Tachyon Condensation and Quark Mass in Modified Sakai-Sugimoto Model

Avinash Dhar

Tata Institute of Fundamental Research, Mumbai

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References

This talk is based on the following work:

- A.D. and Partha Nag, arXiv:0708.3233
- A.D. and Partha Nag, arXiv:0712.nnnn

Related work is:

- Casero, Kiritsis and Paredes, hep-th/0702155
- Bergmann, Seki and Sonnenschein, arXiv:0708.2839

Plan

Yang-Mills

- Yang-Mills part: N_c overlapping $D4$-branes, filling $(3+1)$-d space-time and wrapping a circle of radius R_k with anti-periodic b.c. for fermions

- Energies $<< l_s$: theory on $D4$-branes is $(4+1)$-d pure Yang-Mills with 't Hooft coupling $\lambda_5 = (2\pi)^2 g_s l_s N_c$ of length dimension

- Energies $<<$ KK scale $1/R_k$: theory is pure Yang-Mills in $(3+1)$-d. True only in the weak coupling regime, $\lambda_5 << R_k$

Flavours

- Flavours: come from N_f overlapping $D8$-branes and N_f $\overline{D8}$-branes a distance l_0 apart on the circle. Intersection with $D4$-branes is $(3+1)$-dimensional space-time.
- Quarks: arise as massless open strings between $D4$-branes and $D8$-branes (N_f left-handed flavours) and $\overline{D8}$-branes (N_f right-handed flavours)
- $U(N_f)_L \times U(N_f)_R$ chiral symmetry of QCD is visible on the $D8$-branes and $\overline{D8}$-branes as chiral gauge symmetry.

Brane configuration

Weak coupling: $\lambda_5 << R_k$ - This is described by the following intersecting brane configuration

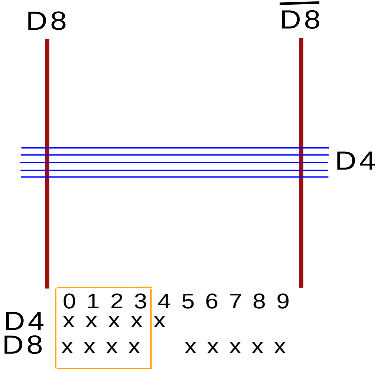

Figure: Sakai-Sugimoto Brane configuration

Background geometry

- Strong coupling: $\lambda_5 >> R_k$ - The appropriate description of the wrapped $D4$-branes is given by the dual background geometry ($R^3 = \frac{\lambda_5 l_s^2}{4\pi}$):

$$ds^2 = (U/R)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)\,(dx^4)^2\right) + (U/R)^{-3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^\phi = g_s(U/R)^{3/4} \qquad F_4 = \frac{2\pi N_c}{V_4} \qquad f(U) = 1 - U_k^3/U^3$$

- conical singularity at $U = U_k$ in the $U - x^4$ plane is absent if

$$R_k = \frac{2}{3}(R^3/U_k)^{1/2}$$

Flavour physics on $D8$-brane probes

Place $D8\ \overline{D8}$-brane pairs in the background geometry at points $x_L^4 = l/2$ and $x_R^4 = -l/2$ respectively on the circle. In this setting χSB has a geometrical description

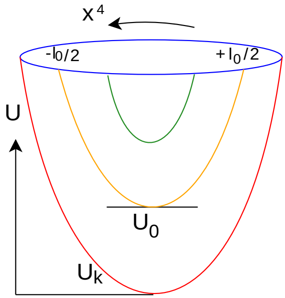

Figure: Brane profile and χSB in the Sakai-Sugimoto model

Problems with Sakai-Sugimoto model

- No bare quark mass: Unlike QCD, there is no bare quark mass parameter in the Sakai-Sugimoto model. There is also no computation to determine the chiral condensate.
- What is l_0? There is no counterpart of this parameter in QCD.
- Tachyon cannot be ignored Ignoring the open string tachyon between $D8$-branes and $\overline{D8}$-branes may be reasonable in the UV where they are well separated, but is not so in the IR where the branes join. In the IR region one expects the tachyon to condense to an infinitely large value in the true ground state!

