MODULI STABILIZATION

AND BRANE INFLATION

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Search for an Inflation Model in String Theory, with all moduli stabilized, is still on!

Massless scalar fields with undetermined VEVs are moduli fields, they donot make specific predictions for many physical quantities.

Moduli stabilization keeps track of the compactification data since moduli describe a geometry and generically arise when solving the Einstein equations.

Any realistic string inspired four dimensional model makes sense if only moduli are stabilized.

Recall Moduli Stabilzation of KKLT

Step-I: Starts with the result (GKP) that, in a flux compactification of Type IIB on Calabi-Yau three-fold, all moduli are fixed except for one Kahler modulus ρ (volume) which survives the compactification.

Leading to a constant superpotential W_0 (GVW flux superpotential) and the Kahler potential

$$\mathcal{K}(\rho, \bar{\rho}) \;=\; -\,3\ln(\rho + \bar{\rho})$$

Step-II: The volume modulus is then fixed through a non-perturbative correction to W_0, of the form:

$$W(\rho) \;=\; W_0 + A\, e^{-b\rho}$$

A is a constant, b is instanton charge for Euclidean D3- brane or $2\pi/n$ for a stack of n D7-branes wrapping a four-cycle.

KKLT studied the potential:

$$V_F \;=\; e^{\mathcal{K}}\;[G^{\bar{i}j}\overline{D_i W}\; D_j W \;-\; 3|W|^2]$$

Minimization of the potential leads to moduli stabilization.

The minimum of the potential turns out to be -ve (AdS)

Step-III: Uplifting:

Adding an anti-D3 brane to the system which sits at the tip of the throat. The warped tension of the anti-D3 branes lifts the AdS minimum to a local metastable dS vacuum.

Brane-anti-brane Inflation :

KKLMMT proposal starts with the one-modulus model of KKLT. Brane dynamics is added by including a mobile D3-brane which is drawn down the throat by it attraction towards the anti-D3 brane.

A D3-brane added to a Type IIB vacuum backreacts on the metric and changes the Kahler potential of the low energy four dimensional supergravity.

KKLMMT considered the modifications to Kahler potential, which depends upon the position, (z^i), of the D3-brane:

$$\mathcal{K}(\rho, \bar{\rho}) \quad = \quad -3 \ln[\rho + \bar{\rho} - K(z, \bar{z})]$$

The non-perturbative potential was kept as in the KKLT model.

The potential $V = V_f + V_D$, where V_D includes the D3-brane interaction with anti-D3-brane (in the warped geometry) was calculated taking $K(z, \bar{z}) \equiv K(\phi, \bar{\phi}) = \phi\bar{\phi}$. It was assumed that such a potential has a dS minimum at some values of ρ and ϕ and the mass of ϕ (D3-brane moduli/ inflaton) was computed in an expansion about this minimum and it turned out that such a mass of the inflaton field lead to a slow-roll parameter, in the inflation model,

$$\eta \quad = \quad 2/3,$$

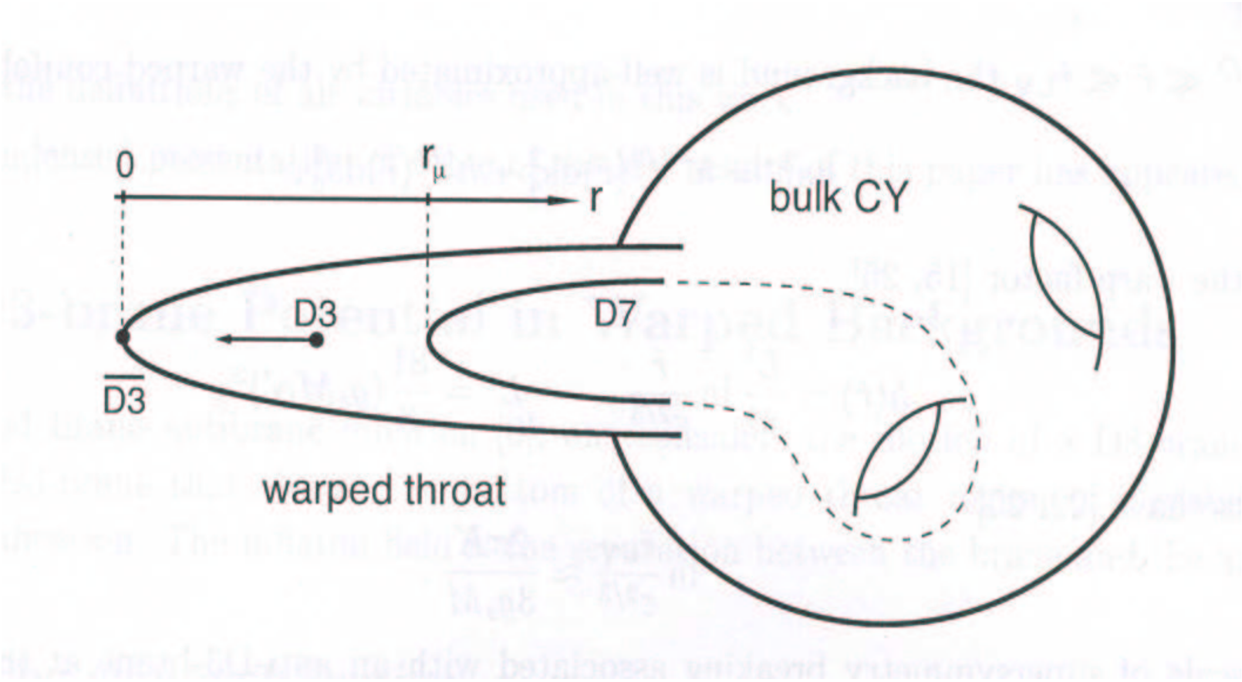
Incompatible with sustained slow-roll inflation.

Presence of D3-brane also modifies the non-perturbative superpotential Giddings and Maharana, hep-th/0507158

Does it solve the η problem?

