Seesaw Mechanism in the Sneutrino sector and its Consequences

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Outline

- The seesaw extended MSSM
  - model parameters and their magnitudes
- The neutrino mass matrix
- The sneutrino squared-mass mass matrix
  - Effective letpon number conserving (M^2_{LC}) and violating (M^2_{LV}) matrices
- Constraints on (M^2_{LC}) from $(g-2)_\mu$ and $\ell^I \to \ell^J\ \gamma$
- Constraints on (M^2_{LV}) from light neutrino masses and mixing
- Sneutrino Oscillations
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Incomplete list of references

- the seesaw-extended MSSM

  Hisano, Moroi, Tobe, Yamaguchi, Yanagita, J. Ellis, Raidal, Shimizu, Masiero, Vives, Farzan, E. Chun, Rossi, Vempati, Demir, Farzan, Arganda, Herrero, Casas, Ibarra

- sneutrino-antisneutrino mixing

  Grossman, Haber, Hirsch, Klapdor-Kleingrothaus, Kovalenko, Hall, Moroi, Murayama, K. Choi, Hwang, W. Song, Bar-Shalom, Eilam, Soni, S. Kolb, Panella, E. Chun

- sneutrino - antisneutrino mixing contributions to ν-masses

  Grossman, Haber, Dedes, Rimmer, Rosiek, Davidson, Losada, Abada, M. Diaz, Hirsch, Porod, Romao, Valle, S. Kang, O. Kong, Allanach, C. Kom.

- lepton flavor violation and associated CP-violating effects in the seesaw-extended MSSM

  Farzan, Peskin, Masina, J. Ellis, Hisano, Raidal, Shimizu, E. Chun, Rossi, Vempati, Demir, Casas, Ibarra, Arganda, Herrero, Antusch, Teixeira

Neutrinos are massless in the minimal version of the SM. Masses can be introduced via a gauge invariant $d = 5$ operator

$$\mathscr{L}_5 = -\frac{f_{IK}}{\Lambda}(\epsilon_{ij} L_i^I H_j)(\epsilon_{k\ell} L_k^K H_\ell) + \mathrm{H.c.}\,,$$

Bounds on light neutrino masses imply that $\Lambda \gtrsim 10^{13}$ GeV. The simplest UV-completion is the seesaw model

$$\mathscr{L}_{\rm seesaw} = -\epsilon_{ij} Y_\nu^{IJ} H_i L_j^I \nu_L^{c\,J} - \frac{1}{2} M^{IJ} \nu_L^{c\,I} \nu_L^{c\,J} + \mathrm{H.c.}$$

where $\nu_L^{c\,I}$ are $SU(2) \times U(1)$ singlet neutrinos. If $\|M\| \sim \Lambda$, one can integrate out the $\nu_L^{c\,I}$ and obtain $\mathscr{L}_5$ above.

Seesaw extended MSSM

In the MSSM ν-masses arise from R-parity symmetry breaking operators. If R-parity is conserved we can introduce a supersymmetric generalization of $\mathscr{L}_{\text{seesaw}}$ via the superfields :

Superfield	hypercharge	Boson Fields	Fermionic Partners
$\widehat{L}^I$	-1	$\widetilde{L}^I_j \equiv (\widetilde{\nu}^I_L\,,\,\widetilde{\ell}^I_L)$	$(\nu^I_L\,,\,\ell^I_L)$
$\widehat{R}^I$	$+2$	$\widetilde{R}^I \equiv (\widetilde{\ell}^I_R)^*$	ℓ^{cI}_L
$\widehat{N}^I$	0	$\widetilde{N}^I \equiv (\widetilde{\nu}^I_R)^*$	ν^{cI}_L
$\widehat{H}^1$	-1	$H^1_j \equiv (H^1_1\,,\,H^1_2)$	$(\widetilde{H}^1_1\,,\,\widetilde{H}^1_2)$
$\widehat{H}^2$	$+1$	$H^2_j \equiv (H^2_1\,,\,H^2_2)$	$(\widetilde{H}^2_1\,,\,\widetilde{H}^2_2)$

Superpotential

$$W = \epsilon_{ij}(\mu \widehat{H}_i^1 \widehat{H}_j^2 - Y_\ell^{IJ} \widehat{H}_i^1 \widehat{L}_j^I \widehat{R}^J + Y_\nu^{IJ} \widehat{H}_i^2 \widehat{L}_j^I \widehat{N}^J) + \frac{1}{2} M^{IJ} \widehat{N}^I \widehat{N}^J$$

where Y_ℓ and Y_ν are complex 3×3 matrices and M is complex symmetric 3×3 matrix and μ a complex parameter