The Action with the Tachyon

Inclusion of tachyon affects the DBI action describing the effective low-energy physics of the flavour brane and antibrane in several ways (for superstring theory the tachyon τ is complex, with $T = |\tau|$):

$$S = -2V_4R^{3/4}\int d^4x\int dU\, V(T)\, U^{13/4} \times\sqrt{f^{-1}(U/R)^{-3/2}+\frac{f}{4}(U/R)^{3/2}l'^2+T'^2+T^2l^2} + \text{C. S. terms}$$

Strictly speaking the above form of action is valid only for brane-antibrane separated along a non-compact direction. However....

The Tachyon Potential

The potential $V(\tau)$ depends only on the modulus T of the complex tachyon τ. It is believed that $V(T)$ satisfies the following general properties:

- $V(T)$ has a maximum at $T = 0$ with $V(0) = \mathcal{T}_8$.
- The normalization of $V(T)$ is fixed by the requirement that the vortex solution on the brane-antibrane system produce the correct relation between Dp and $D(p-2)$-brane tensions.
- In flat space for brane-antibrane on top of each other (i.e. for $l = 0$), the expansion of $V(T)$ around $T = 0$ upto terms quadratic in T gives rise to a tachyon with mass-squared equal to $-\pi$ in our conventions.
- $V(T)$ has a minimum at $T = \infty$ where it vanishes.

The Tachyon Potential

There are several proposals for $V(T)$ which satisfy these requirements, but no rigorous derivation exists. Examples are:

- Potential from boundary string field theory computation:

$$V(T) = \mathcal{T}_8 \, e^{-\frac{\pi}{2} T^2}$$

- Potential from study of decay of unstable D-branes in two-dimensional string theory:

$$V(T) = \mathcal{T}_8 \, \mathrm{sech} \sqrt{\pi} T \sim e^{-\sqrt{\pi} T} (\mathrm{large}\, T)$$

When is Tachyon "tachyonic"?

$V(T)$ has a small T expansion of the form

$$V(T) \sim 1 - \frac{\pi}{2} T^2$$

Thus, effective mass squared of the "tachyon" is

$$m_T^2 \sim \frac{f(U/R)^{3/2} l^2}{1 + f(U/R)^{3/2} l'^2} - \pi$$

- For large U, m_T^2 is positive for any nonzero value of l_0
- If $l \to 0$ at some finite value of U such that $l' \to \infty$, then sufficiently near this point m_T^2 will become tachyonic

Classical Equations

Explicit dependence on R can be removed (except for an overall factor in the action) through a redefinition of variables:

$$U = u/R^3, \quad l(U) = R^3 h(u), \quad U_k = u_k/R^3.$$

In terms of the new variables, the classical eqns for the tachyon $T(u)$ and the brane pofile $h(u)$ are:

$$\begin{aligned} \left(\frac{u^{\frac{13}{4}}}{\sqrt{d_T}} T'\right)' &= \frac{u^{\frac{13}{4}}}{\sqrt{d_T}} \left[h^2 T + \frac{V'(T)}{V(T)} (d_T - T'^2)\right] \\ \left(\frac{u^{\frac{19}{4}}}{\sqrt{d_T}} \frac{f}{4} h'\right)' &= \frac{u^{\frac{13}{4}}}{\sqrt{d_T}} \left[T^2 h - \frac{V'(T)}{V(T)} \frac{f}{4} u^{\frac{3}{2}} h' T'\right] \end{aligned}$$

Approximate Solution - UV region

We seek solutions in which the brane separation approaches a constant in the UV (large u) region. The equation for T linearizes if T is small in this region. Then it can be neglected in the equation for h. The solution is:

$$\begin{aligned} T(u) &= \frac{1}{u^2}\left(T_+ e^{-h_0 u} + T_- e^{h_0 u}\right) + \cdots \\ h(u) &= h_0 - h_1 u^{-9/2} + \cdots \end{aligned}$$

- Solution depends on four parameters $T_+,\ T_-,\ h_0,\ h_1$
- Solution exists for any potential
- $|T_-|$ will be identified with bare quark mass (non-normalizable solution)