This correction, for the warped compactification, has been computed

Bauman, Dymarsky, Klebanov, Maldacena, Mc Allister and Murugan hep-th/0607050


0
r_μ
r
bulk CY
D3
D7
D̄3
warped throat

Result: $W \;=\; W_0 \;+\; W_{np}$

$$W_{np} \;=\; A(z_\alpha)\, e^{-b\rho}$$

$A(z_\alpha)$ depends upon the embedding of D7-branes wrapping the four-cycle and preserving SUSY, specified by $f(z_\alpha) = 0$.

$$A(z_\alpha) = A_0 \left(\frac{f(z_\alpha)}{f(0)} \right)^{1/n}$$

Application to Brane Inflation: BDKMS **Delicate Universe** 0705.3837 (hep-th), BDKM 0706.0360 (hep-th), KP 0705.4682 (hep-th)

Kuperstein embedding: $f(z_1) \;=\; \mu - z_1$

$$K = \frac{3}{2} \left(\sum_{i=1}^{4} |z_i|^2 \right)^{2/3} \;=\; \frac{3}{2} r^2$$

$$\mathcal{K}(\rho, \bar{\rho}, z_\alpha, \bar{z}_\alpha) \;=\; -\,3M_{\rm pl}^2\, \ln[\rho + \bar{\rho} - \gamma K]$$

Taking $z_1 = -r^{3/2}/\sqrt{2}$, σ = Re (ρ) and canonical normalized field $\phi = \sqrt{3T_3/2}r$ with $r_\mu^3 \equiv 2\mu^2$ i.e. $\phi_\mu^2 = 3/2T_3(2\mu^2)^{2/3}$ leads to the **full two-field potential:**

$$\begin{aligned} V(\phi,\sigma) &= \frac{b|A_0|^2}{3M_{\rm pl}^2}\frac{e^{-2b\sigma}}{U^2(\phi,\sigma)}g^{2/n}(\phi)\Bigg\{2b\sigma+6 \\ &- 6e^{b\sigma}\frac{|W_0|}{|A_0|}\frac{1}{g^{1/n}(\phi)}+\frac{3}{n}\left[c\frac{\phi}{\phi_\mu}-\left(\frac{\phi}{\phi_\mu}\right)^{3/2}\right. \\ &\left. - \left(\frac{\phi}{\phi_\mu}\right)^3\right]\frac{1}{g^2(\phi)}\Bigg\}+\frac{D(\phi)}{U^2(\phi,\sigma)}\,, \end{aligned}$$

where

$$\begin{aligned} U(\phi,\sigma) &= 2\sigma-\frac{\gamma}{T_3}\phi^2\,, g(\phi) = 1+\left(\frac{\phi}{\phi_\mu}\right)^{3/2} \\ D(\phi) &= D_0\left(1-\frac{27D_0}{64\pi^2\phi^4}\right),c=1/(6\pi\gamma T_3\phi_\mu^2) \end{aligned}$$

and $D_0 = 2h_0^{-1}T_3$ i.e. twice the warped tension at the tip.

An effective single field potential: $V(\phi) \equiv V(\sigma_*(\phi), \phi)$ where $\partial_\sigma V|_{\sigma_*(\phi)} = 0$ (instantaneous minimum)

Assumes σ is more massive than ϕ and evolves adiabatically while remaining in its instantaneous minimum

$$\sigma_* \approx \sigma_0 \left[1 + c_{3/2} \left(\frac{\phi}{\phi_\mu}\right)^{3/2}\right] .$$

σ_0 related to γ and $c_{3/2}$ is related to n and W_0/A_0 **Numerical simulation does not support the assumption for truely generic configuration of a D3-brane in a compact space**

Working with the approximated expression yields number of e-foldings to be less than 10 even for highly finetuned parameters

σ is not even canonical scalar field but χ is

$$\frac{\chi}{M_{\rm pl}} = \sqrt{\frac{3}{2}} \ln \sigma .$$

Best bet is to consider a two-field inflation with $V(\chi, \phi)$

For flat FRW metric with scale factor a, Eqns of motion:

$$\begin{aligned}
\dot{H} &= -\frac{1}{2M_{\rm pl}^2}(\dot{\phi}^2 + \dot{\chi}^2)\,, \\
&\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0\,, \\
&\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} = 0
\end{aligned}$$

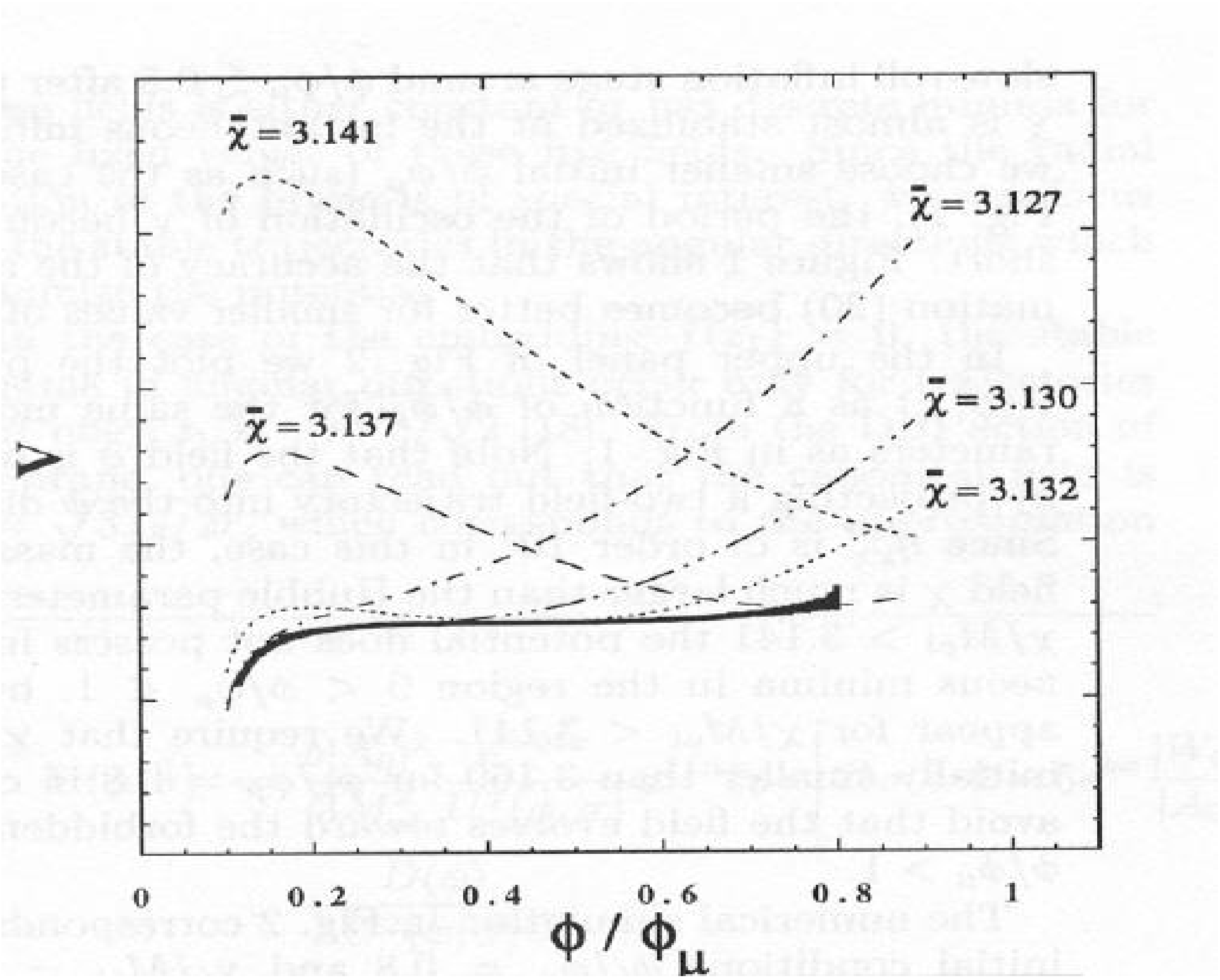
and the constraint eqn:

$$3H^2 = \frac{1}{M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\phi,\chi)\right]$$

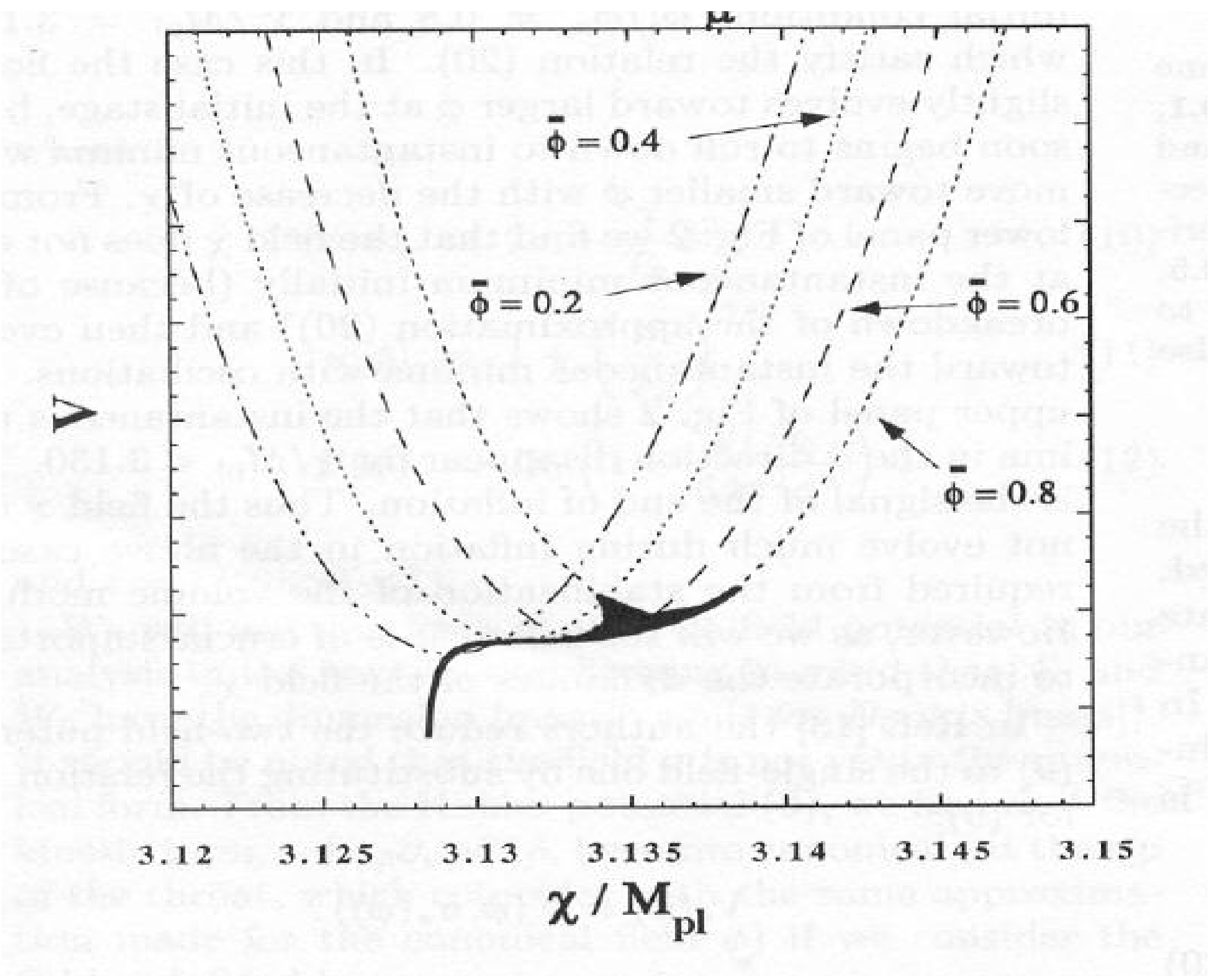
Slow roll parameters:

$$\epsilon_\phi = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2\,, \quad \epsilon_\chi = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\chi}}{V}\right)^2\,,$$