Preferred basis

$$\widehat{L}^I \to V_L^{IJ} \widehat{L}^J \,, \qquad \widehat{R}^I \to V_R^{IJ} \widehat{R}^J \,, \qquad \widehat{N}^I \to V_N^{IJ} \widehat{N}^J$$

where V_L, V_R, V_N are 3×3 unitary matrices

$$\begin{array}{rcll} V_L^T Y_\ell V_R & = & \mathrm{diag}(Y_e \,, Y_\mu \,, Y_\tau) \,, & \qquad \text{SVD} \\ V_N^T M V_N & = & \mathrm{diag}(M_1 \,, M_2 \,, M_3) & \qquad \text{Takagi} \end{array}$$

Note that Y_ν is a general complex 3×3 matrix

Soft-SUSY-breaking Lagrangian

$$
\begin{aligned}
V_{\rm SOFT} &= m_{H_1}^2 H_i^{1*} H_i^1 + m_{H_2}^2 H_i^{2*} H_i^2 + (m_L^2)^{IJ} \widetilde{L}_i^{I*} \widetilde{L}_i^J \\
&+ (m_R^2)^{IJ} \widetilde{R}^{I*} \widetilde{R}^J + (m_N^2)^{IJ} \widetilde{N}^{I*} \widetilde{N}^J \\
&- \left[(m_B^2)^{IJ} \widetilde{N}^I \widetilde{N}^J \; + \; \mathrm{H.c} \right] \\
&- \epsilon_{ij} \left(m_{12}^2 H_i^1 H_j^2 + A_\ell^{IJ} H_i^1 \widetilde{L}_j^I \widetilde{R}^J + A_\nu^{IJ} H_i^2 \widetilde{L}_j^I \widetilde{N}^J + \mathrm{H.c.} \right)
\end{aligned}
$$

where m_L^2, m_R^2 and m_N^2 are hermitian matrices, m_B^2 is a complex symmetric matrix and A_ℓ and A_ν are complex matrices. In general these 3×3 matrices can not further be simplified in the preferred basis.

Expected magnitudes of the model parameters

1. We assume that the Yukawa couplings Y_ν^{IJ} satisfy:

$$\|Y_\nu\| \lesssim \mathcal{O}(1)\,.$$

2. The Majorana mass M is much heavier than the electroweak scale (seesaw mechanism)

$$\|M\| \gg v\,.$$

3. Although μ is a supersymmetric parameter, we require it to be of a similar order to the low-energy supersymmetry-breaking scale, M_{SUSY} :

$$\mu \sim M_{\mathrm{SUSY}}\,.$$

4. The non-singlet soft SUSY-breaking squared-masses are of a similar order to the supersymmetry-breaking scale:

$$\|m_L^2\| \sim \|m_R^2\| \sim M_{\mathrm{SUSY}}^2\,.$$

5. The parameters m_B^2 and A_ν are unconnected to electroweak symmetry breaking at tree-level. However, various constraints on ν-masses lead to

$$\|A_\nu\| \lesssim M_{\rm SUSY}\,, \qquad\qquad \|m_B^2\| \lesssim M_{\rm SUSY}\|M\|\,.$$

6. The singlet soft SUSY-breaking parameter m_N^2 is also unconnected to electroweak symmetry breaking at tree-level.

Thus, we shall present results in this paper that allow for the possibility that:

$$\|m_N^2\| \sim \|M^2\|\ .$$

If this holds, then remnants of the heavy neutrino/sneutrino sector can survive in the effective theory of the light sneutrinos. We explore the origin of this non-decoupling effect.

Terms quadratic in the neutrino fields are given in terms of two-component fermion fields

$$
\begin{aligned}
-\mathscr{L}_{m_\nu} &= \tfrac{1}{2}\left(v_2\sqrt{2}\, Y_\nu^{IJ} \nu_L^I \nu_L^{cJ} \;+\; M^{IJ} \nu_L^{cI} \nu_L^{cJ} + \mathrm{H.c.}\right) \\
&= \tfrac{1}{2}\,(\nu_L^T \quad \nu_L^{cT})\, \mathcal{M}_\nu \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \mathrm{H.c.}
\end{aligned}
$$

The neutrino mass matrix $\mathcal{M}_\nu$ is a 6×6 complex symmetric matrix given in block form by:

$$
\mathcal{M}_\nu \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix},
$$

where m_D is a 3×3 complex matrix

$$
m_D \equiv v_2 Y_\nu / \sqrt{2}
$$

The neutrino mass matrix can be Takagi block-diagonalized. Defining transformed light (ν_ℓ) and heavy (ν_h^c) neutrino states we obtain the effective light and heavy 3×3 neutrino mass matrices :