Approximate Solution - IR region

We seek solutions in which the brane separation goes to zero in the IR (small u) region. We also want the tachyon to diverge in this region. Assume power law behaviour for both $h(u)$ and $T(u)$. The solution is:

$$h(u) = \sqrt{\frac{26}{\pi u_0 f_0}} u_0^{-3/4} (u - u_0)^{1/2} + \cdots$$

$$T(u) = \frac{\sqrt{\pi}}{4} f_0 u_0^{3/2} (u - u_0)^{-2} + \cdots$$

- Solution depends only on one parameter, u_0!
- This solution exists only for the potential $V(T) = \mathcal{T}_8 \mathrm{sech} \sqrt{\pi} T$
- Other potential has no diverging solution for $T(u)$, $u \sim u_0$

Numerical Solutions

- Equations for $h(u)$ and $T(u)$ cannot be solved analytically. To go beyond approximate solutions, one needs to use numerical tools.; results for the second potential, $V(T) = \mathcal{T}_8 \mathrm{sech} \sqrt{\pi} \mathrm{T}$
- Starting from the IR ($u = u_0$) end, evolve towards the UV end. This avoids fine-tuning, necessary if one starts from UV end, where general solution has four parameters, and evolves to the one-parameter subspace at the IR end.
- Choose $u_k = 1$ and use fairly largish values of u_0 such that $f(u) \sim 1$ for all $u \geq u_0 \implies$ the asymptotic separation between the brane and antibrane is small.

Exact solutions

For b.c.'s at the IR, use the approximate solution at a point $u = u_1$ as close to u_0 as allowed by numerics. Starting from the values of $T(u_1)$, $T'(u_1)$, $h(u_1)$ and $h'(u_1)$, evolve the system to larger values of u.

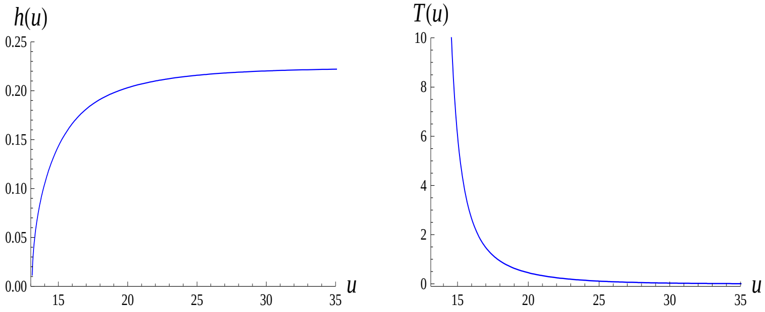

Figure: Solutions for the brane profile and the tachyon. Here $u_k = 1$ and $u_0 = 13.1$

Verification of approximate IR solution

The approximate IR forms for $h(u)$ and $T(u)$ can be verified from the numerical solutions.

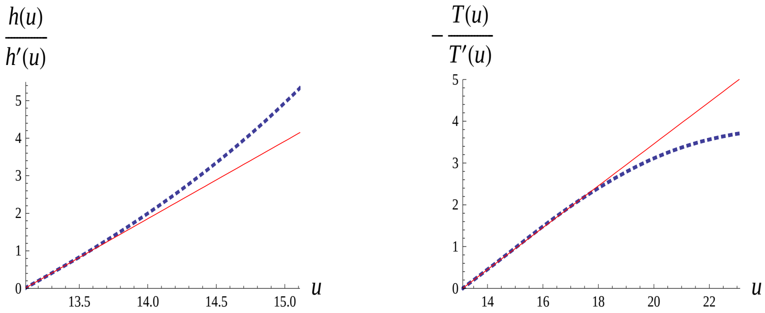

Figure: Numerical verification of exponents in the approximate IR form of brane profile and tachyon solutions. Fits give the two exponents to be 0.50 and -2.07 for $u_0 = 13.1$

Verification of approximate UV solution

The approximate UV forms for $h(u)$ and $T(u)$ can also be verified from the numerical solutions. The fits yield values of the four parameters: $h_0 = 0.224$, $h_1 = -16068$, $T_+ = 26206$, $T_- = -1.4 \times 10^{-4}$.

