Effective Actions in IIB Flux Compactifications

Anshuman Maharana

DAMTP, Cambridge University

Strings to LHC II, Bangalore

Introduction/Motivation

String compactifications with flux have many phenomenologically attractive features

- Moduli Fixing, models with flux have a potential for hidden sector scalars.

- Scalar potential, crucial for input for analysis of models.

- II B string theory, explicit examples of flux compactifications available. We will explore dynamics in these compactifications to study the potential.

Scalars

- Perturbations of g_{mn} are 4D scalars.

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n$$

- Analysis of 10D Einstein Equations (Lichnerovich operator)

$$-\nabla_k \nabla^k h_{mn} - R_{msnt} h^{st} = \lambda h_{mn}$$

- $\lambda = 0$, massless modes.

- $\lambda > 0$, massive modes.

- Massless modes, effective action using zeromodes h_{mn}.

Flux Compactifications

- Higher rank form fields F_{mnp}, F_{mnpq}, H_{mnp} ......

- Flux thread cycles of internal manifold

$$\frac{1}{2\pi\alpha'}\int_{\Sigma_A} F = 2\pi N$$

- Analysis of the small fluctuations shows reduction in the number of zero modes.

- Term in supergravity action

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{g}\frac{G_{mnp}\overline{G}^{mnp}}{12\mathrm{Im}\tau} \qquad G = F - \tau H$$

- Study the spectrum II B in these backgrounds, probe the potential for scalars at the quadratic order.

II-B Flux Compactifications

Data

1. Orientifold of Calabi-Yau, with moduli z_α, ρ_α
   More generally a F-theory base.

2. Closed three form flux satisfying Dirac quantization conditions.

3. Position of D-3 branes.

4. Gauss Law Constraint

$$\int H_3 \wedge F_3 + 2T_3\kappa_{10}^2 Q_3 = 0.$$

Solution

- Metric

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n$$
$$= e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n$$

- Warp factor

$$-\tilde{\nabla}^2 e^{-4A} = \frac{G_{mnp}\overline{G}^{\widetilde{mnp}}}{12\text{Im}\tau} + 2T_3\kappa_{10}^2\tilde{\rho}_3^{loc}$$

- ISD Condition $\tilde{g}_{mn}$ and τ such that

$$G_3 = -i * G_3$$

Condition fixes values of z_α and τ, reduces the number of 4d massless degrees of freedom.

Hierarchy from Fluxes

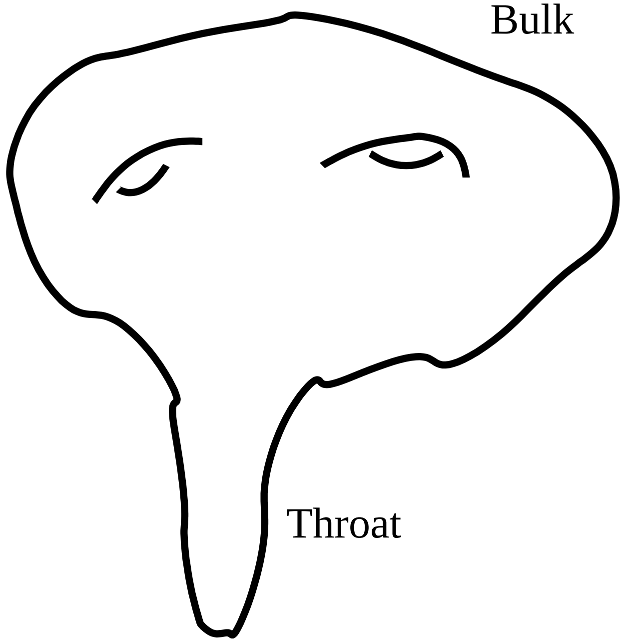
Large but finite redshifts between points on the internal manifold, can be used to explain small numbers

- A conifold singularity on CY_3.

- Much like in the Klebanov-Strassler solution

$$\frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi M \qquad \frac{1}{2\pi\alpha'}\int_B H_3 = -2\pi K$$

- Conifold deformed
  $e^{A_0} \sim$ minimum value of the warp factor,
  Exponentially small in the integer choice of fluxes.