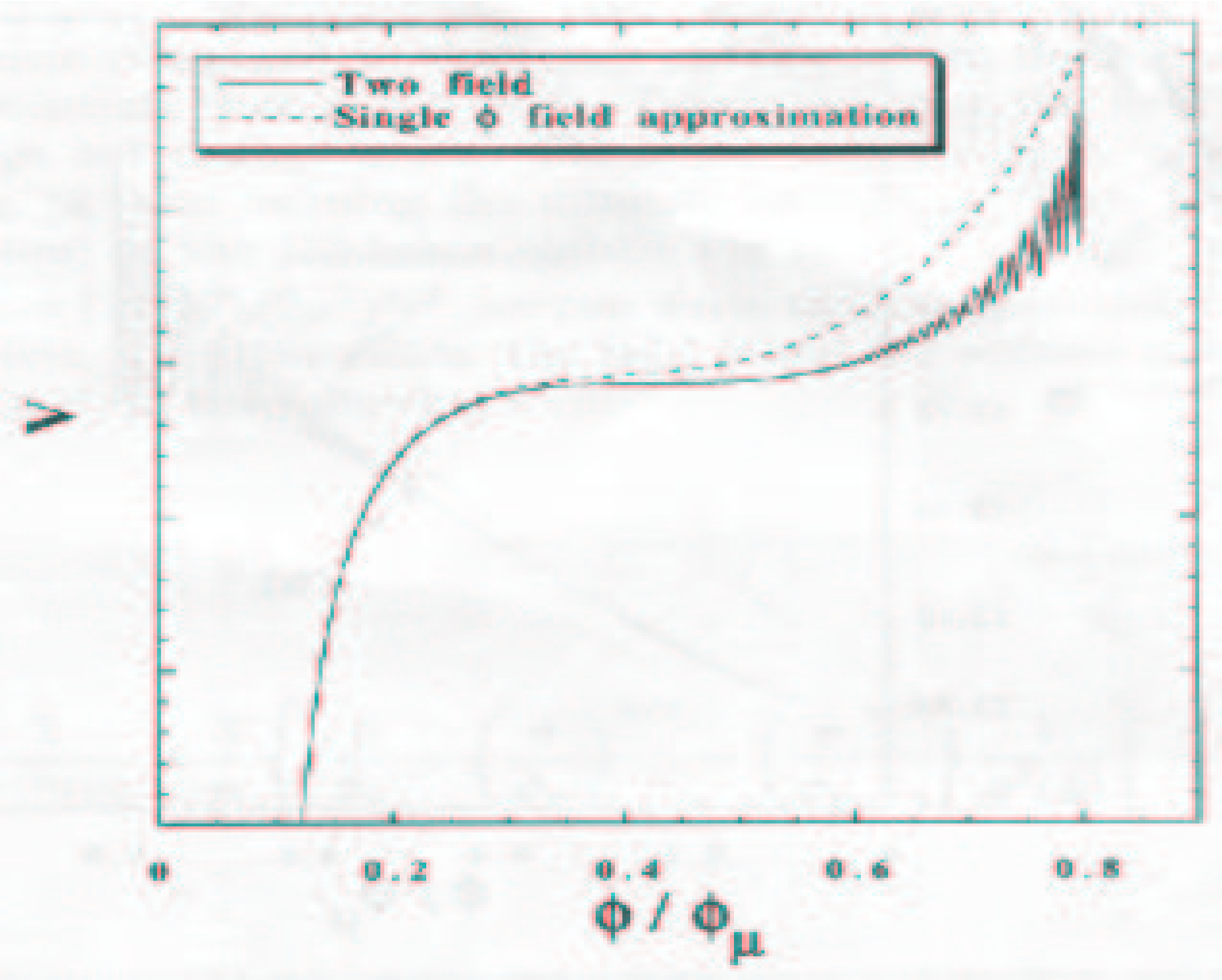
$$\eta_{\phi\phi} = M_{\rm pl}^2\frac{V_{,\phi\phi}}{V}\,, \quad \eta_{\chi\chi} = M_{\rm pl}^2\frac{V_{,\chi\chi}}{V}\,, \quad \eta_{\phi\chi} = M_{\rm pl}^2\frac{V_{,\phi\chi}}{V}\,.$$



Potential vs $\bar{\phi}$ for various fixed $\bar{\chi}$ for $n = 8, A_0 = 1, b\sigma_0 = 10.1, W_0 = 3.496 \times 10^{-4}, D_0 = 1.215 \times 10^{-8}, \phi_\mu = 0.25$ **Solid curve is obtained by solving the background eqn numerically for initial condition**$\phi/\phi_\mu = 0.8$ and $\bar{\chi} \equiv \chi/M_pl = 3.1385$