Light and heavy ν-masses

$$\begin{aligned} M_{\nu_\ell} &= -m_D M^{-1} m_D^T + \mathcal{O}(m_D^4 M^{-3}) \\ M_{\nu_h} &= M + \tfrac{1}{2}(M^{-1} m_D^\dagger m_D + m_D^T m_D^* M^{-1}) + \mathcal{O}(m_D^4 M^{-3}) \end{aligned}$$

To identify the physical light neutrino states, we must perform a Takagi-diagonalization of M_{ν_ℓ}. This is accomplished by introducing the unitary MNS matrix,

$$U_{\rm MNS}^T M_{\nu_\ell} U_{\rm MNS} = {\rm diag}(m_{\nu_{\ell 1}}\,,\, m_{\nu_{\ell 2}}\,,\, m_{\nu_{\ell 3}})\,,$$

where the $m_{\nu_{\ell J}}$ are the (real non-negative) masses of the light neutrino mass eigenstates.

The sneutrino mass matrix

The terms quadratic in the sneutrino fields are

$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \begin{pmatrix} \phi_L^\dagger & \phi_N^\dagger \end{pmatrix} \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LN} \\ (\mathcal{M}^2_{LN})^\dagger & \mathcal{M}^2_{NN} \end{pmatrix} \begin{pmatrix} \phi_L \\ \phi_N \end{pmatrix},$$

where $\phi_L \equiv (\widetilde{L}_1\,,\,\widetilde{L}_1^*)^T$ and $\phi_N \equiv (\widetilde{N}\,,\,\widetilde{N}^*)^T$ are six-dimensional vectors. The 12×12 sneutrino mass matrix, written in terms of 6×6 matrix blocks with estimated magnitudes,

$$\boldsymbol{M}^2_{\tilde{\nu}} \equiv \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LN} \\ (\mathcal{M}^2_{LN})^\dagger & \mathcal{M}^2_{NN} \end{pmatrix} = \begin{pmatrix} \mathcal{O}(v^2) & \mathcal{O}(vM) \\ \mathcal{O}(vM) & \mathcal{O}(M^2) \end{pmatrix},$$

also exhibits a seesaw type behavior, analogous to the seesaw type mass matrix of the neutrino sector.

Using standard methods for block diagonalizing, the effective 6×6 hermitian squared-mass matrix for the light sneutrinos reads:

$$\mathcal{M}^2_{\tilde{\nu}_\ell} \equiv \mathcal{M}^2_{LL} - \mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN}\left(\mathcal{M}^2_{LN}\right)^\dagger + \mathcal{O}(v^4 M^{-2})\ ,$$

analogous to the previously noted light effective neutrino mass matrix. Likewise, the effective 6×6 hermitian squared-mass matrix for the superheavy sneutrinos reads:

$$\begin{aligned} \mathcal{M}^2_{\tilde{\nu}_h} \equiv \mathcal{M}^2_{NN} \quad &+ \quad \frac{1}{2}\left[\mathcal{M}^{-2}_{NN}(\mathcal{M}^2_{LN})^\dagger\mathcal{M}^2_{LN} + (\mathcal{M}^2_{LN})^\dagger\mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN}\right] \\ &+ \quad \mathcal{O}(v^4 M^{-2})\,. \end{aligned}$$

Note that $\mathcal{M}^2_{\tilde{\nu}_\ell} \sim \mathcal{O}(M^2_{SUSY})$ and $\mathcal{M}^2_{\tilde{\nu}_h} \sim \mathcal{O}(M^2)$ as expected.

The effective light sneutrino mass matrix

The effective 6×6 hermitian squared-mass matrix for the light sneutrinos takes a simple form :

$$\mathcal{M}^2_{\tilde{\nu}_\ell} \equiv \begin{pmatrix} M^2_{LC} & (M^2_{LV})^* \\ M^2_{LV} & (M^2_{LC})^* \end{pmatrix} ,$$

where the lepton-number conserving (LC) and lepton-number violating (LV) matrix elements are given by [to order $\mathcal{O}(v^4 M^{-2})$] :

$$\begin{aligned} M^2_{LC} &\equiv m^2_L + \tfrac{1}{2} M^2_Z \cos 2\beta + m^*_D m^T_D - m^*_D M (M^2 + m^2_N)^{-1} M m^T_D \\ M^2_{LV} &\equiv m_D M (M^2 + m^{2\,*}_N)^{-1} m^T_D X^T_\nu + X_\nu m_D (M^2 + m^2_N)^{-1} M m^T_D \\ &\quad - 2 m_D M (M^2 + m^{2\,*}_N)^{-1} m^2_B (M^2 + m^2_N)^{-1} M m^T_D \end{aligned}$$

where

$$X_\nu m_D \equiv \frac{1}{\sqrt{2}} \left(v_2 A_\nu + \mu^* v_1 Y_\nu \right) .$$