- Bulk : warp factor a constant, local geometry $CY_3 \times M_4$

$$\frac{V_{\text{bulk}}}{\alpha'^3} \gg 1$$

- Throat : highly warped, local geometry $X_5 \times AdS_5$

$$\frac{V_{\text{throat}}}{V_{\text{bulk}}} \ll 1$$

- Throats fairly generic in such compactifications.
  Hebecker, Russel hep-th/0607120

- hep-th/0507158
  S.Giddings, AM
- hep-th/0603233
  Andrew Frey, AM
- hep-th/0610255
  C.Burgess, P.Camara, S. De Alwis, S.Giddings, AM, F.Quevedo, K.Suruliz.

Outline

- Volume Modulus.
- Dimensional Reduction, estimates for moduli masses.
- Supersymmetry Breaking.
- Future directions, Summary.

Volume Modulus

Giddings, AM

- Metric and warp factor

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\tilde{g}_{mn}dy^m dy^n$$
$$-\tilde{\nabla}^2 e^{-4A} = \frac{G_{mnp}\overline{G}^{\widetilde{mnp}}}{12\mathrm{Im}\tau} + 2T_3\kappa_{10}^2\tilde{\rho}_3^{loc}$$

- Kahler moduli unfixed.

- Universal Kahler modulus
  - $\tilde{g}_{mn} \to \lambda\tilde{g}_{mn} \Rightarrow e^{-2A} \to \frac{1}{\lambda}e^{-2A}$
  - $e^{-2A}\tilde{g}_{mn}$ is unchanged.

- Modulus is not pure rescaling, but associated with freedom to add homogeneous piece

$$e^{-4A} = e_p^{-4A} + c$$

- Einstein frame metric

$$ds^2 = \frac{1}{c}(e_p^{-4A} + c)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (e_p^{-4A} + c)^{1/2}\tilde{g}_{mn}dy^m dy^n$$

  - e_p^{-4A} is a particular solution, $e_p^{-4A} \to \mathcal{O}(1)$ in the bulk.
  - $c \sim \mathcal{V}^{2/3}$, $\mathcal{V}$ volume in string units.

- Relative redshift (Hierarchy)

$$e^{2H} = \frac{\left(c + e^{-4A_0}\right)^{1/2}}{(c + e_p^{-4A})^{1/2}_{\text{bulk}}}$$

  - $c \to \infty$, $c \gg e^{-4A_0}$

$$e^{2H} \to 1$$

    No Throats (Fluxes α' effects), recover CY.

  - Hierarchy volume dependent

$$e^{2H} \approx \frac{e^{-2A_0}}{c^{1/2}}$$

Dynamics in Infinite Volume Limit

- $c \gg e^{-4A_0}$, warp factor drops out, effective dynamics of moduli

$$W = \int G \wedge \Omega \quad \text{Gukov Vafa Witten Superpotential}$$

$$\mathcal{K} = -\ln\left[-i(\tau-\overline{\tau})\right] - 3\ln\left[-i(\rho-\overline{\rho})\right] - \ln\left[-i\int \Omega \wedge \overline{\Omega}\right]$$

- Small fluctuation analysis of 10D EOM about background,

  Wavefunction of excitation same as zero flux case.

- $$m_{\rm mod} \approx \frac{n}{\mathcal{V}} \quad m_{\rm kk} \approx \frac{1}{\mathcal{V}^{2/3}} \quad \frac{m_{\rm mod}}{m_{\rm kk}} \approx \frac{1}{\mathcal{V}^{1/3}}$$

- What happens at smaller volumes?

Towards Finite Volume

- Essentially an eigenvalue problem, spectrum of Lichnerowicz operator (Δ) obtained from analysis of small fluctuations about our background. Systematics discussed in hep-th/0507158 (Giddings, AM).
- Qualitative Description (using dilaton)
  - True eigenfunction of Δ, different from the zero mode on $\tilde{g}_{mn}$

$$\delta\tau = \delta\tau^0 + a_i \delta\tau_i^{\widetilde{kk}}$$

  - Mixing between z_α and $\delta\tau$.
  - Mixing between fluxes and $\delta\tau$.
- Should not be a surprise, in the presence of flux, excitations are

Massive, Non-Supersymmetric, No Protection

- "small" or "moderate" warping can organize these effects in an expansion in inverse powers of "c".