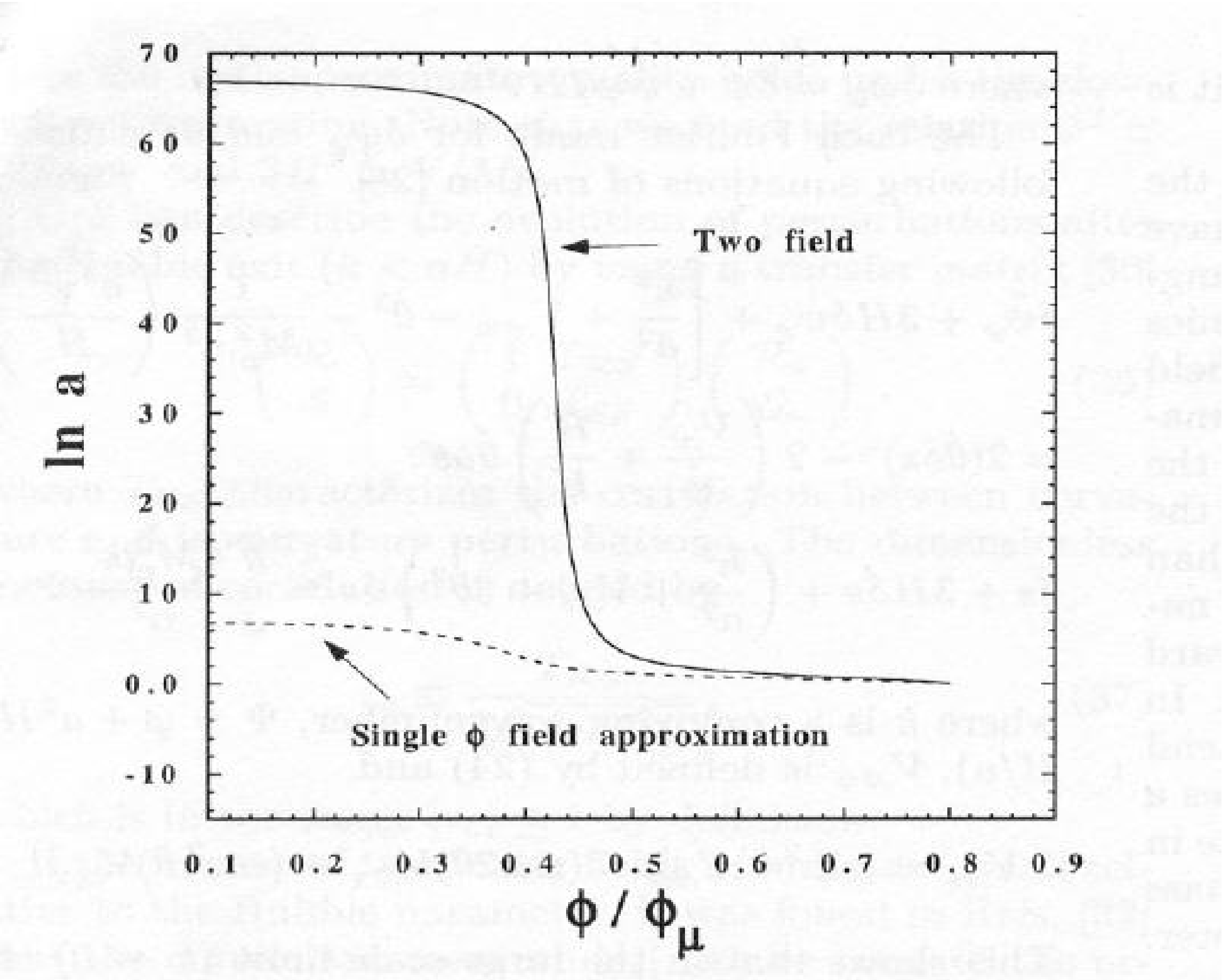


Potential vs $\bar{\chi}$ for same parameters and fixed $\bar{\phi}$



Solid curve is for Potential obtained by numerically solving the background eqns in two-field model

Dotted curve is for $V(\phi, \sigma_*(\phi))$, and same parameters.



Evolution of number of e-foldings as function of ϕ, same model parameters

Acceptable value of Number of e-foldings in two-field model

This discripancy reflects that the background trajectory is not given by the fieldϕ but the combination of both the fields.

We need to address the volume modulus stabilization!

Define the field ψ and s satisfying:

$$\dot{\psi} = (\cos\theta)\dot{\phi} + (\sin\theta)\dot{\chi} \ , \tan\theta = \dot{\chi}/\dot{\phi}\,.$$

$$\dot{s} \equiv -(\sin\theta)\dot{\phi} + (\cos\theta)\dot{\chi} = 0$$

fields donot move to the direction orthogonal to ψ.

S can be thought os as the stabilized volume modulus

Correct single field description of inflation dynamics is in terms of ψ with mass sqared:

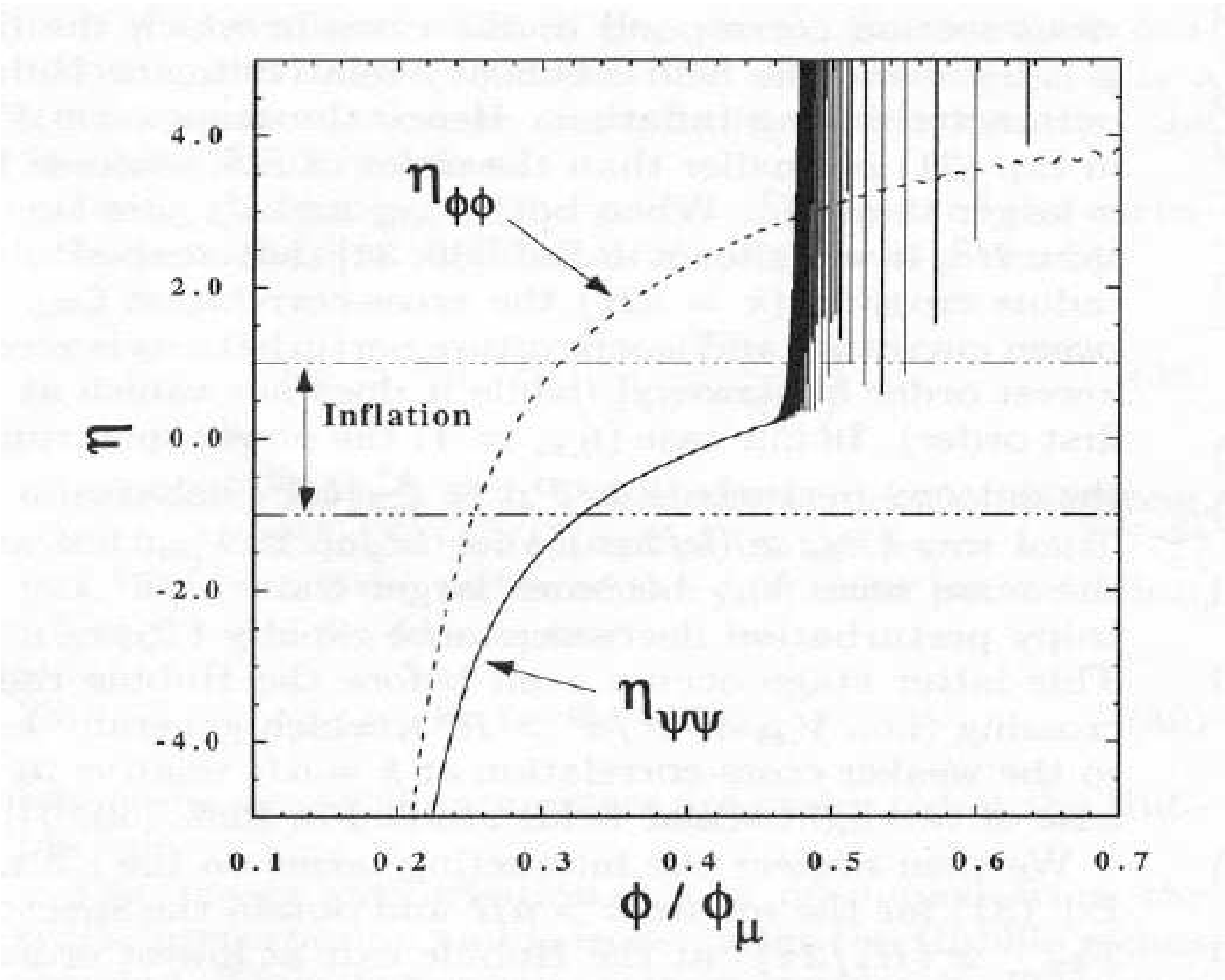
$$V_{,\psi\psi} = (\cos^2\theta)V_{,\phi\phi} + (\sin 2\theta)V_{,\phi\chi} + (\sin^2\theta)V_{,\chi\chi}\,.$$

$$V_{,ss} = (\sin^2\theta)V_{,\phi\phi} - (\sin 2\theta)V_{,\phi\chi} + (\cos^2\theta)V_{,\chi\chi}.$$

Then the slow-roll parameter, $\eta_{\psi\psi} \equiv M_{\rm pl}^2 V_{,\psi\psi}/V$, is

$$\eta_{\psi\psi} = (\cos^2\theta)\eta_{\phi\phi} + (\sin 2\theta)\eta_{\phi\chi} + (\sin^2\theta)\eta_{\chi\chi}\,.$$

Period of inflation is given by $|\eta_{\psi\psi}| < 1$ which is for $0.3 \leq \phi/\phi_\mu \leq 0.5$.