Note that M^2_{LC} is a 3×3 hermitian matrix, and M^2_{LV} is a 3×3 complex symmetric matrix.

A Non-decoupling phenomenon

However, according to our assumptions for the model parameters $m_N^2 M^{-2} \sim \mathcal{O}(1)$ is possible, in which case M_{LC}^2

$$M_{LC}^2 \simeq m_L^2 + \frac{1}{2} M_Z^2 \cos 2\beta + m_D^* M^{-1} m_N^2 M^{-1} m_D^T + \mathcal{O}(v^2 m_N^4 M^{-4})$$

deviates from its MSSM value by a quantity of $\mathcal{O}(v^2)$ *even in the exact decoupling limit of $M \to \infty$.* As a result of this non-decoupling phenomenon, remnants of the heavy sector of the seesaw mechanism may survive in the effective theory of light sneutrinos. These non-decoupling effects can be detected in principle through measurements of the sneutrino and charged slepton properties.

The light sneutrino mass eigenstate

The physical light sneutrino states can be identified by diagonalizing $\mathcal{M}^2_{\tilde{\nu}_\ell}$. Note that if $M^2_{LV} = 0$, then the eigenvalues of $\mathcal{M}^2_{\tilde{\nu}_\ell}$ are doubly degenerate, corresponding to the fact that the conserved lepton number implies that the six light sneutrino states are comprised of three sneutrino antisneutrino pairs. If $M^2_{LV} \neq 0$, then lepton number is violated and the sneutrinos and antisneutrinos can mix.

This mixing splits the degenerate pairs and yields (in general) six non-degenerate light sneutrinos. In particular, the resulting sneutrino mass-eigenstates are self-conjugate real fields, which we denote by $S_1\,, S_2\,, \ldots\,, S_6$ with their masses determined by

$$\mathcal{W}^\dagger \mathcal{M}^2_{\tilde{\nu}_\ell} \mathcal{W} = \mathrm{diag}\,(m^2_{S_1}\,, m^2_{S_2}\,, \ldots\,, m^2_{S_6})\,.$$

We can employ perturbative techniques to evaluate the eigenvalues of $\mathcal{M}^2_{\tilde{\nu}_\ell}$. First, we diagonalize the sub-matrix M^2_{LC}:

$$Q_0^\dagger M^2_{LC} Q_0 = D \equiv \mathrm{diag}(d_1\,,\, d_2\,,\, d_3)\,,$$

where Q_0 is a 3×3 unitary matrix, and the eigenvalues d_I are real. We will argue below that the bounds on the radiative flavor-changing charged lepton decay $\ell^J \to \ell^I \gamma$ imply that matrix M^2_{LC} is very close to a diagonal form. Approximately,

$$(Q_0)_{IJ} \simeq \frac{(M^2_{LC})_{IJ}}{d_J - d_I}\,, \qquad I \neq J\,.$$

Define the unitary matrix :

$$Q = Q_0 T$$

where T is a 3×3 diagonal matrix of phases. Then define B as a 3×3 complex symmetric matrix

$$B \equiv Q^T M^2_{LV} Q\,.$$

As expected, the mass-splittings of the would-be sneutrino–antisneutrino pairs are nonzero due to the presence of the lepton-number violating matrix M^2_{LV}. If we denote the three sneutrino mass-splittings by $(\Delta m_{\tilde{\nu}_\ell})_J \equiv |m_{S_J} - m_{S_{J+3}}|$ (for $J = 1, 2, 3$), then in the non-degenerate case,

Sneutrino-antisneutrino mass difference

$$(\Delta m_{\tilde{\nu}_\ell})_J \simeq \frac{|B_{JJ}|}{\sqrt{d_J}}\,.$$

In the case of degenerate d_I, the mass-splittings $(\Delta m_{\tilde{\nu}_\ell})_J$ also depend on the non-diagonal elements of B.