- "large" warping with throats expansion becomes unreliable
  - $\frac{e_p^{-4A}}{c} \gg 1$
  - strong mixing between $\widetilde{\mathrm{kk}}$ modes and zero modes of $\tilde{g}_{mn}$, disturbing feature from the perspective of 4d effective field theory.

- Expansion in inverse volume may be more subtle, need to try and solve for the spectrum explicitly.

Modeling the Dilaton

Andrew Frey, AM

- Approximate dynamics by keeping dependence on single radial variable, direction of variation of warp factor.
- Equation of motion for the dilaton

$$(\tilde{\nabla}^2 - \frac{e^{4A} G_{mnp} \bar{G}^{\widetilde{mnp}}}{12 \mathrm{Im}\tau}) \delta\tau = -m_n^2 e^{-4A}$$

- After $dw = e^{-2A} dr, \quad \delta\tau = e^{-3A/2}\psi$, can be brought to the Schrodinger form

$$[-\partial_w^2 + V(w)]\psi(w) = m_n^2 \psi(w)$$

with

$$V(w) = e^{-3A/2} \partial_w^2 e^{3A/2} + \frac{e^{2A} G_{mnp} \bar{G}^{mnp}}{12 \mathrm{Im}\tau}$$

local function of the warp factor and the fluxes.

- "c" (volume) a parameter in the potential

$$V(w) = [e_p^{-4A} + c]^{3/8} \partial_w^2 [e_p^{-4A} + c]^{-3/8} + [e_p^{-4A} + c]^{-2} \frac{G_{mnp} \bar{G}^{\widetilde{mnp}}}{12 \mathrm{Im}\tau}$$

- For $c > e^{-4A_0}$, potential uniform on the internal manifold.
- For $c < e^{-4A_0}$, significant relative redshift between points on the internal manifold, the potential develops a distinct well, one would expect lowest "Energy" modes to be localized in the well.

Semiclassical Estimates

- In the throat and bulk region

$$\mathcal{E}_{\text{thrt}} \sim \frac{e^{2A_0}}{n'^2 \mathcal{V}^{2/3}} \qquad \mathcal{E}_{\text{bulk}} \sim \frac{n^2}{\mathcal{V}^2}$$

- Competition between volume and warping.
  For a fairly mild condition

$$e^{-A_0} > \mathcal{V}^{2/3}$$

  wavefunction localizes in the throat and the dilaton has mass

$$m_\tau \approx \frac{e^{A_0}}{n' \mathcal{V}^{1/3}}$$

  n' flux quantum number which sets the size throat at the infrared end.

Spectrum

- No parametric separation with the Kaluza-Klein scale

$$m_{kk} \sim m_0 \approx \frac{e^{A_0}}{n' \mathcal{V}^{1/3}}.$$

- Wavefunction of low lying spectrum localized in the throat.
- $[e^{-A_0}]$ localized modes.
- The warped string scale is

$$M_s^w \sim \frac{e^{A_0}}{\mathcal{V}^{1/3}}$$

- Robust, numerical checks.

Summary of Spectrum and Wavefunctions

- Infinite volume limit, $c \gg e^{-4A_0}$
  Wavefunction uniformly spread in internal directions

$$m_{\mathcal{T}} \sim \frac{n}{\mathcal{V}}, \qquad m_{\rm kk} \sim \frac{1}{\mathcal{V}^{2/3}}$$

- Decrease "c" , keeping flux quanta fixed.
  Mass and wavefunction of excitation change continuously
- Throat develops for $c < e^{-4A_0}$
- For $c < e^{-A_0}$ (long throat)
  - wavefunction localized
  - masses

$$m_{\mathcal{T}} \sim m_{kk} \sim \frac{e^{A_0}}{n' \mathcal{V}^{1/3}}$$

Comments

- Expect phenomenon to be fairly generic, recall dilaton wave-function uniform in the infinite volume limit.
- Gukov Vafa Witten Superpotential
  " Dynamics of moduli with kk modes integrated out."
  Localization under adiabatic change in the volume implies GVW superpotential relevant for the lightest modes, localized in the deepest throats.
  But $m_{\text{mod}} \sim m_{\text{kk}}$, therefore Cmplx. Structure moduli and dilaton should be integrated out with the kk modes.
- Mixing of $\widetilde{\text{kk}}$ modes
  Fourier decomposition of a localized function with have all harmonics, localization in region of large redshift energetically favourable. Thinking in terms of harmonics of $\tilde{g}_{mn}$, breakdown of the moduli space approximation.