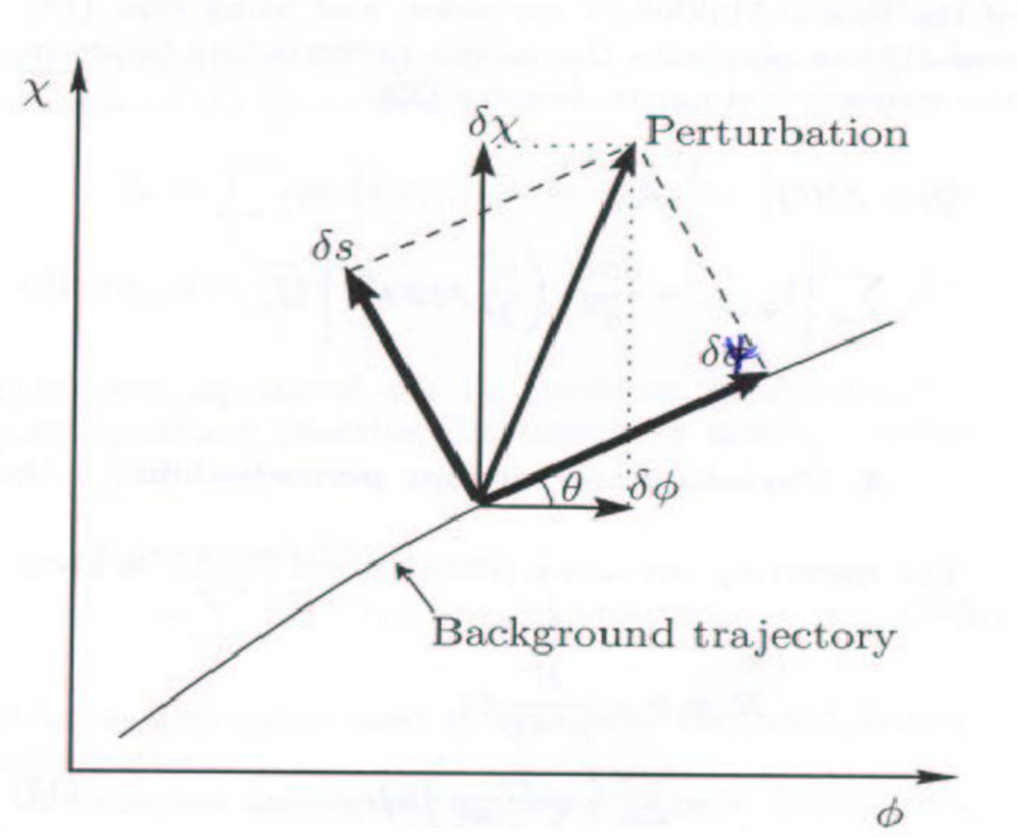
$\eta_{\phi\phi}$ is larger than unity during this period **Possible to have larger number of e-foldings for** $D_0 = 1.218 \times 10^{-8}$ instead of $D_0 = 1.215 \times 10^{-8}$, increases to 148.

If $D_0 = 1.210 \times 10^{-8}$, number of e-foldings decreases to 43

This shows how sensitive it is to model parameters and reflects severe fine tuning

COSMOLOGICAL PERTURBATIONS

In two-field model, density perturbations are different from that of single-field model due to presence of isocurvature (entropy) perturbations. Denote field perturbations in ϕ and χ as $\delta\phi$ and $\delta\chi$ and define, with earlier defn of θ



(Illustration of the decomposition of an arbitrary perturbation into adiabatic ($\delta\psi$) and entropy (δs) componemt.)

$$\delta\psi \equiv (\cos\theta)\delta\phi + (\sin\theta)\delta\chi\,,$$
$$\delta s \equiv -(\sin\theta)\delta\phi + (\cos\theta)\delta\chi$$

Perturb the spacetime around FRW :

$$\begin{aligned} \mathrm{d}s^2 \quad &= \quad -(1+2A)\mathrm{d}t^2 + 2a(\partial_i B - S_i)\mathrm{d}x^i \mathrm{d}t \\ & \qquad +a^2\left[(1-2\varphi)\delta_{ij} + 2\partial_{ij}E + h_{ij}\right]\mathrm{d}x^i \mathrm{d}x^j\,, \end{aligned}$$

The comoving curvature perturbation $\mathcal{R}$ and the isocurvature perturbation $\mathcal{S}$ are defined by

$$\mathcal{R} = \varphi + \frac{H\delta\rho}{\dot{\rho}}\,, \quad \mathcal{S} = \frac{H(\dot{\phi}\delta\chi - \dot{\chi}\delta\phi)}{\dot{\phi}^2 + \dot{\chi}^2}\,,$$

where $\delta\rho$ is the total density perturbation.

$$\mathcal{R} = \frac{H\delta\psi_\varphi}{\dot{\psi}}\,, \quad \mathcal{S} = \frac{H\delta s}{\dot{\psi}}\,,$$

where $\delta\psi_\varphi = \delta\psi + \dot{\psi}\,\varphi/H$.

The each Fourier mode for $\delta\psi_\varphi$ and δs satisfies the following equations of motion

$$\delta\ddot{\psi}_\varphi + 3H\delta\dot{\psi}_\varphi + \left[\frac{k^2}{a^2} + V_{,\psi\psi} - \dot{\theta}^2 - \frac{1}{M_{\rm pl}^2 a^3}\left(\frac{a^3\dot{\psi}^2}{H}\right)^{\cdot}\right]\delta\psi_\varphi$$

$$= 2(\dot{\theta}\delta s)^{\cdot} - 2\left(\frac{V_{,\psi}}{\dot{\psi}} + \frac{\dot{H}}{H}\right)\dot{\theta}\delta s\,,$$

$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + V_{,ss} + 3\dot{\theta}^2\right)\delta s = \frac{\dot{\theta}}{\dot{\psi}}\frac{4M_{\rm pl}^2 k^2}{a^2}\Psi\,,$$

where k is a comoving wavenumber and $\Psi = \varphi + a^2 H(\dot{E} - B/a)$.