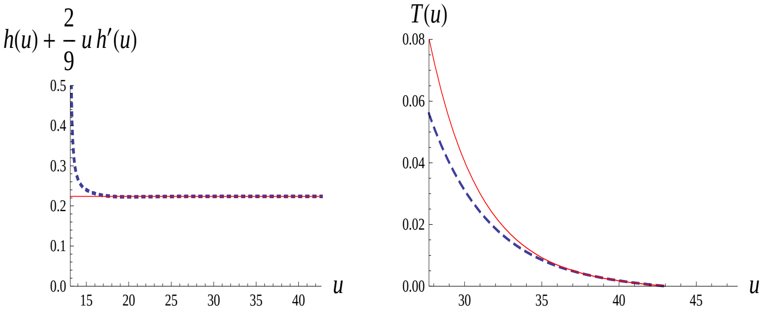

Figure: Numerical verification of the approximate UV forms of the brane profile and the tachyon solution

Role of T_-

T_- is the coefficient of the rising exponential in $T(u) \implies$ for a sufficiently large value of u this term dominates. Eventually $T(u)$ begins to rise and becomes so large that the conditions under which the approximate solutions in the UV were obtained no longer apply.

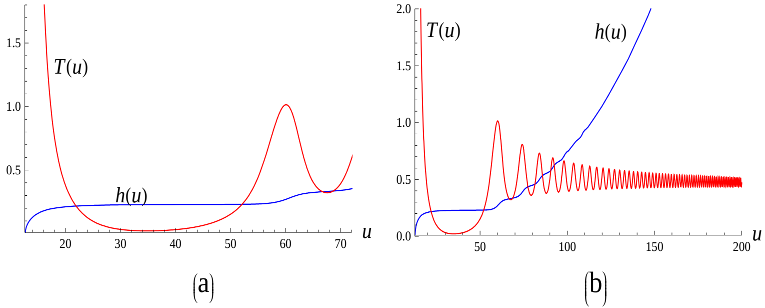

Figure: Solutions for two different large values of u for $u_0 = 12.7$

T_- as a function of u_0

The value of T_- decreases with increasing u_0. Marginally increasing the value of u_0 from $u_0 = 13$ to $u_0 = 13.0878$ dramatically decreases T_-. Fine-tuning u_0 such that T_- is precisely zero is hard.

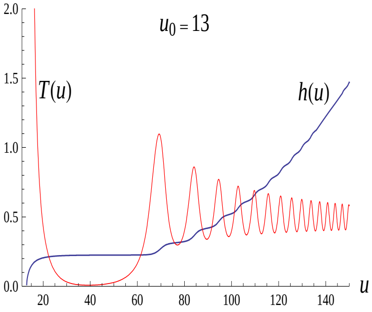

Figure: Solutions for increasing values of u_0 for positive T_-

T_- as a function of u_0

Increasing u_0 beyond this value makes T_- negative.

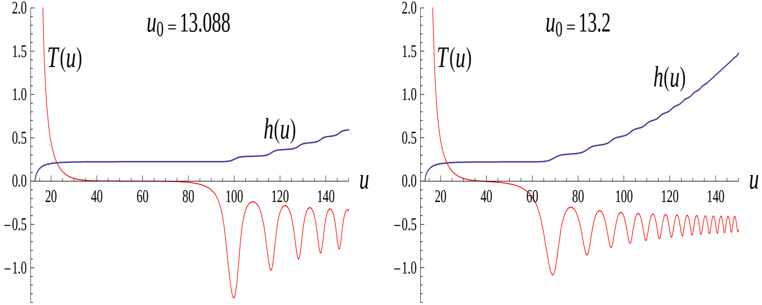

Figure: Solutions for increasing values of u_0 for negative T_-

T_- is quark mass

Change of T_- with u_0 is monotonic, as far as we can tell. The parameter u_0 can be traded for T_-, which we want to identify with bare quark mass.