Example : One generation model

In this case, M_{LC}^2 and $B \equiv M_{LV}^2$ are just numbers. The corresponding sneutrino mass-splitting, $\Delta m_{\tilde{\nu}_\ell} \equiv |m_{S_2} - m_{S_1}|$, is given by

$$\frac{\Delta m_{\tilde{\nu}_\ell}}{m_{\nu_\ell}} = \frac{2M^2}{m_{\tilde{\nu}_\ell}(M^2 + m_N^2)} \left| X_\nu - \frac{M m_B^2}{M^2 + m_N^2} \right| , \tag{1}$$

where $m_{\nu_\ell} \equiv |m_D|^2/M$ is the mass of the light neutrino and $m_{\tilde{\nu}_\ell} \equiv \frac{1}{2}(m_{S_1} + m_{S_2})$ is the average light sneutrino mass.

Assuming that $m_B^2 = m_0\ M$, and $A_\nu = A_0$ it follows that both terms on the right hand side of eq. (1) are of the same order, which implies that $\Delta m_{\tilde{\nu}_\ell} \sim \mathcal{O}(m_{\nu_\ell})$. This can be enhanced by at most three orders of magnitude but not more.

We study all possible corrections to neutrino masses. The dominant one reads :

$$\begin{aligned}
\delta M^{IJ}_{\nu_\ell} &\equiv (M^{1-\mathrm{loop}}_{\nu_\ell})^{IJ} - m_{\nu_{\ell I}} \delta^{IJ} \\
&\simeq \frac{-1}{32\pi^2} \sum_{i,K,M} m_{\chi^0_i} \mathrm{Re} \left[(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2 \, U^{KI}_{\mathrm{MNS}} U^{MJ}_{\mathrm{MNS}} \, (M^2_{LV})_{KM} \right] \left(\frac{\Delta B_0}{\Delta m^2} \right)_{iKM},
\end{aligned}$$

where

$$\left(\frac{\Delta B_0}{\Delta m^2} \right)_{kIJ} \equiv \begin{cases} \dfrac{B_0(m_{\chi^0_k}, m_{\tilde{\nu}^I_\ell}) - B_0(m_{\chi^0_k}, m_{\tilde{\nu}^J_\ell})}{m^2_{\tilde{\nu}^I_\ell} - m^2_{\tilde{\nu}^J_\ell}}, & \text{for } I \neq J\,, \\[2ex] \dfrac{\partial B_0(m_{\chi^0_k}, m_{\tilde{\nu}^I_\ell})}{\partial m^2_{\tilde{\nu}^I_\ell}}, & \text{for } I = J\,. \end{cases}$$

As expected, this contribution is finite and is explicitly lepton number violating, as it is proportional to the matrix M^2_{LV}.

Sneutrino-mass-difference and ν-masses

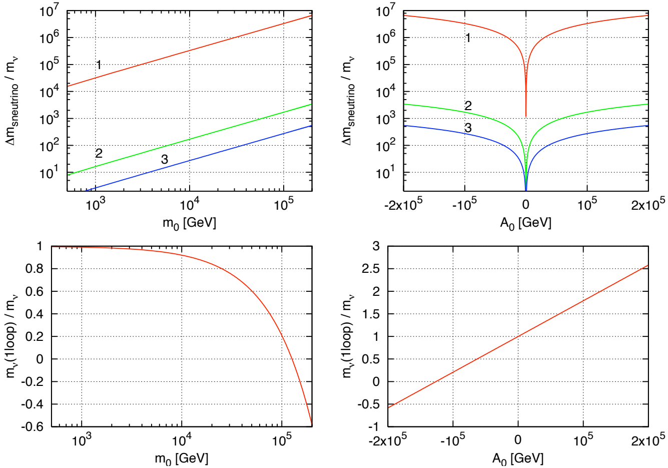

Figure: Predictions for the ratios $(\Delta m_{\tilde{\nu}_\ell}/m_{\nu_\ell})_I$ and $(m_{\nu_\ell}^{(1-\mathrm{loop})}/m_{\nu_\ell})_I$ for the three neutrino states ($I = 1, 2, 3$) as functions of the soft SUSY-breaking parameters m_0 and A_0. When varying m_0 [left panels] we set $A_0 = 0$ and when varying A_0 [right panels] we set $m_0 = 0$.