We find that since($\eta_{,ss} \gg 1$) the power spectrum of the entropy perturbation, $\mathcal{P}_{\delta s} \equiv \frac{k^3}{2\pi^2}|\delta s|^2$, behaves in the usual way ($\mathcal{P}_{\delta s} \simeq (k/2\pi a)^2$) for $k^2/a^2 \gg V_{,ss}$, but after the mass term $V_{,ss}$ becomes larger than k^2/a^2 the entropy perturbation decreases more rapidly ($\mathcal{P}_{\delta s} \propto a^{-3}$). This latter stage occurs even before the Hubble radius crossing (i.e., $V_{,ss} > k^2/a^2 > H^2$) implying a weaker Cross Correlation (compared to two light fields) between curvature and isocurvature perturbations.

For the modes $k > aH$ we obtain the spectrum $\mathcal{P}_{\delta\psi_{\varphi *}} \simeq (H_*/2\pi)^2$ at the Hubble exit at lowest order in slow-roll of the field ψ (in what follows we use the symbol $*$ to represent the quantities at the Hubble radius crossing, $k = aH$). Hence the power spectrum of the curvature perturbation at $k = aH$ is given by

$$\mathcal{P}_{\mathcal{R}_*} \simeq \left(\frac{H^2}{2\pi\dot{\psi}}\right)^2_* \simeq \left(\frac{V}{24\pi^2\epsilon M_{\rm pl}^4}\right)_* ,$$

where

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\psi}}{V}\right)^2 \simeq \epsilon_\phi + \epsilon_\chi \,. \qquad (1)$$

Here the last approximate equality holds under the slow-roll approximation.

We can describe the evolution of perturbations after the Hubble exit ($k < aH$) by using a transfer matrix

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{RS}} \\ 0 & T_{\mathcal{SS}} \end{pmatrix} \begin{pmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{pmatrix} ,$$

where $T_{\mathcal{RS}}$ characterizes the correlation between curvature and isocurvature perturbations.

The dimensionless measure of correlation is defined by

$$r_c \equiv \frac{T_{\mathcal{RS}}}{\sqrt{1 + T_{\mathcal{RS}}^2}} \, ,$$

which is in the range $|r_c| \leq 1$ by definition.

If the masses of two scalar fields ϕ and χ are small relative to the Hubble parameter, the correlation measure $|r_c|$ can be close to the order of 1

Since in our case one of the fields is heavy, the amplitude of δs exponentially decreases ($|\delta s| \propto a^{-3/2}$), which results in a very weak correlation ($|r_c| \ll 1$) and hence it is a good approximation to neglect the correlation after the Hubble radius crossing.

Thus, the power spectrum of the curvature perturbation at the end of inflation is given by

$$\mathcal{P}_{\mathcal{R}} \simeq \mathcal{P}_{\mathcal{R}_*} \simeq \left(\frac{V}{24\pi^2 \epsilon M_{\rm pl}^4} \right)_* ,$$

which holds for $|r_c| \ll 1$.

The spectrum index of the curvature perturbation, $n_{\mathcal{R}} \equiv 1 + {\rm d} \ln \mathcal{P}_{\mathcal{R}} / {\rm d} \ln k$, is given by

$$n_{\mathcal{R}} \simeq 1 - 6\epsilon + 2\eta_{\psi\psi} ,$$

where slow-roll parameters are evaluated at the Hubble exit.

The tensor perturbation h_{ij} is decoupled from the scalar perturbation and is frozen after the Hubble radius crossing. Thus its power spectrum is given by

$$\mathcal{P}_T = \mathcal{P}_{T_*} \simeq \frac{2V_*}{3\pi^2 M_{\rm pl}^4} \,,$$

with the spectral index

$$n_T \equiv \frac{{\rm d}\ln P_T}{{\rm d}\ln k} = -2\epsilon \,. \tag{2}$$

We also obtain the tensor to scalar ratio

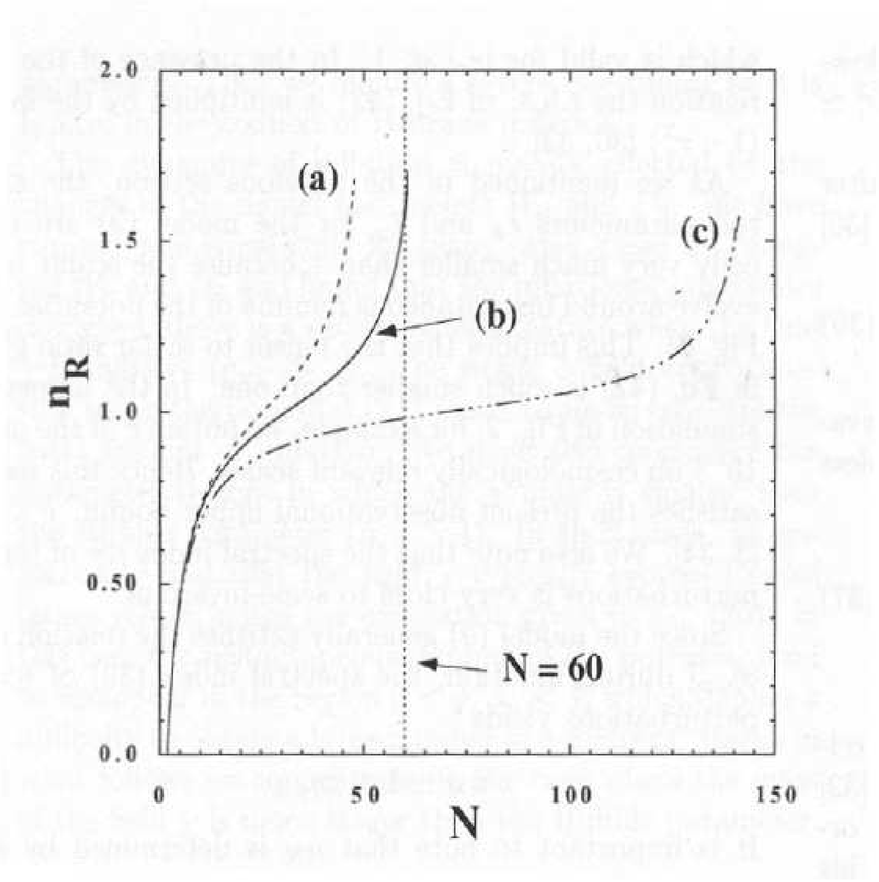
$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} \simeq 16\epsilon \simeq 16(\epsilon_\phi + \epsilon_\chi) \,,$$

which is valid for $|r_c| \ll 1$.