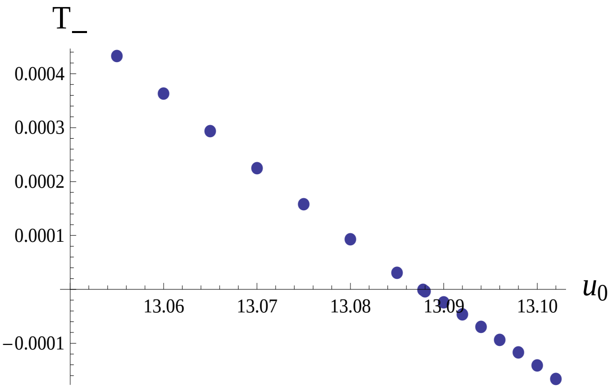

Figure: T_- as a function of u_0

Need for a cut-off in u

- At large U of order $N^{4/3}$, string coupling becomes large and one needs to go over to M-theory description of the background. However, there is no known lift of $D8$-brane to M-theory. So, with $D8$-branes present, we need to impose UV cut-off, $u_{\max}$

- The precise value of the cut-off $u_{\max}$ is dictated by the values of T_+ and T_-. The approximate UV solution is valid if the following condition holds:

$$T_+^2 e^{-2h_0 u_{\max}} + T_-^2 e^{2h_0 u_{\max}} << \frac{u_{\max}^{5/2}}{2h_0^2}$$

Spectra for low-spin mesons

Action for quadratic fluctuations of the gauge fields on $D8$-brane and $\overline{D8}$-brane:

$$\begin{aligned}\Delta S_{\text{gauge}} &= -\int d^4x \int_{u_0}^{\infty} du \Big[a(u)A_u^2 + b(u)A_\mu^2 \\ &+c(u)\left((F_{\mu\nu}^V)^2 + (F_{\mu\nu}^A)^2\right) \\ &+d(u)\left((F_{\mu u}^V)^2 + (F_{\mu u}^A)^2\right) + e(u)F_{\mu u}^A A^\mu \Big]\end{aligned}$$

- F^V: Field strength for the vector field, $V = (A_{D8} + A_{\overline{D8}})$
- F^A: Field strength for the gauge-invariant axial vector field, $A = (A_{D8} - A_{\overline{D8}} - \partial\theta)$

Massless pseudoscalar

Sakai-Sugimoto model has $a(u) = b(u) = e(u) = 0$. This gives rise to a massless pseudoscalar state as follows.

- Fix the gauge $V_u(x, u) = 0$, $A_u(x, u) = 0$. This leaves the u-independent transformations as residual gauge symmetries
- Impose the IR b.c.: $A_\mu(x, u)|_{u=u_0} = 0$. This implies u-independent gauge transformation for the axial-vector is an x-independent constant
- Impose the UV b.c: $F^V(x, u)|_{u=\infty} = 0$, $F^A(x, u)|_{u=\infty} = 0$. This means $V_\mu(x, \infty)$ and $A_\mu(x, \infty)$ are pure gauge. Residual gauge symmetry for the vector field implies we can set $V_\mu(x, \infty) = 0$. Pure gauge part of the axial vector (at $u = \infty$) gives rise to a massless pseudoscaler (from the mass term).

Pseudoscalar masses

Expanding the gauge-field $A_u(x,u)$ in modes in $(3+1)$-dim space-time, $A_u(x,u) = \sum_m \phi^{(m)}(x) S_m(u)$, the pseudoscalar masses λ_m^{ϕ} are given by

$$\lambda_m^{\phi} = 2 \int du \; a(u) (S_m(u))^2$$

The mode functions $S_m(u)$ satisfy certain eigenvalue equations and orthonormality conditions

Lightest pseudoscalar

- The lightest pseudoscalar corresponds to $m = 0$. It can be shown that λ_0^ϕ vanishes unless T_- is nonzero
- The leading term in the limit of small T_- gives $\lambda_0^\phi \sim T_- T_+$ (after ensuring that the mass vanishes for $T_- = 0$ when the cut-off is removed). This is essentially the Gell-Mann-Oakes-Renner relation of quark mass parameter and chiral condensate with the pion mass.

Summary

- It is necessary to modify the Sakai-Sugimoto model to take into account the open string tachyon stretching between the flavour branes and anti-branes.
- Tachyon condensation in the bulk provides the necessary mechanism to join the branes and anti-branes, which breaks chiral symmetry of the boundary theory. It also provides the quark mass parameter, giving rise to a massive pion
- The modified model provides a geometric picture of χSB in a holographic QCD setting which is complete in all its well-known qualitative features.