Constraints on M^2_{LC} from $(g-2)_\mu$ and $\ell^J \to \ell^I \gamma$

The anomalous magnetic moment and electric dipole moment (EDM) of the leptons and the lepton flavor violating decays $\ell^J \to \ell^I \gamma$ are derived from the following terms of an effective Hamiltonian

$$\mathscr{H} \;=\; e\,\bar{\ell}^{\,J} \sigma_{\mu\nu} \Big[\mathrm{Re} C^{JJ}_{1L} + m_{\ell^J} (C^{JJ}_{4L} + C^{JJ}_{4R}) - i \mathrm{Im} C^{JJ}_{1L} \gamma_5 \Big] \ell^{\,J} F^{\mu\nu} \;,$$

which can be extracted from the computation of the one loop vertex for $\ell^J \to \ell^I \gamma$.

By matching to the standard form

$$\mathscr{H} \quad = \quad -\frac{e}{4m_{l^J}} a_J \bar{\ell}^{\,J} \sigma_{\mu\nu} \ell^{\,J} F^{\mu\nu} + \frac{i d_{\ell^J}}{2} \bar{\ell}^{\,J} \sigma_{\mu\nu} \gamma_5 \ell^{\,J} F^{\mu\nu} \ ,$$

where $a_J \equiv (g_J - 2)/2$ is the magnetic moment anomaly and d_{ℓ^J} is the EDM of the lepton, one can extract the expressions for the electron EDM, d_e, and for $g_\mu - 2$,

$(g-2)_\mu$ and the electron EDM

$$d_e \quad = \quad -2e\,\mathrm{Im} C_{1L}^{11} \ ,$$

$$a_\mu \quad = \quad -4m_\mu \left[\mathrm{Re} C_{1L}^{22} \ + \ m_\mu (C_{4L}^{22} + C_{4R}^{22}) \right] \ .$$

- Constraints from e-EDMs : NONE at 1-loop, if μ and M_2 are real.
- Constraints from a_μ : i.e., $(m_L)^{min} \gtrsim 300$ GeV for $\tan\beta = 10$ and $M_2 = 100$ GeV.

$\ell^J \to \ell^I \gamma$ decay width

$$\Gamma(\ell^J \to \ell^I \gamma) = \frac{e^2 m_{l^J}^3}{4\pi} \left(|C_L^{IJ}|^2 + |C_R^{IJ}|^2 \right) .$$

Thus, for $J > I$ and neglecting terms proportional to the lighter lepton mass, one arrives at the simple result:

$$\begin{aligned} C_L^{IJ} &\simeq 0 \,, \\ C_R^{IJ} &\simeq C_{1R}^{bIJ} + m_{\ell^J} C_{4L}^{bIJ} \\ &\simeq \frac{m_{\ell^J}}{(4\pi)^2} \frac{e^2}{2 s_W^2} (M_{LC}^2)^{IJ} \left(|Z_+^{1i}|^2 \left(\frac{\Delta C_{23}}{\Delta m^2} \right)_{iIJ} - \frac{\sqrt{2}}{\cos\beta} \frac{m_{\chi_i^+}}{M_W} Z_+^{1i*} Z_-^{2i*} \left(\frac{\Delta C_{11}}{\Delta m^2} \right. \right. \end{aligned}$$

where the $Z_\pm$ are the chargino mixing matrices defined in,

$$\left(\frac{\Delta C_{ij}}{\Delta m^2} \right)_{kIJ} \equiv \begin{cases} \dfrac{C_{ij}(m_{\chi_k^+}, m_{\tilde{\nu}_\ell^I}) - C_{ij}(m_{\chi_k^+}, m_{\tilde{\nu}_\ell^J})}{m_{\tilde{\nu}_\ell^I}^2 - m_{\tilde{\nu}_\ell^J}^2} \,, & \text{for } I \neq J \,, \\ \dfrac{\partial C_{ij}(m_{\chi_k^+}, m_{\tilde{\nu}_\ell^I})}{\partial m_{\tilde{\nu}_\ell^I}^2} \,, & \text{for } I = J \,. \end{cases}$$

and $m_{\tilde{\nu}_\ell^I}$ are the three "CP-averaged" sneutrino masses.

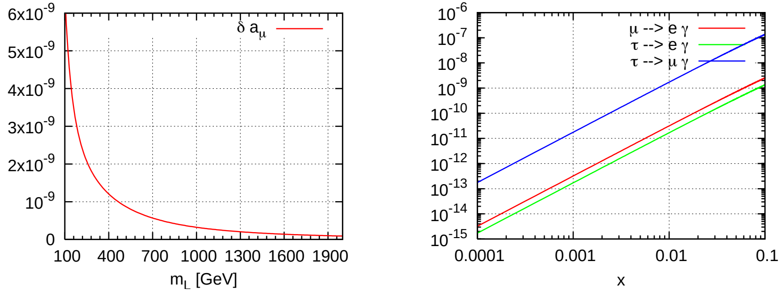

Figure: (a) In the left panel, the SUSY contribution to the muon anomalous magnetic moment as a function of $m_L = m_R$ is exhibited. (b) In the right panel, the prediction for BR($\ell^J \to \ell^I \gamma$) is shown as a function of the parameter $x = m_N^2/M^2$.