We obtain r of the order 10^{-5} on cosmologically relevant scales. Hence this model satisfies the present observational upper bound: $r < 0.3$. We also note that the spectral index n_T of tensor perturbations is very close to scale-invariant.

Since our model generally satisfies the relation $\epsilon \ll |\eta_{\psi\psi}|$ during inflation, the spectral index of scalar perturbations yields

$$n_{\mathcal{R}} \simeq 1 + 2\eta_{\psi\psi} \,.$$



Spectral index as a function of e-foldings from end of inflation: (a) $D_0 = 1.213 \times 10^{-4}$ (b) $D_0 = 1.215 \times 10^{-4}$ and (c) $D_0 = 1.218 \times 10^{-4}$, other parameters same as earlier

Curve (a) does not reach cosmologically relevant scales

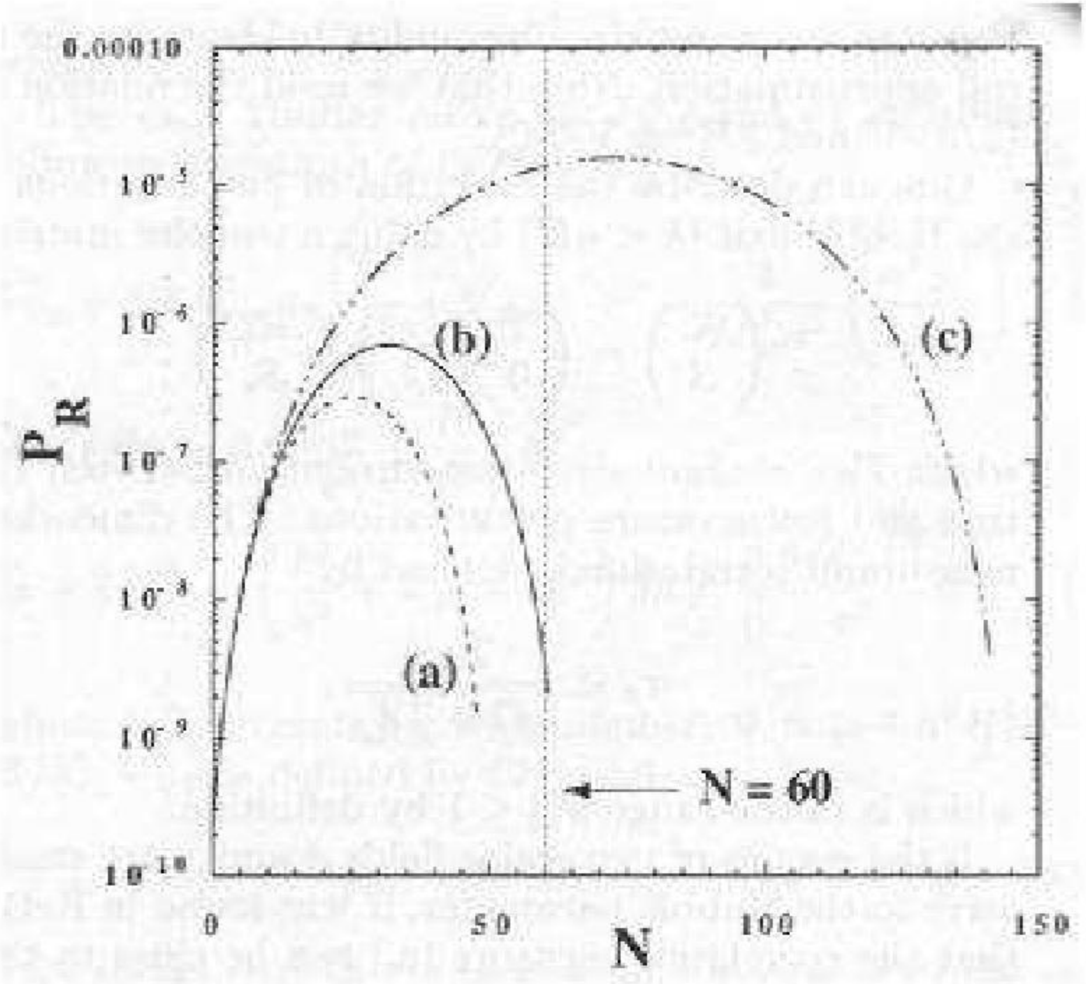
For curve (b) $n_{\mathcal{R}} = 1.6$ for the COBE scale of $N = 60$, too large since observation imply $n_{\mathcal{R}} = 0.97 - 1.21$.

For curve (c), we have red-tilted spectrum $n_{\mathcal{R}} = 0.98$ for $N = 60$

The model has to satisfy the condition of COBE normalization

$$\mathcal{P}_{\mathcal{R}} \simeq 2.4 \times 10^{-9} \,,$$

on cosmologically relevant scales observed by COBE. We find that curve (b) can satisfy this condition and not curve (c).



Results

(1) Tensor to scalar ratio is found to be 10^{-5}, consistent with observational bound < 0.3.

(2) The spectral index of scalar perturbation turns out to be $n_{\mathcal{R}} \simeq 1 + 2\eta_{\psi\psi}$ i.e determined by $\eta_{\psi\psi}$ instead of $\eta_{\phi\phi}$, consistent with our observation that background trajectory is along ψ direction.

(3) However, we found that when the spectrum approaches the scale invarint value $n_{\mathcal{R}} = 1$, the amplitude $\mathcal{P}_{\mathcal{R}}$ tends to be larger than the COBE normalized value (2.4×10^{-9}) by about three order of maginude.

Thus the correction to non-perturbative superpotential, found as yet, is not adequate!

More massaging needed