Supersymmety Breaking

Burgess, Camara, De Alwis, Giddings, AM, Quevedo, Suruliz

- In these constructions, SUSY broken by (0,3) flux, gravitino acquires a mass by the superhiggs effect.
- II B gravitino equations of motion

$$\Gamma^{MNP}\hat{D}_N\Psi_P = -\frac{i}{2}\Gamma^P\Gamma^M\hat{\lambda}^*P_P - \frac{i}{48}\Gamma^{NPQ}\Gamma^M\hat{\lambda}G^*_{NPQ}$$

Relevant dynamical equation, $\Psi(x,y) = \psi \otimes \eta(y)$

$$\mathcal{V}^{2/3}\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho \otimes \eta(y) + \frac{1}{24}\gamma^{\mu\nu}\psi^*_\nu \otimes e^A G_{mnp}\gamma^{mnp}\eta^* = 0$$

- Again for $e^{-A_0} > \mathcal{V}^{2/3}$, localization and

$$m_{3/2} \sim \frac{e^{A_0}}{\mathcal{V}^{1/3}}$$

Kahler Potential

- Formula for gravitino mass in N=1 formalism $m_{3/2} = e^{K/2}|W|$
  In the infinite volume limit $c \gg e^{-4A_0}$,

$$m_{3/2} \sim \frac{|W|}{\mathcal{V}} \Rightarrow K \sim -2\ln\mathcal{V} = -3\ln[-i(\rho - \bar{\rho})]$$

- Assume : Gukov Vafa Witten Superpotential receives no finite volume correction

$$m_{3/2} \sim \frac{e^{A_0}|W|}{\mathcal{V}} \Rightarrow K \sim 2A_0 - \frac{2}{3}\ln\mathcal{V}$$

- – No success in getting an analytic expression so far.
  – Expansion around the infinite volume $\mathcal{O}(\frac{e^{-A_0}}{\mathcal{V}^{2/3}})$

$$\alpha' \qquad N\alpha' \qquad e^{-A_0}\alpha'$$

Soft Supersymmetry Breaking

- $W_0 \sim 1$, Susy breaking 10d.
- In the limit of $W_0 \to 0$, can use $\mathcal{N} = 1$ formalism to study soft terms on brane.
- For instance, soft susy breaking masses for scalars in terms of hidden sector auxiliary vevs F^m (with diagonal flavor mass matrix)

$$m_Y^2 = m_{3/2}^2 - F^m F^{\bar{n}} \partial_m \partial_{\bar{n}} \ln K_Y$$

- Alternatively can explicitly compute them using DBI action in the highly warped background, this yields

$$m_Y \sim \frac{e^{A_0}}{\mathcal{V}^{1/3}}$$

- With our prescription of adding a piece linear in A_0 to the Kahler potential, powers of warpfactor agree for both computations.

Applications/Future Directions

Throats generic, used for large number of phenomenological applications.

- Cosmological Moduli Problem, for towers localized in the throats.
- When the effects that fix the Kahler moduli are incorporated to the kahler moduli, do they exhibit localization ?
- Given the redshifted masses of the complex structure moduli and dilaton, compare the scales associated with the kahler moduli and complex complex structure moduli.
- Effects of warping in brane inflation models.

Summary

- Warping generic feature of IIB flux compactifications.
- For fairly mild conditions on the volume,

  $$\mathcal{V}^{2/3} < e^{-A_0}$$

  Some new features -

  – Localization of moduli wave functions.

  – Redshifting of moduli masses.

- Much more to explore in the rich dynamics of flux compactifications.