Bounds on m_N

$$x \equiv \frac{||m_N^2||}{||M^2||} \lesssim 10^{-2}$$

Bounds on M^2_{LC}

$$M^2_{LC} = \begin{pmatrix} \gtrsim 270^2 & \lesssim 4^2 & \lesssim 11^2 \\ ... & \gtrsim 270^2 & \lesssim 31^2 \\ ... & ... & \gtrsim 270^2 \end{pmatrix}$$

Typical Bounds on the structure of the matrix elements of M^2_{LC} for $M_2 = \mu = 200$ GeV and $\tan\beta = 10$. All masses are given in GeV.

Bounds on M^2_{LV}

As an example, under the assumption of tri-bimaximal mixing of Perkins et.al and model parameters given in the paper

Bounds on M^2_{LV}

$$M^2_{LV} \;\lesssim\; \left(\begin{array}{ccc} 2\times 10^{-9} & \dots & \dots \\ \dots & 2\times 10^{-6} & \dots \\ \dots & \dots & 10^{-5} \end{array} \right) \;\mathrm{GeV}^2 \quad ,$$

where the dots indicate elements with similar bounds as the diagonal ones. The significant suppression of the lepton number violating matrix elements of M^2_{LV} relative to the lepton number conserving matrix elements $M^2_{LC} \sim \mathcal{O}(v^2)$ is particularly noteworthy.

Sneutrino Oscillation Phenomena

- Sneutrino-antisneutrino oscillations
- Sneutrino-flavour oscillations

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Sneutrino-antisneutrino oscillations

Also referred to as CP-driven oscillations; analogous to $K - \bar{K}$ ones.
In $e^+ + e^- \to \widetilde{\nu}_I + \widetilde{\nu}_I^*$ let $N_{\ell-}$ $[N_{\ell+}]$ be the number of negatively and positively charged leptons due to decay $\tilde{\nu} \to l^- + \chi^+$ $[\tilde{\nu}^* \to l^+ + \chi^-]$. Then the asymmetry,

$$A_l = \frac{N_{l-} - N_{l+}}{N_{l-} + N_{l+}} ,$$

is not zero due to $\tilde{\nu} - \tilde{\nu}^*$ oscillations and is proportional to the quantum interference term. This can be measured if $\Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}} \sim O(1)$.
Unfortunately for typical seesaw paramaters, with $m_0 \lesssim 10^5$ GeV, we obtain $\Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}} \lesssim 2.7 \times 10^{-6}$.

To sufficiently suppress the sneutrino width, one would have to suppress (or make kinematically inaccessible) the dominant two-body decay rates. Three body phase space provides a sufficient suppression, although without a further extension of the model, the latter would require a stable (or very long-lived) charged slepton, in conflict with astrophysical data.

Sneutrino-flavour oscillations

To a good approximation, and for $I \neq J$

$$P_{\tilde{\nu}_I \to \tilde{\nu}_J} \quad \approx \quad 2e^{-t/\tau} \left[|Q_0^{IJ}|^2 - \mathrm{Re}(Q_0^{IJ})^2 \cos \Delta m_{IJ} t \right]$$

where $\Delta m_{IJ} \equiv m_{\tilde{\nu}_I} - m_{\tilde{\nu}_J}$ and $Q_0^{IJ} \simeq (M_{LC}^2)^{IJ} / (m_{\tilde{\nu}_J}^2 - m_{\tilde{\nu}_I}^2)$.
For flavour oscillations the condition $\Delta m_{\tilde{\nu}} / \Gamma_{\tilde{\nu}} \sim O(1)$ is more easily satisfied, as mass splittings of order 10 GeV are not uncommon (if we use the two-body decay channel). Unfortunately the small off-diagonal elements of M_{LC}^2 suppress the probability of flavour oscillations.
Typically we find :

$$P_{\tilde{\nu}_\mu \to \tilde{\nu}_e} \approx \mathcal{O}(10^{-5}) \, [1 \; - \; \cos(\Delta m_{12} \tau)] \; .$$

Thus, in the see-saw extended MSSM, sneutrino flavor oscillations are difficult to observe at colliders. To avoid this negative conclusion, one requires some (unnatural?) cancellation mechanism that would permit an enhancement of the off-diagonal elements of M_{LC}^2 relative to the previously quoted bounds.

