neutron" star is quark matter, then it IS a color superconductor.

$$T_{\text{star}} \sim \text{keV} \ll T_c$$

- Not known whether neutron stars have quark matter cores. Goal: understand observational consequences, so we can find out.

FIRST: Can we discover whether there is a crystalline color superconductivity window?

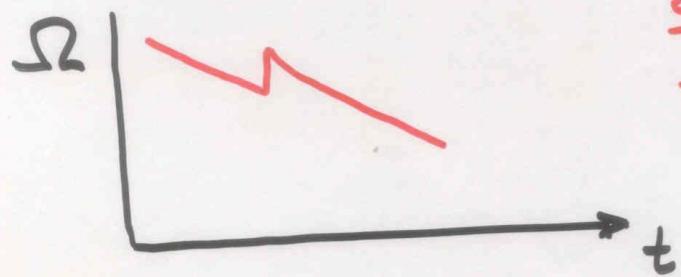
- As a function of increasing depth, $m_s^2/\mu\Delta$ decreases.

\therefore LOFF WINDOW \rightarrow LOFF SHELL

THEN: List other examples of ways to answer this question.

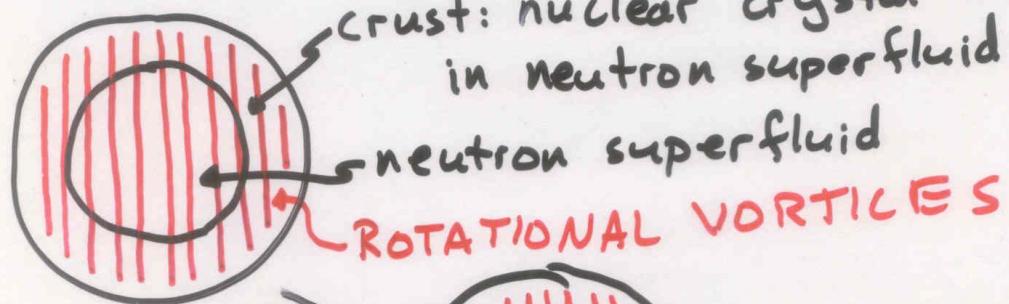
GLITCHES

Pulsars glitch:

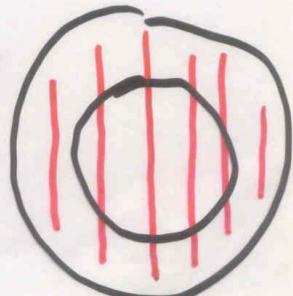
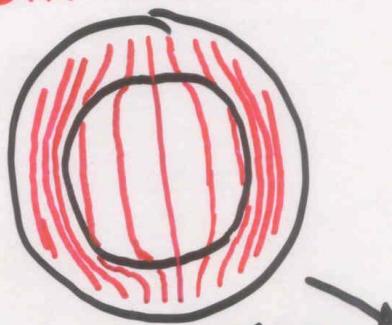


$$\frac{\delta \Omega}{\Omega} \sim 10^{-9} \rightarrow 10^{-6}$$

Conventional mechanism:

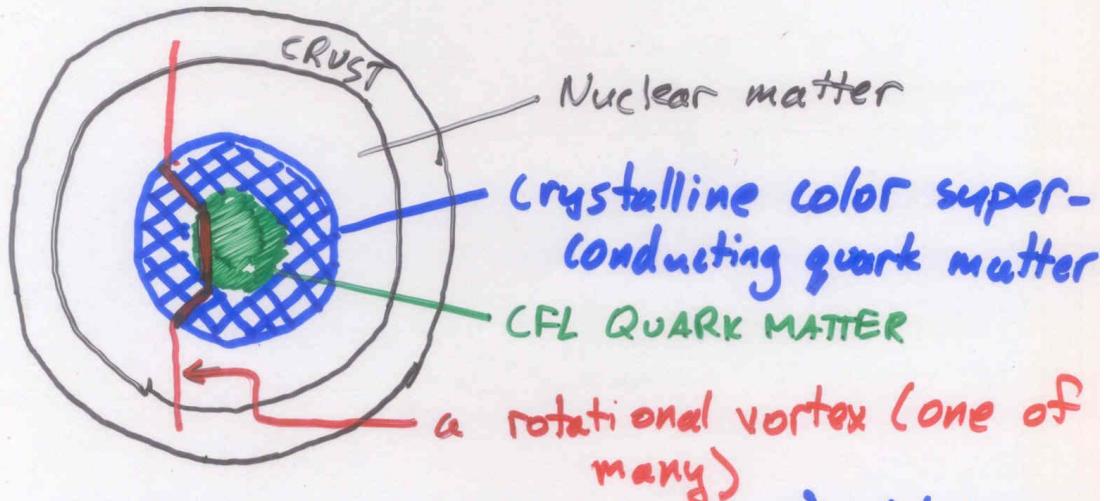


Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.
∴ thought impossible in QM.



GLITCHES IN QUARK MATTER?

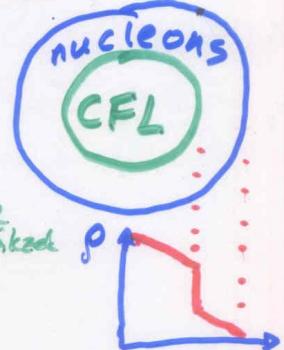
- crystalline condensate may pin vortices,
∴ they prefer to follow intersections of
nodal planes



- Could some (eg the smaller?) glitches originate in a crystalline layer in core?
- Would features of observed glitches rule out existence of crystalline layer?
- Serious glitch phenomenology requires calculation of pinning force and shear modulus. Both can be calculated once neutral, 3-flavor, crystalline phase with real crystal structure is in hand.

ASTROPHYSICAL CONSEQUENCES IF NEUTRON STARS HAVE CFL CORES

- For given M , R a little smaller.
But, uncertainty in R still ^{Alford Reddy} dominated by nuclear outer layer.
- At a sharp interface, big <sup>Alford KR
Reddy et al 2004</sup> density step. \rightarrow LIGO signal
- If spherical stars have CFL cores but oblate stars do not, \rightarrow unusual spin-up history. ^{Glendenning, Weber; Baecklin, Grigorian, Pogosyan}
- Transparent insulator. $\rightarrow \vec{B}$ in core not in flux tubes; not frozen. $\rightarrow \vec{B}$ evolution governed by outer layer.
- For $T <$ few MeV:
 - very small specific heat, neutrino <sup>Page Prakash
Lattimer Steiner</sup> emissivity, neutrino opacity. <sup>Jalikumer Prakash
Schaefer</sup>
 - superfluidity \rightarrow very large ^{Shovkovy Ellis} thermal conductivity
 \Rightarrow cooling of star controlled by nuclear outer layer
- During supernova, $T \sim$ tens of MeV $>$ meson mass
 - mesons emit and scatter neutrinos <sup>Reddy
Sekharikowski
Tachibana;
Carter Reddy</sup>
 - and, also, may be phase transitions
 \rightarrow signals in time distribution of supernova V
- Bare quark star would be nice. **NOT seen...**



GOALS

PUZZLE: If non-CFL quark matter intervenes between CFL & nuclear, what are its properties?

HINTS: gCFL instability \Rightarrow crystalline condensate

COMING: neutral, 3-flavor crystalline color superconductor, with realistic crystal structure: does it have lower energy than gCFL?

LONGER TERM: Improve calculations of properties and consequences of these phases, allowing observations to rule their presence within neutron stars out or in. Eg:

- pinning force & shear modulus of X-tal
in glitches
- almost no limit to possible improvement in calculation of CFL properties
- new data coming on M, R, V-cooling, SN-V, LIGO,