Conclusions

- In the seesaw-extended MSSM, the light sneutrino sector is governed by a $\Delta L = 0$ hermitian matrix, M^2_{LC} , and a $\Delta L = 2$ complex symmetric matrix, M^2_{LV}.
- Corrections from the heavy sneutrino sector can yield non-decoupling corrections to M^2_{LC} (which modify the charged slepton-sneutrino mass relation), and are responsible for additional contributions to $\ell^J \to \ell^I \gamma$.
- Limits on rare flavor-changing charged lepton decays yield strong upper bounds on the off-diagonal elements of M^2_{LC}. Lower bounds on the diagonal elements of M^2_{LC} (as a function of chargino parameters) can be derived from the observed value of $(g-2)_\mu$.
- Significant radiative corrections to the tree-level neutrino mass matrix could provide the dominant source of neutrino flavor texture. The one-loop sneutrino-neutralino graph (governed by sneutrino-antisneutrino mixing) can yield neutrino mass shifts of order the tree-level masses and provide upper limits on the elements of M^2_{LV}.
- Both sneutrino-antisneutrino and sneutrino flavor oscillations are possible, although $\Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}} \sim O(1)$ (required for observability of the effect) cannot be simultaneously satisfied in both cases. Due to constraints on M^2_{LC} and M^2_{LV} , these phenomena are probably not observable at future colliders for generic SUSY-seesaw parameters.

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Understanding the non-decoupling effect

In the limit $||M|| \to \infty$ with $||m_N^2 M^{-2}||$=fixed, the Lepton number Conserving sneutrino mass sub-matrix becomes

$$M_{LC}^2 \simeq m_L^2 + \frac{1}{2} M_Z^2 \cos 2\beta + \; m_D^* M^{-1} m_N^2 M^{-1} m_D^T \; + \mathcal{O}(v^2 m_N^4 M^{-4})$$

Thus, we have a non-decoupling correction to the usual MSSM result of $\mathcal{O}(m_N^2 M^{-2})$ as previously noted in this talk.

To understand the origin of this non-decoupling phenomenon, we formally integrate out the heavy sector and obtain the effective theory of the light sneutrinos. Thus,

$$\widetilde{N}^I = \tilde{\nu}_h^I - \epsilon_{kn} [(M^2 + m_N^2)^{-1} M Y_\nu^T]^{IJ} \widetilde{L}_n^J H_k^2 \,, \tag{3}$$

before electroweak symmetry breaking, where we have used $\widetilde{N}^I \equiv \tilde{\nu}_R^{I\,*}$.

To obtain the relevant operator that survives in the low-energy effective theory, we insert eq. (3) for $\widetilde{N}^I$ in the scalar potential, and then take the limit as $\|M\| \to \infty$, In addition, we set $\tilde{\nu}_h = 0$. The end result is: $[m_N^2 < M^2]$,

$$\epsilon_{kn} \epsilon_{ij} [Y_\nu^* M^{-1} m_N^2 M^{-1} Y_\nu^T + \mathcal{O}(m_N^4 M^{-4})]^{JK} \widetilde{L}_n^{J\,*} \widetilde{L}_j^K H_k^{2\,*} H_i^2 \,.$$

For significant contributions to the Higgs mass must be $x \gtrsim 0.1$. But as we showed, for typical SUSY parameters $x \lesssim 0.01$ from $\mu \to e\gamma$ bound.

Naturalness constraints on m_N^2

Large $x \sim 0.01$ seems to be phenomenologically viable. However, if one imposes the usual fine-tuning (or naturalness) requirements for the stability of the electroweak scale, one can show that $\|m_N^2\|$ cannot be significantly larger than $\mathcal{O}(v^2)$.

This can be verified by computing the one-loop correction to the H_2^2 self-energy. The surviving contribution to the squared-mass term of H_2^2 is of the form

$$m_N^2 |Y_\nu|^2 \mathcal{I}(M^2, m_N^2) |H_2^2|^2 \,,$$

where $\mathcal{I}$ is a logarithmically divergent integral.

We now add this one-loop result to the corresponding tree-level contribution to the scalar potential:

$$(m_{H_2}^2 + |\mu|^2) |H_2^2|^2 \,.$$

In order to achieve successful electroweak symmetry breaking with $v = 246$ GeV, the complete coefficient multiplying $|H_2^2|^2$ must be of $\mathcal{O}(v^2)$. Thus :

$$m_N^2 \sim \mathcal{O}(v^2) \,.$$